ACHROMATIC NUMBER IS NP-COMPLETE FOR COGRAPHS AND INTERVAL GRAPHS

Hans L. Bodlaender

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Rijksuniversiteit Utrecht

Vakgroep informatica

Padualaan 14 3584 CH Utrecht
Corr. adres: Postbus 80.089, 3508 TB Utrecht
Telefoon 030-531454
The Netherlands

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Department of Computer Science
University of Utrecht
P.O. Box 80.089, 3508 TB Utrecht
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2 Definitions.

In this section we give a few definitions.

Definition.
A complement-reducible graph, or, in short, a cograph is a graph that can be obtained with the following rules.

1. A graph with one vertex and no edges is a cograph.
2. If $G = (V, E)$ is a cograph, then the complement of G, $G^C = (V, E^C)$, $E^C = \{(v,w)|v,w \in V, v \neq w, (v,w) \notin E\}$ is a cograph.
3. If graphs $G', \ldots, G^r$ are cographs, then the disjoint union of $G', \ldots, G^r$ is a cograph.

Alternatively, one can define the class of cographs to consist of the graphs that do not have P_4, a path with 4 vertices as an induced subgraph.

Definition.
A graph $G = (V, E)$ is an interval graph, if one can associate to each vertex $v \in V$ an interval $[a_v, b_v] \subseteq \mathbf{R}$, such that $(v, w) \in E \Longleftrightarrow [a_v, b_v] \cap [a_w, b_w] \neq \emptyset$.

In this paper, we use the following description of the Achromatic Number problem:

> [**Achromatic Number**]
> **Instance:** Graph $G = (V, E)$, positive integer $k \leq |V|$.
> **Question:** Is there a positive integer $k \geq K$, and a function $f : V \rightarrow \{1, 2, \ldots, k\}$, such that
>
> 1. for every edge $(v, w) \in E : f(v) \neq f(w)$ and
> 2. for every $i, j \in \{1, 2, \ldots, k\} : i \neq j \Rightarrow \exists v, w \in V : (v, w) \in E \cap f(v) = i \cap f(w) = j$?

It is easy to see that this formulation is equivalent to the formulation in [5]. Achromatic number is NP-complete, even if G is the complement of a bipartite graph (and hence every color $\in \{1, \ldots, k\}$ can be used for at most 2 vertices) (cf. [5], p.191). We call functions $f : V \rightarrow \{1, \ldots, k\}$, fulfilling properties (1) and (2) from the description of the Achromatic Number problem *correct colorings*.

3 Main results.

First we prove NP-completeness for Achromatic Number on graphs that are cographs and interval graphs, but do not need to be connected. Later we give an easy transformation to the connected case.