Homework Exam 2 2019

Deadline: 11 December 2019, 15:00

This homework exam has 5 questions for a total of 90 points. You can earn 10 additional points by a careful preparation of your hand-in: using a good layout, good spelling, good figures, no sloppy notation, no statements like "The algorithm runs in $n \log n$." (forgetting the $O(..)$ and forgetting to say that it concerns time), etc. Use lemmas, theorems, and figures where appropriate. Your final grade will be the number of points divided by 10. Unless stated otherwise, both randomized and deterministic solutions are allowed. In case you are asked to analyze the running time and your algorithm is randomized, analyze its expected running time.

Question 1 *(15 points)*

Let $\mathcal{S}$ be a planar subdivision with n vertices, represented as a DCEL. Give pseudo-code for an algorithm that, given a pointer to a vertex v, test if v is incident to a face that is a triangle.

Your algorithm should use the 'Twin', 'NextEdge', 'PrevEdge', etc. fields to navigate (i.e. you cannot assume you can directly access a list of vertices). Describe in a few sentences the main idea of your algorithm and give its running time.

Question 2

Let P be a set of n points in $\mathbb{R}^2$. The *diameter* $diam(P)$ of P is the maximum pairwise distance, $\max_{(p,q)\in P\times P} \|pq\|$, and the pair of points (p, q) that realizes the diameter is the *diametral pair*.

(a) *(15 points)*
Prove that p and q appear on the convex hull $CH(P)$ of P.

(b) *(10 points)*
Develop an $O(n \log n)$ time algorithm to compute the diametral pair p, q. Prove that your algorithm is correct and achieves the stated running time. You may assume that all distances are unique, and that no three points are colinear.

Question 3 *(20 points)*

Let P be a set of n points in $\mathbb{R}^2$, let ρ be a ray (oriented half-line) that starts in one of the points $z \in P$. Develop an algorithm that, given P and ρ, can find the edge of $CH(P)$ hit by ρ. You may assume that no three points in P are colinear, and that ρ contains no points of P other than z. Prove that your algorithm is correct, and analyze its running time. Aim for the fastest possible solution.

The number of points awarded for this question depends on the running time of your solution.

Question 4 *(15 points)*

Prove that a triangulation of a polygon with $n \geq 9$ vertices may consist of more than n triangles. That is, describe a construction that, given a number $n \geq 9$, constructs a polygon P with n vertices that can be triangulated into more than n triangles.

Question 5 *(15 points)*

Let P be a simple polygon with n vertices, and let T be an equilateral triangle that has a horizontal side of length one.

Sketch an $O(n^2 \log n)$ time algorithm to test if there is a translation T^* of T that is completely contained in P, i.e. such that $T^* \subseteq P$.

Argue briefly that your solution is correct and achieves the stated running time. However, you do not have to give the full proof(s).