# Open Problems of the Lorentz Workshop "Enumeration Algorithms using Structure"

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**Abstract**

This is a collection of open problems presented at the Lorentz Workshop "Enumeration Algorithms using Structure" which took place at the Lorentz Center of the University of Leiden (The Netherlands), August 24 - 30, 2015. The workshop brought together researchers interested in various aspects of enumeration algorithms; in particular classical output-sensitive enumeration including output-polynomial and polynomial delay algorithms and the input-sensitive enumeration mainly dealing with exact exponential time enumeration algorithms and combinatorial bounds.

# *Endre Boros*

## Disjoint coverings of linear orders

Let $V = \{1, 2, ..., n\}$, where $n$ is a positive integer, $K(V) = (V \times V) \setminus \{(u, u) \mid u \in V\}$ be the set of all ordered pairs, and denote by $\mathbb{S}_V$ the set of permutations of $V$.

For a subset $A \subseteq K(V)$ and a permutation $\pi \in \mathbb{S}_V$ we say that $\pi$ is a *linear extension of the directed graph* $G(A) = (V, A)$ if for all $(u, v) \in A$ we have $\pi(u) < \pi(v)$.

A family $A_i \subseteq K(V)$, $i = 1, ..., m$ is called a *disjoint cover* of $\mathbb{S}_V$ if for every $\pi \in \mathbb{S}_V$ there **exists a unique index** $i$ such that $\pi$ is a linear extension of $G(A_i)$. (Clearly, a necessary condition for this is that $G(A_i)$, $i = 1, ..., m$ are all acyclic, and that $G(A_i \cup A_j)$ has a directed cycle for all pairs $1 \leq i < j \leq m$.)

We would like to generate all disjoint covers of $\mathbb{S}_V$ in an output efficient (say incremental polynomial) way. The corresponding decision problem arises:

NEXT-DC: Given subsets $A_i \subseteq K(V)$, $i = 1, ..., m$ such that

(i) $G(A_i)$ is acyclic for all $i = 1, ..., m$, and

(ii) $G(A_i \cup A_j)$ contains a directed cycle for all $1 \leq i < j \leq m$,

is there a permutation $\pi \in \mathbb{S}_V$ that is not a linear extension of any of these directed graphs?

The complexity of problem NEXT-DC is open.

**Remark 1**

- *If condition (ii) does not hold, then the problem is NP-hard, in general.*
- *If the number of linear extensions of any directed graph obtainable from $G(A_i)$ by a series of contractions can be computed in poly(n) time for all $i = 1, ..., m$ then the problem can be solved in poly(n,m) time. For instance, if $|A_i| \leq k$ for some constant $k$ for all $i = 1, ..., m$ then the problem is polynomial.*

## Maximal negative cycle free subgraphs

Given a directed graph $G = (V, A)$ and an arc weight $w : A \mapsto \mathbb{R}$, a directed cycle $C \subseteq A$ is called a negative cycle if $w(C) < 0$.

Is it possible to generate in an output efficient way (say in incremental polynomial time) all maximal subsets $F \subseteq A$ of the arcs such that $(V, F)$ does not have a negative cycle with respect to the given arc weight $w$?

## Vertices of bounded polyhedra

Consider bounded polyhedra given by a system of linear inequalities: $P = \{x \mid Ax \leq b,\ c \leq x \leq d\}$, where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, and $c, d \in \mathbb{R}^n$, $c \leq d$.

Is it possible to generate in an output efficient way (say in incremental polynomial time) all vertices of such polyhedra?

## Anti-simplices

Consider a finite point set $P \subseteq \mathbb{R}^d$ in an Euclidean space, and set $\mathbf{1} = (0, ..., 0) \in \mathbb{R}^d$. For a subset $X \subseteq P$ we denote by $\mathbf{c}onv(X)$ the convex hull of the points in $X$. Let us call a subset $X \subseteq P$ *covering* if $\mathbf{1} \in \mathbf{c}onv(X)$, and *noncovering* otherwise. A maximal noncovering subset is also called an anti-simplex.

Is it possible to generate in an output efficient way (say in incremental polynomial time) all maximal noncovering subsets of $P$?

# *Khaled Elbassioni*

## Complexity of Dualization over Products of Partially Ordered Sets

Let $\mathcal{P} = \mathcal{P}_1 \times \ldots \times \mathcal{P}_n$ be the product of $n$ partially ordered sets (posets). Denote by $\preceq$ the precedence relation in $\mathcal{P}$ and also in $\mathcal{P}_1, \ldots, \mathcal{P}_n$, i.e., if $p = (p_1, \ldots, p_n) \in \mathcal{P}$ and $q = (q_1, \ldots, q_n) \in \mathcal{P}$, then $p \preceq q$ in $\mathcal{P}$ if and only if $p_1 \preceq q_1$ in $\mathcal{P}_1$, $p_2 \preceq q_2$ in $\mathcal{P}_2, \ldots$, and $p_n \preceq q_n$ in $\mathcal{P}_n$. For a set $\mathcal{A} \subseteq \mathcal{P}$, let $\mathcal{A}^+ = \{x \in \mathcal{P} \mid x \succeq a, \text{ for some } a \in \mathcal{A}\}$ and $\mathcal{A}^- = \{x \in \mathcal{P} \mid x \preceq a, \text{ for some } a \in \mathcal{A}\}$ denote respectively the ideal and filter generated by $\mathcal{A}$. Any element in $\mathcal{P} \setminus \mathcal{A}^+$ is called *independent of* $\mathcal{A}$. Let $\mathcal{I}(\mathcal{A})$ be the set of all maximal independent elements for $\mathcal{A}$, also called the *dual* of $\mathcal{A}$ in $\mathcal{P}$:

$$\mathcal{I}(\mathcal{A}) \stackrel{\text{def}}{=} \{p \in \mathcal{P} \mid p \notin \mathcal{A}^+ \text{ and } (q \in \mathcal{P},\ q \succeq p,\ q \neq p \ \Rightarrow\ q \in \mathcal{A}^+)\}.$$

Then we have the following decomposition of $\mathcal{P}$

$$\mathcal{A}^+ \cap \mathcal{I}(\mathcal{A})^- = \emptyset, \qquad \mathcal{A}^+ \cup \mathcal{I}(\mathcal{A})^- = \mathcal{P}.$$

Call $\mathcal{A}$ an *antichain* if no two elements are comparable in $\mathcal{P}$. We are concerned with the following *dualization* problem:

***DUAL***$(\mathcal{P}, \mathcal{A}, \mathcal{B})$**:** *Given an antichain $\mathcal{A} \subseteq \mathcal{P}$ in a poset $\mathcal{P}$ and a collection of maximal independent elements $\mathcal{B} \subseteq \mathcal{I}(\mathcal{A})$, either find a new maximal independent element $x \in \mathcal{I}(\mathcal{A}) \setminus \mathcal{B}$, or state that the given collection is complete: $\mathcal{B} = \mathcal{I}(\mathcal{A})$.*

If $\mathcal{P}$ is the Boolean cube, i.e., $\mathcal{P}_i = \{0, 1\}$ for all $i = 1, \ldots, n$, the above dualization problem reduces to the well-known *hypergraph transversal* problem, which calls for enumerating all minimal subsets $X \subseteq V$ that intersect all edges of a given hypergraph $\mathcal{H} \subseteq 2^V$. The complexity of this problem is still an important open question, for which the currently best known algorithm [3] runs in quasi-polynomial time $poly(n, m) + m^{o(\log m)}$, where $m = |\mathcal{A}| + |\mathcal{B}|$, providing strong evidence that the problem is unlikely to be NP-hard. More generally, when each $\mathcal{P}_i$ is a chain, that is, a totally ordered set, the problem was considered in [1], where it was shown that the algorithms of [3] can be extended to work in quasi-polynomial time, regardless of the chains sizes. It is natural to investigate whether these results can be extended further to wider classes of partially ordered sets. In [2], we achieve this for the cases when each $\mathcal{P}_i$ is

(i) a join (or meet) semi-lattice with constant width,

(ii) a forest, that is a poset in which the underlying undirected graph of the precedence graph is acyclic, and in which either the in-degree or the out-degree of each element is bounded by a constant, and

(iii) the lattice of intervals defined by a set of intervals on the real line $\mathbb{R}$.

**Open question:** Is problem $DUAL(\mathcal{P}, \mathcal{A}, \mathcal{B})$ NP-hard for the more general case of products of arbitrary posets? Can the problem can be solved in quasi-polynomial time, at least for posets $\mathcal{P}_i$ of small size?

# *Mamadou Kanté*

The vertex set of a graph $G$ is denoted by $V(G)$, its edge set by $E(G)$, and their sizes are denoted respectively by $n$ and $m$. Similarly, the vertex set of a hypergraph $\mathcal{H}$ is denoted by $V(\mathcal{H})$ and its hyperedge set by $\mathcal{E}(\mathcal{H})$.

## Enumeration of Minimal Connected Dominating Sets

Let $G$ be a graph. A subset $D$ of $V(G)$ is a connected dominating set if $D$ is a dominating set of $G$ and $G[D]$ is connected. We denote by $\mathcal{CD}(G)$ the set of (inclusion-wise) minimal connected dominating sets in $G$. Observe that $D$ is a minimal connected dominating set if for each $x \in D$ either $D \setminus x$ is not connected or $N[D \setminus x] \neq V(G)$. It is worth noticing that $\mathcal{CD}(G)$ is different from the set of minimal dominating sets that are connected. We are interested in the enumeration of $\mathcal{CD}(G)$ in output-polynomial time (CDom-Enum).

$S \subset V(G)$ is a *separator* of a graph $G$ if $G \setminus S$ is not connected and we denote by $\mathcal{S}(G)$ the set of (inclusion-wise) minimal separators of $G$. $T \subseteq V(\mathcal{H})$ is a *transversal* of a hypergraph $\mathcal{H}$ if $T$ intersects every hyperedge of $\mathcal{H}$ and we denote by $tr(\mathcal{H})$ the set of (inclusion-wise) minimal transversals of $\mathcal{H}$. The following are proved in [8].

1. For every graph $G$, $\mathcal{CD}(G) = tr(\mathcal{S}(G))$.
2. For every hypergraph $\mathcal{H}$, one can construct a split graph $G(\mathcal{H})$ such that $tr(\mathcal{H}) = \mathcal{CD}(G(\mathcal{H}))$. The same holds if we replace split graph by co-bipartite graph.

From (1) and (2) we know that for some graph classes the Hypergraph Dualisation problem and the CDom-Enum are equivalent problems, *e.g.*, chordal graphs, and for others the Hypergraph Dualisation is at least harder, *e.g.*, circular-arc graphs, circle graphs, etc. Because of (1) we suspect the following conjecture.

**Conjecture 1** *There is no output-polynomial algorithm for enumerating the minimal connected dominating sets in graphs, unless* $P = NP$.

Since it is unlikely that there is a graph class $\mathcal{C}$ for which $|\mathcal{CD}(G)| = poly(n, m)$ for each $G \in \mathcal{C}$, and the computation of a minimum connected dominating set is $NP$-complete on graphs in $\mathcal{C}$, we conjecture the following (under Conjecture ).

**Conjecture 2** *Let* $\mathcal{C}$ *be a class of graphs. If there is a polynomial* $p(x, y)$ *such that for each graph* $G \in \mathcal{C}$*, we have* $|\mathcal{CD}(G)| \leq p(n, m)$*, then there is a polynomial time algorithm for enumerating the minimal connected dominating sets of graphs in* $\mathcal{C}$.

## Enumeration of (Non) Locally Equivalent Graphs

Given a graph $G$ and a vertex $x$ of $G$, the *local complementation of* $G$ *at* $x$ is the graph, denoted by $G * x$, obtained from $G$ by replacing $G[N(x)]$ by its complement. Of course, $G * x * x = G$. A graph $H$ is *locally equivalent* to a graph $G$ if $H$ can be obtained from $G$ by a sequence of local complementations [5, 6, 7]. Observe that being locally equivalent is an equivalence relation. One can check in polynomial time whether two graphs on the same vertex set are locally equivalent in polynomial time [4, 7]. The local complementation operation is used to define the quasi-order *vertex-minor*[1] on finite graphs and is related to the complexity measure *rank-width* [9]. For instance circle graphs are characterised by a finite list of graphs to exclude as a vertex-minor [5], and graphs of bounded rank-width are characterised by a finite list of graphs to exclude as vertex-minors [9]. We are interested in the following questions

> **Problem.** LC-EQUIVALENT (resp. ISO-LC-EQUIVALENT)
> **Input.** A graph $G$.
> **Output.** The set of (resp. non-isomorphic) graphs locally equivalent to $G$.

> **Problem.** GEN-LC-EQUIVALENT (resp. GEN-ISO-LC-EQUIVALENT)
> **Input.** A positive integer $n$.
> **Output.** The set of (resp. non isomorphic) non locally equivalent graphs with $n$ vertices.

If $H$ is locally equivalent to $G$, then the *distance between $H$ and $G$* is the minimum number of local complementations to apply to $G$ to obtain $H$. From [6] the distance between two graphs is bounded by $\max\{n+1, \frac{10}{9}n\}$. Therefore, LC-EQUIVALENT admits an output-polynomial time algorithm. Similarly for ISO-LC-EQUIVALENT if $G$ belongs to a graph class closed under local complementation and admitting a polynomial time ISOMORPHISM TESTING algorithm. However, the algorithm uses exponential space and consists in doing the following: Generate all graphs locally equivalent to $G$ at distance 1, then those at distance 2, and so on until the generation of graphs at distance $\max\{n+1, \frac{10}{9}n\}$; keep a graph (or one representative of each isomorphism class) only if it was not generated on previous steps.

We are interested in the following questions.

**Question 1.** Does LC-EQUIVALENT admit a polynomial delay and polynomial space algorithm? Is it the case also for ISO-LC-EQUIVALENT if we assume that the input is in a graph class closed under local complementation and admitting a polynomial time ISOMORPHISM TESTING?

**Question 2.** Are GEN-LC-EQUIVALENT and GEN-ISO-LC-EQUIVALENT doable with polynomial delay and polynomial space in graph classes closed under local complementation and admitting a polynomial time ISOMORPHISM TESTING algorithm?

**Question 3.** Is ISOMORPHISM TESTING a bottleneck for obtaining an output-polynomial algorithm for ISO-LC-EQUIVALENT, GEN-LC-EQUIVALENT and GEN-ISO-LC-EQUIVALENT?

[1] A graph $H$ is a *vertex-minor* of a graph $G$ if $H$ is an induced subgraph of a graph locally equivalent to $G$.

# *Mikko Koivisto*

## Enumerating disjoint pairs

**Problem:** Given a family $\mathscr{F}$ of subsets of $\{1, 2, \ldots, n\}$, enumerate the *disjoint pairs* of $\mathscr{F}$, that is, the elements of $\mathscr{D}(\mathscr{F}) := \{(X, Y) \in \mathscr{F} \times \mathscr{F} : X \cap Y = \emptyset\}$.

**Open question:** Can the problem be solved in time $O^*(|{\downarrow}\mathscr{F}| + |\mathscr{D}(\mathscr{F})|)$, where the family ${\downarrow}\mathscr{F}$ consists of the members of $\mathscr{F}$ and their subsets, and $O^*$ hides factors polynomial in $n$?

**Remarks:**

1. The problem can be solved trivially in time $O^*(|\mathscr{F}|^2)$.
2. *Cannot decide* whether $\mathscr{D}(\mathscr{F})$ is empty in time $O^*(|\mathscr{F}|^p)$ for any $p < 2$, unless the strong exponential time hypothesis fails [ R. Williams, *A new algorithm for optimal 2-constraint satisfaction and its implications.* Theor. Comput. Sci. 348 (2005) 357–365 ].
3. *Can count* the pairs (i.e., compute $|\mathscr{D}(\mathscr{F})|$) in time $O^*(|{\downarrow}\mathscr{F}|)$ [ A. Björklund, T. Husfeldt, P. Kaski, M. Koivisto, *Counting paths and packings in halves.* ESA 2009, pp. 578–586 ].

# *Janne Korhonen*

## The number of connected sets

**Connected sets.** Let $G = (V, E)$ be a graph. We say that a vertex set $U \subseteq V$ is *connected* if the the subgraph $G[U]$ induced by $U$ is connected. Let $\mathcal{C} \subseteq 2^V$ be the family of connected sets in the graph $G$.

In general, the number $|\mathcal{C}|$ can be very large; for instance, a clique on $n$ vertices has $2^n$ connected sets, and a star has $2^{n-1}+n-1$. Here we are interested in bounding $|\mathcal{C}|$ on suitable graph classes. In particular, this is motivated by the work of Björklund et al. [11], who show that the classic Bellman–Held–Karp dynamic programming algorithm [10, 12] for the travelling salesman problem can be modified to run in time $|\mathcal{C}|\, n^{O(1)}$, and give bounds of form $c^n$ with $c < 2$ for $|\mathcal{C}|$ on *bounded-degree graphs* and *triangle-free bounded-degree graphs*. These bounds were recently improved slightly by Kangas et al. [13] for small maximum degree bounds $\Delta$; see Table 1 for the the best known bounds.

Our open problems concern both improving the above bounds on bounded-degree graphs, and generalising this framework to other graph classes:

**Question 1:** What are the tight bounds for the number of connected sets on graphs with maximum degree $\Delta$?

**Question 2:** Are there other graph classes where we can bound the number of connected sets by $c^n$ for $c < 2$, and does this give improved TSP algorithms on those graph classes?

The currently known bounds are based on application of Shearer's entropy lemma, which seems to be difficult to apply if the graph does not have bounded degree. This raises the question whether there are other techniques in the enumeration algorithmics toolbox that can be used to improve the known bounds.

Table 1: Best known upper bounds and lower bounds for maximum $|\mathcal{C}|$ in bounded-degree graphs. Smaller additive terms are omitted for clarity; see Kangas et al. [13] for details.

| | Maximum degree | | | |
|---|---|---|---|---|
| | 3 | 4 | 5 | $\Delta$ |
| Upper bound | $1.9351^n$ | $1.9812^n$ | $1.9940^n$ | $(2^{\Delta+1} - 1)^{n/\Delta+1}$ |
| Lower bound | $1.5537^n$ | $1.6180^n$ | $1.7320^n$ | — |

**Variants.** Besides connected sets, there are two other set families where answering the analogs of Questions 1 and 2 would give also improved bounds for TSP [11]:

(1) The number of *connected dominating sets* can be used to bound the running time of polynomial-space TSP algorithms for graphs with bounded integer weights.

(2) Let $s, u \in V$ be two distinct vertices. We say that set $U \subseteq V$ is *transient with starting point $s$ and endpoint $u$* if $U$ is connected and for each vertex $v \notin N(s) \cup N(u)$ we have that

  (a) if $v \in U$, then at least two neighbours of $v$ are also in $U$, and

  (b) if $v \notin U$, then at least two neighbours of $v$ are also not in $U$.

  The number of transient sets with a fixed starting point $s$ also gives running time upper bounds for TSP algorithms; intuitively, the transient sets are sets that can appear in the Bellman–Held–Karp dynamic programming as the ground sets of partial solutions.

# *Dieter Kratsch*

## Upper bounds on the number of vertex subsets with property $P$

Let $G = (V, E)$ be an undirected graph and $P$ be a property of vertex subsets of graphs. We are interested in $\#P(n)$ which is the maximum number of of subsets of property $P$ in an $n$-vertex graph. There is a trivial upper bound of $\#P(n) \leq 2^n$. If $P$ is the property of being a maximal independent set, a maximal clique and a maximal vertex cover then $\#P(n) = 3^{n/3}$ and this result is known since more than 50 years [14].

Recently motivated by interest in exact exponential algorithms to enumerate graphical objects other properties like minimal dominating set, minimal connected dominating set, minimal connected vertex cover, minimal feedback vertex set have been studied both for general graphs and also for a variety of graph classes.

Here are some of the challenging open questions:

Let $P_{mcds}(n)$ be the property of being a minimal connected dominating set. Improve upon the trivial bound by showing that $\#P_{mcds} \leq c^n$ for some $c < 2$ either via a branching enumeration algorithm or in a combinatorial (non-algorithmic) way.

Let $P_{mds}(n)$ be the property of being a minimal dominating set. It is known that $\#P_{mds}$ on chordal graphs is at most $1.6181^n$. No chordal graph with more than $3^{n/3}$ minimal dominating sets is known. Find the exact value $c$ such that $P_{mds}(n) = c^n$ on chordal graphs.

Let $Q$ be any MSOL property of a vertex set and $P$ the property minl-Q or maxl-Q, i.e. a vertex set has property $P$ if it is a minimal (maximal) set with property $Q$. Develop possibly general methods to lower or upper bound $\#P(n)$ on graphs of bounded treewidth. (Remark: Such results are not even known for trees.)

# *Arnaud Mary*

## Enumeration of maximal stories

Let $G := (B \cup W, A)$ be a bicolored digraph. A Story $S$ is a partial subgraph that is a DAG and such that each vertex of $V(S) \cap W$ has at least one incoming arc and one outgoing arc in $S$.

**Enumeration of maximal stories**
**input:** A bicolored digraph $G$.
**output:** All maximal stories of $G$.

Comment:
This problem has applications in biology [16]. We don't know the complexity of the problem. It is closely related to the enumeration of the maximal DAG (or the minimal feedback arc sets) of a digraph for which a polynomial delay algorithm can be found in [15]. Adding the degree constraints on the white vertices seems to make the problem much harder.

# *Takeaki Uno*

## Maximal Irredundant Set Enumeration and Minimal Redundant Set Enumeration

Let $(V, \mathcal{F})$ be a hypergraph. For a hyperedge set $S$ and its hyperedge $F$, a *private neighbor* of $F$ is a vertex of $V$ such that $v \in F$ and $v \not\in F'$ for any $F' \neq F, F' \in S$. $S$ is said to be *redundant* If some hyperedges of $S$ have no private neighbor, and *irredundant* otherwise.

**Problem: Maximal Irredundant Set Enumeration**
For given hypergraph, enumerate all its maximal irredundant sets.

**Problem: Minimal Redundant Set Enumeration**
For given hypergraph, enumerate all its minimal redundant sets.

For the latter problem, in this workshop, we could find a simple proof such that any hypergraph dualization problem can be reduced to this problem, so it is harder than the dualization.

**Proof** For given a hypergraph $(U, \mathcal{H})$, we construct a hypergraph $(V, \mathcal{F})$ such that $V = U \cup \mathcal{H}$ and any hyperedge $H'$ of $\mathcal{F}$ is constructed from a hyperedge $H \in \mathcal{H}$ by adding the "vertex" $H \in V$ that is the hyperedge $H$ in $V$. We further regard $U$ as a hyperedge, and add it to $\mathcal{F}$. Since vertex $H$ is included only in the hyperedge $H'$, we can see that any redundant set $H$ must include the hyperedge $U$, and $U$ has to have no private neighbor. Thus, any redundant set $S$ of $(V, \mathcal{F})$ satisfies $U \subseteq \bigcup_{F \in S} F$. It implies that $S$ corresponds to a hitting set in $(U, \mathcal{H})$. There is a one to one correspondence between minimal hitting sets in $(U, \mathcal{H})$ and minimal redundant sets in $(V, \mathcal{F})$.

In particular, we consider the problems on graph classes, by restricting the input hypergraphs on the set of closed neighbors of all the vertices of a graph. We can then consider the problems on many graph classes. So the open problem is to clarify the difficulty of the problems in many graph classes.

For both problems, at least, for interval graphs (circular arc graphs) and permutation graphs, we can apply dynamic programming algorithms, as same as minimal dominating set enumeration. We can also apply the same algorithm scheme as the minimal dominating set enumeration to k-degenerate graphs, bounded tree-width, and bounded clique-width. The idea is in some sense straightforward, but details are non-trivial. For split graphs, we can see that any maximal irredundant set is a minimal dominating set, thus we can enumerate them in polynomial time delay.

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