Universiteit Utrecht

[Faculty of **Science**
Information and Computing Sciences]

Applied Functional Programming

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C5. Lazy evaluation and profiling

C5.1 Reduction

Reduction

A subexpression that can be reduced is called a **redex**.

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Question

What if there are multiple redexes?

Multiple redexes

Many terms have multiple redexes.

How many redexes are in the following term?

$$\mathsf{id}\ (\mathsf{id}\ (\lambda \mathsf{z} \rightarrow \mathsf{id}\ \mathsf{z}))$$

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$(\lambda \mathsf{x} \rightarrow \lambda \mathsf{y} \rightarrow \mathsf{x} * \mathsf{x})\ (1 + 2)\ (3 + 4)$

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$$\begin{aligned} &(\lambda \mathsf{x} \to \lambda \mathsf{y} \to \mathsf{x} * \mathsf{x})\ (1+2) \\ &\qquad\qquad\qquad\qquad (1+2) \\ &\qquad\qquad\qquad\qquad\qquad (3+4) \end{aligned}$$

Operations such as $+$ and $*$ that require their arguments to be (partially) evaluated are called strict in their arguments.

Example

$$\text{head}\ (\text{repeat}\ 1)$$

Example

$$\text{minimum xs} = \text{head (sort xs)}$$

$$\text{minimum } [4, 1, 3]$$

C5.2 Evaluation strategies

Haskell's lazy evaluation

In Haskell,

- expressions are only evaluated if actually required,
- the leftmost outermost redex is chosen to achieve this,
- sharing is introduced in order to prevent evaluating expressions multiple times.

If no redexes are left, an expression is in **normal form**. If the top-level of an expression is a constructor or lambda, then the expression is in **(weak) head normal form**.

Common evaluation strategies

Call by value / strict evaluation

Most common. Arguments are reduced as far as possible before reducing a function application, usually left-to-right.

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Functions are reduced before their arguments. Used by some macro languages (TEX, for instance).

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Call by name

Functions are reduced before their arguments. Used by some macro languages (TEX, for instance).

Call by need / lazy evaluation

Optimized version of "Call by name": function arguments are only reduced when needed, but shared if used multiple times.

> $\lambda f\ g\ x \rightarrow \text{combine}\ (f\ x)\ (g\ x)$

Church-Rosser

Theorem (Church-Rosser)

If a lambda term e *can be reduced to* e_1 *and* e_2*, there is a term* e_3 *such that both* e_1 *and* e_2 *can be reduced to* e_3*.*

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Corollary

Each lambda term has at most one normal form.

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Corollary

Each lambda term has at most one normal form.

Theorem

If a term has a normal form, then lazy evaluation reaches this normal form.

Non-termination

- In lambda calculus, it is possible to encode recursion and therefore nonterminating (diverging) terms.
- A nonterminating value is usually written $\perp$.
- In Haskell, $\perp$ is available as undefined :: a.
- Error values are also often denoted $\perp$, and error :: String $\rightarrow$ a can be seen as an optimization of a diverging computation that prints an error message and breaks off program execution rather than actually diverging.
- In the presence of $\perp$, a function f is called strict if f $\perp \equiv \perp$. In other words, a strict function diverges if applied to a diverging argument.

Examples of lazy evaluation

1. $(\lambda \mathsf{x} \to \mathsf{x})\ \mathsf{True} \quad \rightsquigarrow^*$
2. $(\lambda \mathsf{x} \to \mathsf{x})\ \bot \quad \rightsquigarrow^*$
3. $(\lambda \mathsf{x} \to ())\ \bot \quad \rightsquigarrow^*$
4. $(\lambda \mathsf{x} \to \bot)\ () \quad \rightsquigarrow^*$
5. $(\lambda \mathsf{x}\ \mathsf{f} \to \mathsf{f}\ \mathsf{x})\ \bot \quad \rightsquigarrow^*$
6. $\bot_1\ \bot_2 \quad \rightsquigarrow^*$
7. $\mathsf{length}\ (\mathsf{map}\ \bot\ [1, 2]) \quad \rightsquigarrow^*$

Examples of lazy evaluation

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5. $(\lambda x\ f \to f\ x)\ \bot \quad \rightsquigarrow^* \quad \lambda f \to f\ \bot$
6. $\bot_1\ \bot_2 \quad \rightsquigarrow^* \quad \bot_1$
7. $\mathsf{length}\ (\mathsf{map}\ \bot\ [1, 2]) \quad \rightsquigarrow^* \quad 2$

Forcing evaluation

- Haskell has the following primitive function

```
seq :: a → b → b   -- primitive
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  evaluates x before returning y.

- The function seq can be used to define strict function application:

```
($!) :: (a → b) → a → b
f $! x = x `seq` f x
```

  Recall sharing!

Examples of forcing

Forcing only evaluates to weak head normal form (i.e., a lambda abstraction, literal or constructor application).

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1. $(\lambda \mathsf{x} \to ())\ \$!\ \bot \quad \rightsquigarrow^*$
2. $\mathsf{seq}\ (\bot_1, \bot_2)\ () \quad \rightsquigarrow^*$
3. $\mathsf{snd}\ \$!\ (\bot_1, \bot_2) \quad \rightsquigarrow^*$
4. $(\lambda \mathsf{x} \to ())\ \$!\ (\lambda \mathsf{x} \to \bot) \quad \rightsquigarrow^*$
5. $\bot_1\ \$!\ \bot_2 \quad \rightsquigarrow^*$
6. $\mathsf{length}\ \$!\ \mathsf{map}\ \bot\ [1, 2] \quad \rightsquigarrow^*$
7. $\mathsf{seq}\ (\bot_1 + \bot_2)\ () \quad \rightsquigarrow^*$
8. $\mathsf{seq}\ (\mathsf{foldr}\ \bot_1\ \bot_2)\ () \quad \rightsquigarrow^*$
9. $\mathsf{seq}\ (1 : \bot)\ () \quad \rightsquigarrow^*$

Examples of forcing

Forcing only evaluates to weak head normal form (i.e., a lambda abstraction, literal or constructor application).

1.	$(\lambda \mathsf{x} \to ())\ \$!\ \bot$	$\rightsquigarrow^*$	$\bot$
2.	$\mathsf{seq}\ (\bot_1, \bot_2)\ ()$	$\rightsquigarrow^*$	$()$
3.	$\mathsf{snd}\ \$!\ (\bot_1, \bot_2)$	$\rightsquigarrow^*$	$\bot_2$
4.	$(\lambda \mathsf{x} \to ())\ \$!\ (\lambda \mathsf{x} \to \bot)$	$\rightsquigarrow^*$	$()$
5.	$\bot_1\ \$!\ \bot_2$	$\rightsquigarrow^*$	$\bot_2$
6.	$\mathsf{length}\ \$!\ \mathsf{map}\ \bot\ [1, 2]$	$\rightsquigarrow^*$	2
7.	$\mathsf{seq}\ (\bot_1 + \bot_2)\ ()$	$\rightsquigarrow^*$	$\bot_1$
8.	$\mathsf{seq}\ (\mathsf{foldr}\ \bot_1\ \bot_2)\ ()$	$\rightsquigarrow^*$	$()$
9.	$\mathsf{seq}\ (1 : \bot)\ ()$	$\rightsquigarrow^*$	$()$

C5.3 Heap profiling

Space leaks

- Recall how Haskell data is represented at run-time.

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- Recall how Haskell data is represented at run-time.
- A space leak is a data structure which grows bigger, or lives longer, than might be expected.
- Such unexpected memory use can cause a program to require more garbage collections or to run out of heap.
- More garbage collections cost a significant amount of time, so space performance is related to time performance!

Computing a large sum

$$
\begin{aligned}
&\mathsf{main} = \mathsf{print}\ (\mathsf{sum}_1\ [1\,..\,1000000]) \\
&\mathsf{sum}_1\ [\,] \qquad\quad = 0 \\
&\mathsf{sum}_1\ (\mathsf{x} : \mathsf{xs}) = \mathsf{x} + \mathsf{sum}_1\ \mathsf{xs}
\end{aligned}
$$

- A straight-forward definition: this is how sum is usually defined in an introductory course.
- What is the problem?

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- A straight-forward definition: this is how sum is usually defined in an introductory course.
- What is the problem?
- The correct answer (500000500000) is computed: this takes a significant amount of time, and more than 10MB of memory.
- The same is happening if sum_1 is defined using foldr.

Heap profiling

- Compile and run the program with profiling enabled:

```
$ ghc --make -prof -auto-all Sum1
$ ./Sum1 +RTS -K100M -p -hc
```

- The flag `+RTS` is special and supported by all GHC programs. Subsequent flags are not passed to the program, but to the GHC runtime system.
  - The flag `-K100M` increases the stack size.
  - The flag `-p` enables basic profiling.
  - The flag `-hc` enables heap profiling.
- Two files are produced by running the program: `Sum1.prof` and `Sum1.hp`.
- The heap profile data can be transformed into a PostScript file using the tool `hp2ps` which is included in the GHC distribution:

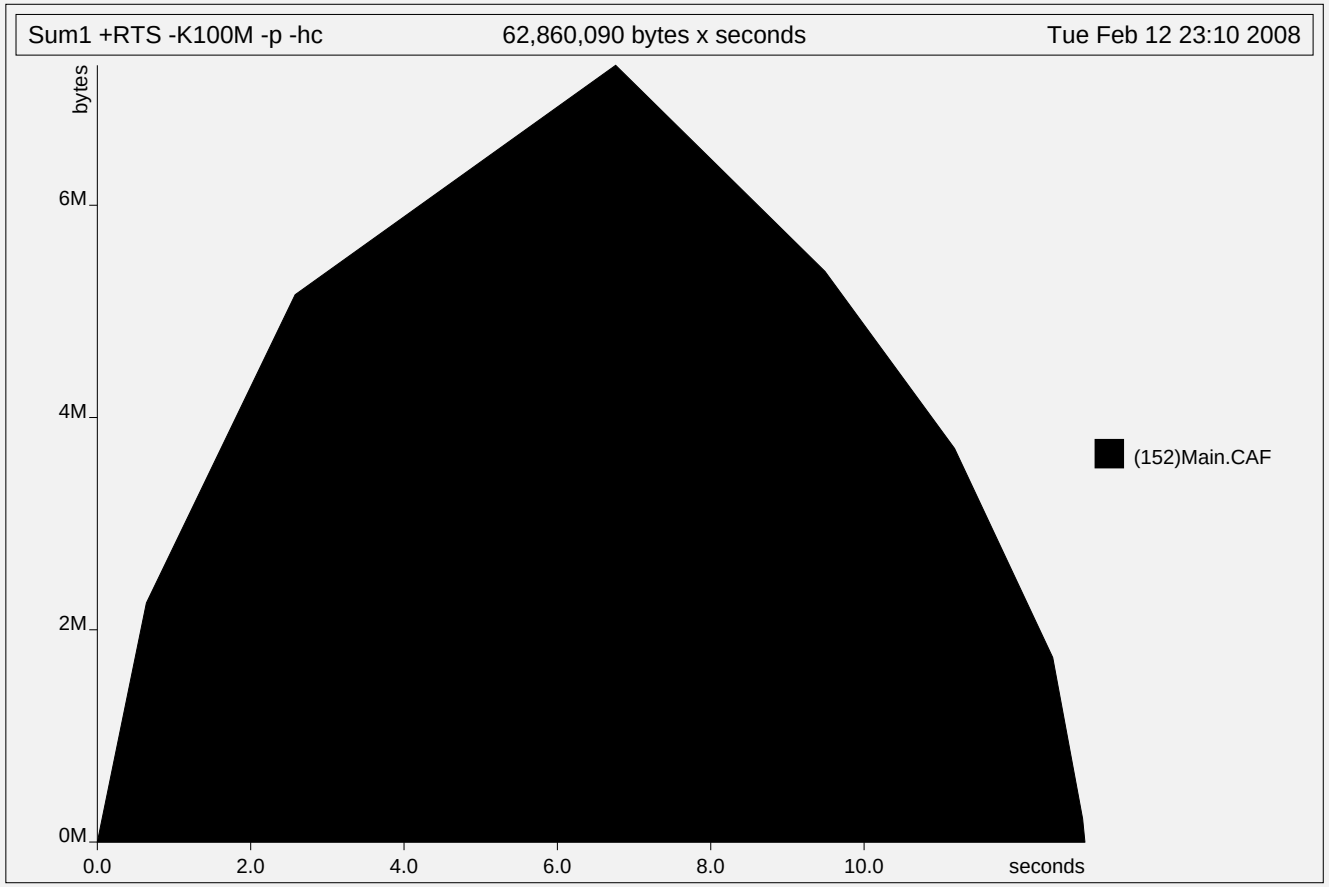
```
hp2ps Sum1.hp
```

The problem

$$
\begin{array}{ll}
 & \mathsf{sum}_1\ [1,2,3,4,\ldots] \\
\equiv & \quad \{\ \text{Definition of } \mathsf{sum}_1\ \} \\
 & 1 + \mathsf{sum}_1\ [2,3,4,\ldots] \\
\equiv & \quad \{\ \text{Definition of } \mathsf{sum}_1\ \} \\
 & 1 + (2 + \mathsf{sum}_1\ [3,4,\ldots]) \\
\equiv & \quad \{\ \text{Definition of } \mathsf{sum}_1\ \} \\
 & 1 + (2 + (3 + \mathsf{sum}_1\ [4,\ldots])) \\
\equiv & \\
 & \ldots
\end{array}
$$

The whole recursion has to be unfolded before the first addition can be reduced!

Heap profile for sum_1



Attempting a tail-recursive version

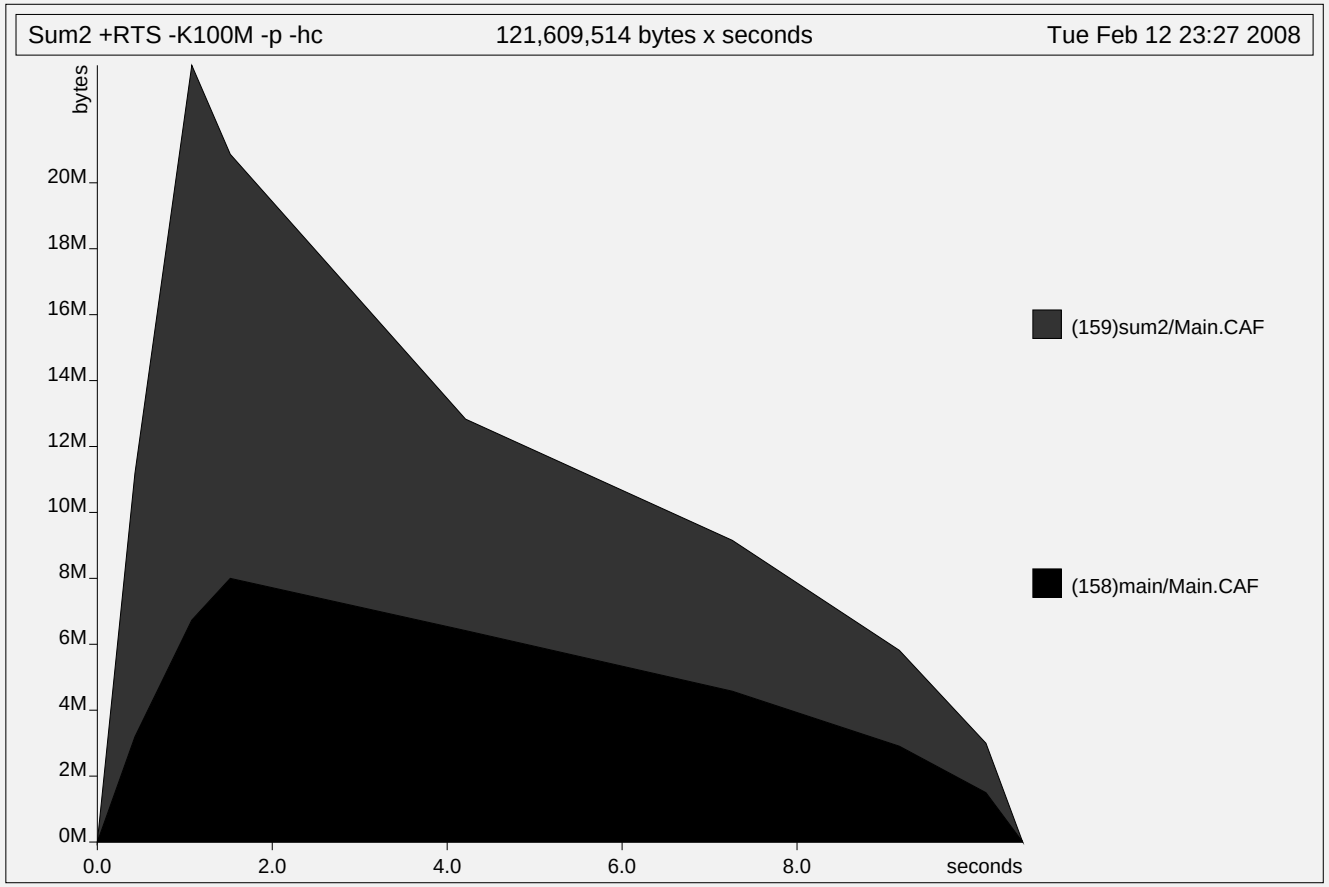
$$
\begin{aligned}
&\mathsf{main} = \mathsf{print}\ (\mathsf{sum}_2\ [1\,..\,1000000]) \\
&\mathsf{sum}_2\ \mathsf{xs} = \mathsf{sum}'_2\ 0\ \mathsf{xs} \\
&\quad \textbf{where} \\
&\qquad \mathsf{sum}'_2\ \mathsf{acc}\ [\,] \qquad\quad = \mathsf{acc} \\
&\qquad \mathsf{sum}'_2\ \mathsf{acc}\ (\mathsf{x} : \mathsf{xs}) = \mathsf{sum}'_2\ (\mathsf{acc} + \mathsf{x})\ \mathsf{xs}
\end{aligned}
$$

- This is more or less equivalent to defining the function via foldl:

$$
\begin{aligned}
&\mathsf{foldl} :: (\mathsf{b} \to \mathsf{a} \to \mathsf{b}) \to \mathsf{b} \to [\mathsf{a}] \to \mathsf{b} \\
&\mathsf{foldl}\ \mathsf{op}\ \mathsf{e}\ [\,] \qquad\quad = \mathsf{e} \\
&\mathsf{foldl}\ \mathsf{op}\ \mathsf{e}\ (\mathsf{x} : \mathsf{xs}) = \mathsf{foldl}\ \mathsf{op}\ (\mathsf{op}\ \mathsf{e}\ \mathsf{x})\ \mathsf{xs} \\
&\mathsf{sum}_2 = \mathsf{foldl}\ (+)\ 0
\end{aligned}
$$

- We hope that tail-recursion improves stack usage, and might thereby improve space behaviour as well, but . . .

Heap profile for sum_2



The new problem

$$
\begin{array}{ll}
 & \mathsf{sum}_2\ [1,2,3,4,\ldots] \\
\equiv & \quad \{\ \text{Definition of } \mathsf{sum}_2\ \} \\
 & \mathsf{sum}_2'\ 0\ [1,2,3,4,\ldots] \\
\equiv & \quad \{\ \text{Definition of } \mathsf{sum}_2\ \} \\
 & \mathsf{sum}_2'\ (0+1)\ [2,3,4,\ldots] \\
\equiv & \quad \{\ \text{Definition of } \mathsf{sum}_2\ \} \\
 & \mathsf{sum}_2'\ ((0+1)+2)\ [3,4,\ldots] \\
 & \ldots \\
\equiv & \\
 & \mathsf{sum}_2'\ (\ldots((0+1)+2)\ldots)\ [\,] \\
\equiv & \quad \{\ \text{Definition of } \mathsf{sum}_2\ \} \\
 & (\ldots(0+1)+2)\ldots)
\end{array}
$$

We still build up the whole addition, but now in an accumulating argument!

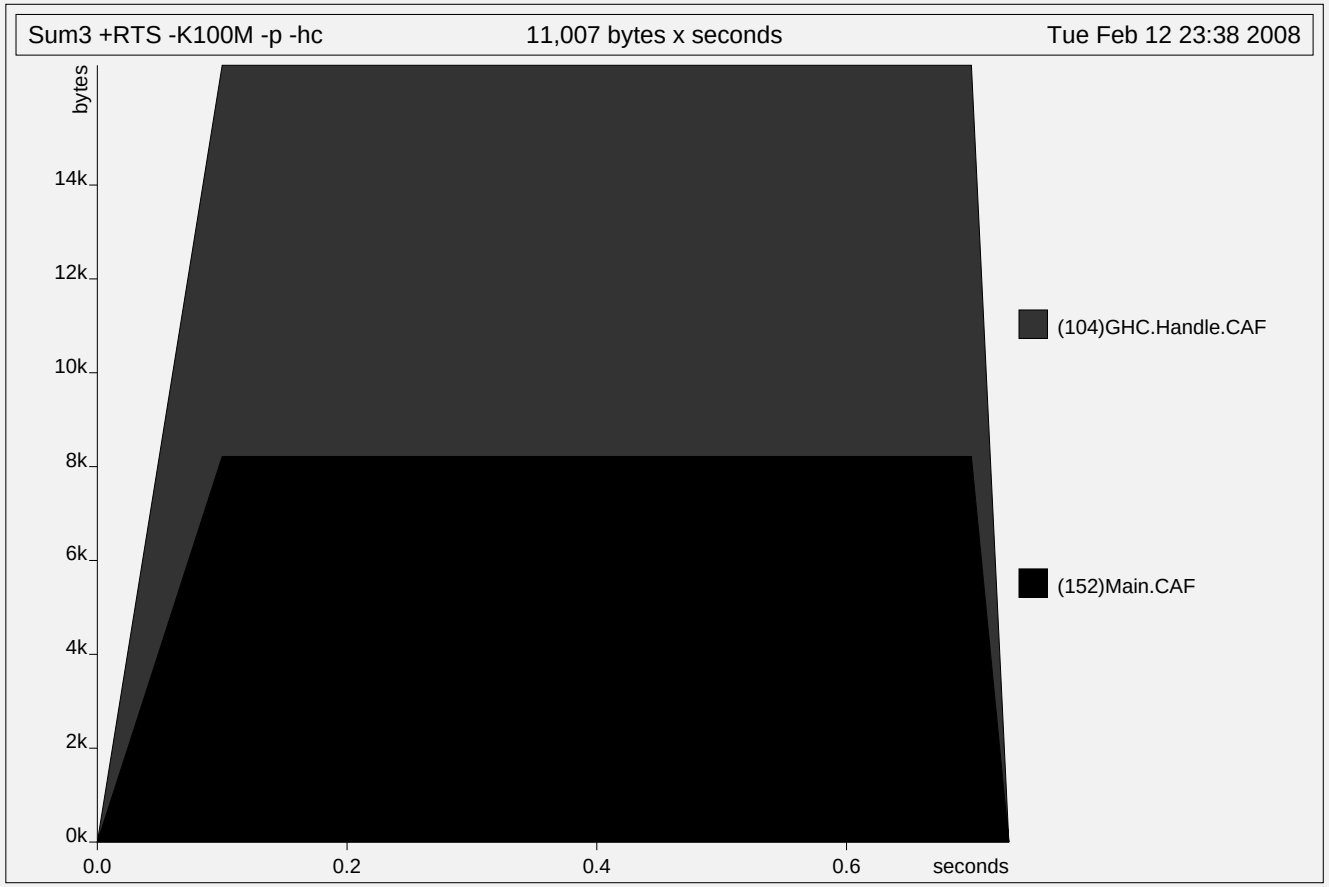
Forcing the addition

$$
\begin{aligned}
&\mathsf{main} = \mathsf{print}\ (\mathsf{sum}_3\ [1\,..\,1000000]) \\
&\mathsf{sum}_3\ \mathsf{xs} = \mathsf{sum}'_3\ 0\ \mathsf{xs} \\
&\quad \textbf{where} \\
&\qquad \mathsf{sum}'_3\ \mathsf{acc}\ [\,] \qquad\quad = \mathsf{acc} \\
&\qquad \mathsf{sum}'_3\ \mathsf{acc}\ (\mathsf{x} : \mathsf{xs}) = (\mathsf{sum}'_3\ \$!\ \mathsf{acc} + \mathsf{x})\ \mathsf{xs}
\end{aligned}
$$

- This is equivalent to using Data.List.foldl′, which is also used in the Prelude version of sum:

$$
\begin{aligned}
&\mathsf{foldl}' :: (\mathsf{b} \to \mathsf{a} \to \mathsf{b}) \to \mathsf{b} \to [\mathsf{a}] \to \mathsf{b} \\
&\mathsf{foldl}'\ \mathsf{op}\ \mathsf{e}\ [\,] \qquad\quad = \mathsf{e} \\
&\mathsf{foldl}'\ \mathsf{op}\ \mathsf{e}\ (\mathsf{x} : \mathsf{xs}) = (\mathsf{foldl}\ \mathsf{op}\ \$!\ \mathsf{op}\ \mathsf{e}\ \mathsf{x})\ \mathsf{xs} \\
&\mathsf{sum}_3 = \mathsf{foldl}'\ (+)\ 0
\end{aligned}
$$

Heap profile for sum_3



Another possibility: strict datatypes

```
data StrictInteger = SI !Integer
main = print (sum4 [1..1000000])
sum4 xs = sum'4 (SI 0) xs
  where
    sum'4 (SI acc) []       = acc
    sum'4 (SI acc) (x : xs) = sum'4 (SI (acc + x)) xs
```

- The 'bang' is a strictness annotation. It can be used on all fields of a datatype and makes the constructors strict in the respective arguments.
- Note that bangs in general only force evaluation to weak head normal form.

Question

Is using foldl/strictness always preferable?

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What about map?

Rules of thumb

- If you expect partial results or want to use infinite lists, use foldr.
  Examples: map, filter.
- If the operator is strict, use foldl′.
  Examples: sum, product.
- Otherwise, use foldl.
  Examples: reverse.
- Use the GHC optimizer by passing `-O`. GHC performs strictness analysis to optimize your code.