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A formalisation of argumentation schemes for case-based reasoning in ASPIC+

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Abstract

In this paper we offer a formal account of reasoning with legal cases in terms of argumentation schemes. These schemes, and undercutting attacks associated with them, are formalised as defeasible rules of inference within the ASPIC+ framework. We begin by modelling the style of reasoning with cases developed by Alevén and Ashley in the CATO project, which describes cases using factors, and then extend the account to accommodate the dimensions used in Rissland and Ashley’s earlier HYPO project. Some additional scope for argumentation is then identified and formalised.

Keywords: argumentation, legal case based reasoning

1 Introduction

Legal case-based reasoning (LCBR) has long been a topic of interest in AI and Law, and a variety of approaches have evolved. One important line of work on LCBR began with HYPO [?], developed by Edwina Rissland and her student, Kevin Ashley at Amherst. HYPO represented reasoning with legal cases as the exchange of arguments and counter arguments based on *dimensions*, legally significant aspects of the cases. Subsequently the ideas of HYPO were further developed by Ashley at Pittsburgh where he worked with his student, Vincent Alevén, on CATO [?], which introduced the notions of *factors* and a *factor hierarchy*, and with another student, Steffi Bruninghaus, on IBP [?] which attempted to predict case outcomes instead of simply identifying the arguments for the two sides. Like HYPO these systems were applied to US Trade Secrets law. Meanwhile Rissland stayed at Amherst where she worked with her student, David Skalak, on CABARET [?], which was based on Home Office Deduction cases and embedded the case based reasoning within a structure of rules modelling the relevant legislation, and with Skalak and Timur Friedman on BankXX [?]. which generated arguments about Home Office Deduction through heuristic search. The model of case based reasoning used in this paper is largely based on the model developed in CATO, although we shall also draw on these other systems where convenient. More theoretically-oriented research related to this general approach appears in [?], [?], and [?]. In all these approaches, a current undecided case is decided by comparing and contrasting features in the current case against precedent cases in a case-base that have similar features. The decision in the “best” precedent case is then taken as the decision into the current case following the legal reasoning principle of *stare decisis*.

In [?], a number of novel argumentation schemes designed to reflect reasoning with factors as in Alevén and Ashley’s CATO [?] were described, where the focus is to determine how and in what way a precedent case does (or does not) argue in support of a determination in the current case. However, the presentation in [?] was semi-formal and not set in an analytic framework which supports reasoning about these schemes. In this paper, we reanalyse and formalise legal case-based argumentation schemes in terms of the formal

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argumentation framework of ASPIC+ [?]. Formalising these schemes clarifies them and makes them more precise, while formalising them in ASPIC+ makes the metatheory of the ASPIC+ framework available for our account. In particular, we shall use its metatheory to prove that our specification satisfies the rationality postulates of [?]. A formalisation in ASPIC+ also illustrates the potential of that framework for formalising reasoning with argumentation schemes.

[?] was also limited in that it did not consider additional aspects about reasoning with factors, nor reasoning with dimensions as in HYPO. An attempt to rectify these deficiencies was made in [?], but this paper represents a rewritten, revised and extended version which improves markedly on both the formal representation and the analysis.

The current paper advances the state-of-the-art in several respects: legal case-based reasoning with factors is clarified, defeasible legal case-based reasoning is represented and formalised in argumentation schemes, the arguments are compatible with and evaluated in a formally defined argumentation framework, and the analysis presents a well-developed and justified instantiation of defeasible argumentation schemes in a formal framework. Furthermore, the analysis provides a uniform representation language into which various alternative proposals for LCBR can be cast, compared, integrated, and reasoned with. Finally, the paper as a whole provides a demonstration of how an aspect of domain expertise, in this case reasoning with legal precedents, can be fruitfully captured and represented as a set of argumentation schemes, and the specific domain conceptualisation required to support them. This technique is generally applicable to expertise which comprises the ability to reason in a particular way.

The organisation of the paper is as follows. We first set out elements of the formal framework for argumentation that we are assuming. In section 3, we discuss case-based reasoning as in CATO. We introduce our running example and the elements of the language we need for the argumentation schemes before presenting CATO style argumentation schemes in the formal framework. In section 4, the analysis is extended to reasoning about dimensions, the relationship between facts and factors, and factor incompatibility. Section 5 offers some additional discussion.

2 The Formal Setting

We first briefly summarise the formal frameworks used in this paper. An *abstract argument framework*, as introduced by Dung, [?] is a pair $AF = \langle \mathcal{A}, \text{defeat} \rangle$, where \mathcal{A} is a set of arguments and *defeat* a binary relation on \mathcal{A} . A subset \mathcal{B} of \mathcal{A} is said to be *conflict-free* if no argument in \mathcal{B} defeats an argument in \mathcal{B} and it is said to be *admissible* if it is both conflict-free and also defends itself against any attack, i.e., if an argument A_1 is in \mathcal{B} and some argument A_2 in \mathcal{A} but not in \mathcal{B} defeats A_1 , then some argument in \mathcal{B} defeats A_2 . A *preferred extension* is then a maximal (with respect to set inclusion) admissible set. Dung defines several other types of extensions but they are not used in our model.

Dung's arguments are entirely abstract, with no features other than the defeat relation. A general framework for giving structure to arguments is the ASPIC framework, most fully defined as ASPIC+ in [?]. The ASPIC+ framework first defines the notion of an *argumentation system*, which consists of a logical language \mathcal{L} with a binary contrariness relation \neg and two sets of inference rules \mathcal{R}_s and \mathcal{R}_d of *strict* and *defeasible inference rules* defined over \mathcal{L} , written as $\varphi_1, \dots, \varphi_n \rightarrow \varphi$ and $\varphi_1, \dots, \varphi_n \Rightarrow \varphi$. Informally, that an inference rule is strict means that if its antecedents are accepted, then its consequent must be accepted *no matter what*, while that an inference rule is defeasible means that if its antecedents are accepted, then its consequent must be accepted *if there are no good reasons not to accept it*.

In the present paper we use an argumentation system in which \mathcal{L} is a first-order language with equality further specified in the coming sections, its contrariness relation corresponds to classical negation, the strict rules \mathcal{R}_s are all valid first-order inferences over \mathcal{L} and the defeasible rules \mathcal{R}_d are as specified in the coming sections.

Arguments are in ASPIC+ constructed from a knowledge base \mathcal{K} , which contains two disjoint kinds of formulas: the *axioms* \mathcal{K}_n and the *ordinary premises* \mathcal{K}_p . The formal definition of an argument is as follows:

Definition 2.1 [Argument] An *argument* A on the basis of a knowledge base \mathcal{K} in an argumentation system $(\mathcal{L}, \neg, \mathcal{R}_s, \mathcal{R}_d)$ is:

1. φ if $\varphi \in \mathcal{K}$ with: $\text{Prem}(A) = \{\varphi\}$; $\text{Conc}(A) = \varphi$; $\text{Sub}(A) = \{\varphi\}$; $\text{TopRule}(A) = \text{undefined}$.

$$\frac{\frac{P1 \quad P2}{P3} (A1) \quad P4 (A2)}{C1}$$

Figure 1: An argument

2. $A_1, \dots, A_n \rightarrow/\Rightarrow \psi$ if A_1, \dots, A_n are arguments such that there exists a strict or a defeasible rule $\text{Conc}(A_1), \dots, \text{Conc}(A_n) \rightarrow/\Rightarrow \psi$ in $\mathcal{R}_s/\mathcal{R}_d$.
 $\text{Prem}(A) = \text{Prem}(A_1) \cup \dots \cup \text{Prem}(A_n)$; $\text{Conc}(A) = \psi$; $\text{Sub}(A) = \text{Sub}(A_1) \cup \dots \cup \text{Sub}(A_n) \cup \{A\}$;
 $\text{TopRule}(A) = \text{Conc}(A_1), \dots, \text{Conc}(A_n) \rightarrow/\Rightarrow \psi$.

An argument is *strict* if all its inference rules are strict and *defeasible* otherwise, and it is *firm* if all its premises are in \mathcal{K}_n and *plausible* otherwise.

Arguments can be displayed as inference trees. An example argument, A_2 , is shown in Figure 1. A_2 has premises P_1, P_2, P_3 , and conclusion C_1 . A single and double bar stand for, respectively, a strict and defeasible inference. Argument A_2 has three subarguments, namely A_1 , which has premises P_1 and P_2 and conclusion P_3 , and the formulas P_1, P_2 and P_4 as atomic subarguments.

An argumentation system and a knowledge base are combined with an *argument ordering* into an *argumentation theory*. The argument ordering could be defined in any way, for example, in terms of orderings on \mathcal{R}_d and \mathcal{K}_p .

Definition 2.2 [Argumentation theories] An *argumentation theory* is a triple $AT = (AS, \mathcal{K}, \preceq)$ where AS is an argumentation system, \mathcal{K} is a knowledge base in AS and \preceq is a partial preorder on the set of all arguments on the basis of \mathcal{K} in AS (below denoted by \mathcal{A}_{AT}).

Arguments can be attacked in three ways: attacking a conclusion of a defeasible inference, attacking the defeasible inference itself, or attacking a premise. To define how a defeasible inference can be attacked, a function n is assumed that assigns to each element of \mathcal{R}_d a well-formed formula in \mathcal{L} . Informally, $n(r)$ (where $r \in \mathcal{R}_d$) means that r is applicable. For our argumentation system, ASPIC+'s definitions of attack can be simplified as follows:¹

Definition 2.3 [attacks] A attacks B iff A *undercuts*, *rebuts* or *undermines* B , where:

- A *undercuts* argument B (on B') iff $\text{Conc}(A) = -n(r)$ for some $B' \in \text{Sub}(B)$ such that B' 's top rule r is defeasible.
- A *rebuts* argument B (on B') iff $\text{Conc}(A) = -\varphi$ for some $B' \in \text{Sub}(B)$ of the form $B''_1, \dots, B''_n \Rightarrow \varphi$.
- Argument A *undermines* B (on φ) iff $\text{Conc}(A) = -\varphi$ for some ordinary premise φ of B .

In Figure 1, argument A_2 can only be rebutted or undercut on its defeasible subargument A_1 .

Attacks combined with the preferences defined by an argument ordering yield three kinds of defeat.

Definition 2.4 [Successful rebuttal, undermining and defeat]

- A *successfully rebuts* B if A rebuts B on B' and $A \not\prec B'$.
- A *successfully undermines* B if A undermines B on φ and $A \not\prec \varphi$.
- A *defeats* B iff A undercuts or successfully rebuts or successfully undermines B .

The success of rebutting and undermining attacks thus involves comparing the conflicting arguments at the points where they conflict. The definition of successful undermining exploits the fact that an argument premise is also a subargument. For undercutting attack no preferences are needed to make it succeed, since undercutters state exceptions to the rule they attack.

ASPIC+ thus defines a set of arguments with a binary relation of defeat, that is, it defines abstract argumentation frameworks in the sense of [?]. Formally:

¹In the definitions below, $-\neg\varphi$ denotes φ , while if φ does not start with a negation, $-\varphi$ denotes $\neg\varphi$.

Definition 2.5 [Argumentation framework] An *abstract argumentation framework* (AF) corresponding to an *argumentation theory* AT is a pair $\langle \mathcal{A}, Def \rangle$ such that:

- \mathcal{A} is the set of arguments on the basis of AT as defined by Definition 2.1,
- Def is the relation on \mathcal{A} given by Definition 2.4.

Thus any semantics for abstract argumentation can be applied to ASPIC+.

3 CATO Argumentation Schemes

In this section, formal argumentation schemes for CATO style case-based reasoning are provided. We give a brief overview of CBR as represented in CATO in section 3.1, introduce our running example in section 3.2, present elements of the language in section 3.3, formalise the argumentation schemes in 3.4, and report the results with respect to our example in 3.5. In section 3.6 we prove that our ASPIC+ argumentation theories satisfy [?]’s rationality postulates of strict closure and consistency.

3.1 Case-based Reasoning as in CATO

CATO [?], which we focus on in this section, analyses cases in terms of *factors*, where a factor is a prototypical fact situation that predisposes the decision in favour of one party or the other in the case; for trade secret law, the domain CATO is designed for, the factors concern trade secret misappropriation and are derived from *Restatement of Torts First, Sec. 757* and the *Uniform Trade Secret Act* (see [? ?]). As different precedents have different distributions of factors, finding and reasoning about precedents with respect to a current case requires one to examine the combinations of and counter-balancing between, factors in the cases. In addition to the factors themselves, there is a *factor hierarchy* in which an *abstract factor* has factors as children; in reasoning with the abstract factors and the factors of a case, differences between the cases can sometimes be reconciled. The argumentation schemes discussed in this paper make such reasoning patterns explicit and formal.

A case comparison method for LCBR was introduced in [?], where cases are analysed in terms of *partitions* of case factors. Various distributions of factors amongst the partitions can be used to support or undermine the plaintiff’s argument that the current case should be decided in the plaintiff’s favour. [?] provided some informally expressed *argumentation schemes* for this partition method, where the schemes are *defeasible reasoning patterns* and the partitions are sets of CATO factors and the factor hierarchy is used. This paper formalises, articulates, and extends this line of research on LCBR.

3.2 Running Example

To clarify the discussion, we provide a running example using *Mason v Jack Daniels Distillery* (indicated with *Mason*) and *M. Bryce and Associates v Gladstone* (indicated with *Bryce*) as analysed in CATO, based on the factors and factor hierarchy in [?].

*Mason v Jack Daniels Distillery*², is a well known case, so well known that an episode of the Simpsons³ was based on it. A bartender, Tony Mason, invented a cocktail, Lynchburgh Lemonade comprising Jack Daniel’s whiskey, Triple Sec, sweet and sour mix, and 7-Up. It proved surprisingly popular. Mason met Winston Randle, a sales representative for Jack Daniel Distillery, and they talked about the drink, and its possible use in a promotion. Approximately one year later the defendants were developing a national promotion campaign for Lynchburgh Lemonade. Mason claimed that he had parted with the recipe because he had been told that his band would be used in the promotion. In fact Mason received nothing. The jury found for the plaintiff, but awarded only a dollar in damages. Here we will treat *Mason* as the current case under consideration.

²518 So.2d 130, 1987 Ala. Civ. App.

³Flaming Moe’s, the tenth episode of the third season.

In *M. Bryce and Associates v Gladstone*⁴ Bryce was a software company with a product bearing the registered trademark “Pride”. “Pride” is a complete methodology for the design, development and implementation of an information system. Bryce made a presentation of “Pride” to the defendants, hoping to make a sale, after which the defendants designed and implemented a manual that duplicated its procedures, forms and standards. *Bryce* thus also involves disclosure in negotiations and was found for the plaintiff, and so can serve as a possible precedent.

We give the factors for each case, as used in CATO, (the factor identifiers, *F1* and so on are those used in [?] and adopted in other work discussing CATO). We also indicate the side favoured by the factors:

- *Mason*
 - *F1 Disclosure-In-Negotiations* (*d*)
 - *F6 Security-Measures* (*p*)
 - *F15 Unique-Product* (*p*)
 - *F16 Info-Reverse-Engineerable* (*d*)
 - *F21 Knew-Info-Confidential* (*p*)
- *Bryce*
 - *F1 Disclosure-In-Negotiations* (*d*)
 - *F4 Agreed-Not-To-Disclose* (*p*)
 - *F6 Security-Measures* (*p*)
 - *F18 Identical-Products* (*p*)
 - *F21 Knew-Info-Confidential* (*p*)

In subsequent sections, we illustrate the formalism with this example.

3.3 Elements of a Language

We begin by defining the language that we shall use to talk about our cases and which will be used in our underlying knowledge base. We assume a many-sorted first-order language with sorts for parties, cases (with subsorts for current cases and precedents), factors and factor sets. We trust that the types of the terms and predicate and function symbols will be clear from the context and wording.

We first discuss some preliminaries. To correctly represent and reason with set-theoretic expressions, the following definitions are assumed to be in \mathcal{K}_n :

1. $\forall s, s'. (s \subseteq s' \equiv \forall x (x \in s \implies x \in s'))$ ⁵
2. $\forall x, s, s'. (x \in s \cap s' \equiv (x \in s \wedge x \in s'))$
3. $\forall x, s, s'. (x \in s \cup s' \equiv (x \in s \vee x \in s'))$
4. $\forall x, s, s'. (x \in s \setminus s' \equiv (x \in s \wedge x \notin s'))$

In expressions like $pFactors(Mason) = \{F6, F15, F21\}$ the brackets { and } are together a function symbol operating on the terms *F6*, *F15* and *F21*. To preserve the meaning of the function symbol the following axiom is added to \mathcal{K}_n :

5. $\forall s, x_1, \dots, x_n. s = \{x_1, \dots, x_n\} \equiv \forall y (y \in s \equiv (y = x_1 \vee \dots \vee y = x_n))$

Here the variable *s* ranges over sets. This definition assumes that sets are finite, which in our domain is a safe assumption.

Factors are in \mathcal{K}_p declared to be either pro-plaintiff or pro-defendant, with:

⁴107 Wis. 2d 241

⁵In this paper the long double arrow \implies denotes the material implication.

- $pFactor(factor)$, meaning that $factor$ is a pro-plaintiff factor;
- $dFactor(factor)$, meaning that $factor$ is a pro-defendant factor.

In our running example we have at least the following formulas in \mathcal{K}_p :

- $pFactor(F4)$
- $pFactor(F6)$
- $pFactor(F15)$
- $pFactor(F18)$
- $pFactor(F21)$
- $dFactor(F1)$
- $dFactor(F16)$

No factor can be both pro-plaintiff and pro-defendant, expressed by adding to \mathcal{K}_n :

6. $\forall factor \neg (pFactor(factor) \wedge dFactor(factor))$

We next turn to the representation of cases. We will not talk about cases directly, but cases as analysed for use by the CATO system, which is the system that provides the paradigm on which our argumentation will be based. For CATO, a case has a name, a set of factors in favour of the plaintiff, a set of factors in favour of the defendant, and (if the case is a precedent) an outcome, which is one of plaintiff or defendant. We describe cases as follows. First for each case the sets of (plaintiff and defendant) factors in the case are specified with the following predicates:

- $hasFactor(case, factor)$, meaning that $factor$ is a factor in $case$.
- $hasPfactor(case, factor)$, meaning that $factor$ is a plaintiff factor in $case$.
- $hasDfactor(case, factor)$, meaning that $factor$ is a defendant factor in $case$.

Moreover, the following definitions are added to \mathcal{K}_n .

7. $\forall case, factor. hasPfactor(case, factor) \equiv hasfactor(case, factor) \wedge pFactor(factor)$
8. $\forall case, factor. hasDfactor(case, factor) \equiv hasfactor(case, factor) \wedge dFactor(factor)$

The idea is that $hasFactor(case, factor)$ statements are added to \mathcal{K}_p and that they together with the specifications of the types of factors in \mathcal{K}_p give rise to strict arguments for $hasPfactor(case, factor)$ and $hasDfactor(case, factor)$ conclusions. In our running example we thus have strict arguments for the following conclusions:

- $hasPfactor(Mason, F6)$
- $hasPfactor(Mason, F15)$
- $hasPfactor(Mason, F21)$
- $hasDfactor(Mason, F1)$
- $hasDfactor(Mason, F16)$
- $hasPfactor(Bryce, F4)$
- $hasPfactor(Bryce, F6)$

- $hasPfactor(Bryce, F18)$
- $hasPfactor(Bryce, F21)$
- $hasDfactor(Bryce, F1)$

To ensure that a factor belongs to a case if and only if specified as such, the predicate completions of the predicate $hasFactor$ and the unique-names and domain-closure axioms for objects satisfying these predicates are added to \mathcal{K}_n . This makes, for example, the following pairs of formulas mutually inconsistent:

- $pFactors(Mason) = \{F6, F15, F21\}$ and $pFactors(Mason) = \{F4, F21\}$
- $pFactors(Mason) = \{F6, F15, F21\}$ and $hasPfactor(Mason, F4)$

In our running example the predicate completion formulas are as follows:

- $\forall case, factor. hasFactor(case, factor) \equiv ((case = Mason \wedge (factor = F1 \vee F6 \vee factor = F15 \vee F1 \vee factor = F21)) \vee (case = Bryce \wedge (factor = F1 \vee F4 \vee factor = F6 \vee factor = F18 \vee factor = F21)))$

The unique-names and domain closure axioms are:

- $\forall case. case = Mason \vee case = Bryce$
- $Mason \neq Bryce$
- $\forall factor. factor = F1 \vee \dots \vee factor = F115$
- $F1 \neq \dots \neq F115$

The following three function expressions are used to denote a case's sets of pro-plaintiff and pro-defendant factors and its outcome:

- $pFactors(case) = setOfFactors.$
- $dFactors(case) = setOfFactors.$
- $outcome(case) = party.$

We then add the following axioms to \mathcal{K}_n to link expressions with the $hasPfactor$, $hasDfactor$ and $hasfactor$ predicates to expressions with the \in symbol. It is these axioms that enable set-theoretic operations on factors and factor sets.

9. $\forall case, f. f \in Factors(case) \equiv hasFactor(case, factor)$
10. $\forall case, f. f \in pFactors(case) \equiv hasFactor(case, factor) \wedge pFactor(factor)$
11. $\forall case, f. f \in dFactors(case) \equiv hasFactor(case, factor) \wedge dFactor(factor)$

With respect to our running example, we then have strict arguments for the following conclusions:

- $pFactors(Mason) = \{F6, F15, F21\}$
- $dFactors(Mason) = \{F1, F16\}$
- $pFactors(Bryce) = \{F4, F6, F18, F21\}$
- $dFactors(Bryce) = \{F1\}$

We also have in \mathcal{K}_p :

- $outcome(Bryce) = Plaintiff$

Additionally a feature of CATO is that factors are organised into a factor hierarchy, with factors being the children of more abstract factors. Thus for every factor we can have relations of the form:

- $parentFactor(factor, abstractFactor)$

While CATO has some intermediate layers in the factor hierarchy, we omit some of them for our current purposes as well as the label of these higher level factors. The abstract factors are also associated with a side, with the following formulas in \mathcal{K}_p :

- $pFactor(F102)$
- $pFactor(F115)$
- $dFactor(F105)$

Note that cases are described only in terms of base level factors: thus $pFactors(case)$ and $dFactors(case)$ do not return any abstract factors. The factor hierarchy was originally built by Alevel starting from the base level factors, and in principle it would be possible to construct different factor hierarchies, using different abstract factors and/or different $parentFactor$ relations, in which case paternity could even be the subject of dispute, and $parentFactor(factor, abstractFactor)$ would need to be the conclusion of some rule, rather than a premise. We will, however, consider the factor hierarchy to be fixed to that used in [?], and use $parentFactor$ only as it is defined there.

- $parentFactor(F1, F102)$
- $parentFactor(F4, F102)$
- $parentFactor(F4, F115)$
- $parentFactor(F6, F102)$
- $parentFactor(F15, F105)$
- $parentFactor(F16, F105)$
- $parentFactor(F21, F115)$

A factor hierarchy can be specified by adding a formula of the following form to \mathcal{K}_p :

- $\forall factor1, factor2. parentFactor(factor1, factor2) \equiv (factor1 = F_{i1} \wedge factor2 = F_{j1}) \vee \dots \vee (factor1 = F_{in} \wedge factor2 = F_{jn})$

If desired, axioms can be added to \mathcal{K}_n to exclude cycles in the factor hierarchy, but multiple parents must be allowed to represent the hierarchy of [?].

Cases are compared with one another in terms of their factors. This gives rise to a further six functions of the following type:

$$commonPfactors : currentcases \times precedents \longrightarrow 2^{factors}$$

The six functions are defined as follows as elements of \mathcal{K}_n .

$\forall f, curr, prec:$

12. $commonPfactors(curr, prec) = pFactors(curr) \cap pFactors(prec)$
13. $commonDfactors(curr, prec) = dFactors(curr) \cap dFactors(prec)$
14. $currPfactors(curr, prec) = pFactors(curr) \setminus commonPfactors(curr, prec)$
15. $currDfactors(curr, prec) = dFactors(curr) \setminus commonDfactors(curr, prec)$
16. $precPfactors(curr, prec) = pFactors(prec) \setminus commonPfactors(curr, prec)$

$$17. \text{precDfactors}(\text{curr}, \text{prec}) = \text{dFactors}(\text{prec}) \setminus \text{commonDfactors}(\text{curr}, \text{prec})$$

With respect to our running example, we have:

- $\text{commonPfactors}(\text{Mason}, \text{Bryce}) = \{F6, F21\}$
- $\text{commonDfactors}(\text{Mason}, \text{Bryce}) = \{F1\}$
- $\text{currPfactors}(\text{Mason}, \text{Bryce}) = \{F15\}$
- $\text{currDfactors}(\text{Mason}, \text{Bryce}) = \{F16\}$
- $\text{precPfactors}(\text{Mason}, \text{Bryce}) = \{F4, F18\}$
- $\text{precDfactors}(\text{Mason}, \text{Bryce}) = \emptyset$

These relations are the building blocks for our arguments. The first two are the basis for a comparison and represent what is common between the two cases. The remaining four represent differences, and their effect will depend on the outcome of the previous case and the side for which we are arguing. Suppose we are arguing for the plaintiff: then we can only use precedents with the outcome plaintiff. For such cases, currPfactors and precDfactors will strengthen the plaintiff's position, since they represent, respectively, plaintiff reasons in curr not available in the prec and defendant reasons in the prec which are not available in curr . On the other hand, currDfactors and precPfactors weaken the plaintiff's position in curr . Similarly, if arguing for the defendant in the curr , currDfactors and precPfactors strengthen the position and currPfactors and precDfactors weaken it. The precise nature of the strengthening and weakening will be made clear when we consider the argumentation schemes based on these different partitions.

Next we need to express that one set of factors, factorSet1 , is preferred over another, factorSet2 .

- $\text{preferred}(\text{factorSet1}, \text{factorSet2})$.

In our analysis, the preference is the claim of a defeasible argumentation scheme CS2, which only appears later. We cannot, then, straightforwardly provide the preference in our running example until section 3.4.

Finally, we need another relation between factors. If factors for a given party share the same ancestor, then both factors get their force from the fact that the same abstract factor is present in the case. This means that, if they favour the same party to the case, it may be possible to substitute one for another. Similarly if they favour different parties, they may cancel each other out so as to remove the abstract factor from the case. Therefore we have two additional predicates:

- $\text{substitutes}(\text{factor1}, \text{factor2})$
- $\text{cancels}(\text{factor1}, \text{factor2})$

To define these predicates, the following definition of the ancestor relation between factors is added to \mathcal{K}_n :

$$18. \forall f_1, f_2. \text{ancestor}(f_1, f_2) \equiv \text{parentFactor}(f_1, f_2) \vee \exists f_3 (\text{ancestor}(f_1, f_3) \wedge \text{parentFactor}(f_3, f_2))$$

We define substitution and cancellation of factors that would benefit the plaintiff as follows, where substitutions apply *between* cases and cancellations apply *within* cases. Substitutions and cancellations for the defendant would be similar, though switching the predicates (and factor sets). The following definitions are in \mathcal{K}_n :

$$19. \forall f_1, f_2. \text{substitutes}(f_1, f_2) \equiv ((p\text{Factor}(f_1) \wedge p\text{Factor}(f_2)) \vee (d\text{Factor}(f_1) \wedge d\text{Factor}(f_2))) \wedge \exists f_3 (\text{ancestor}(f_1, f_3) \wedge \text{ancestor}(f_2, f_3))$$

$$20. \forall f_1, f_2. \text{cancels}(f_1, f_2) \equiv ((p\text{Factor}(f_1) \wedge d\text{Factor}(f_2)) \vee (d\text{Factor}(f_1) \wedge p\text{Factor}(f_2))) \wedge \exists f_3 (\text{ancestor}(f_1, f_3) \wedge \text{ancestor}(f_2, f_3))$$

In our running example, we have:

- *substitutes*($F4, F6$) since $precPfactors(Mason, Bryce) = \{F4, F18\}$ and $F4 \in \{F4, F18\}$ and $pFactors(Mason) = \{F6, F15, F21\}$ and $F6 \in \{F6, F15, F21\}$ and $parentFactor(F4, F102)$ and $parentFactor(F6, F102)$.
- *cancels*($F15, F16$) since $currPfactors(Mason, Bryce) = \{F15\}$ and $F15 \in \{F15\}$ and $dFactors(Mason) = \{F1, F16\}$ and $F16 \in \{F16, F16\}$ and $parentFactor(F15, F105)$ and $parentFactor(F16, F105)$.

Intuitively, we want to argue that we should decide *Mason* for the plaintiff on the basis of *Bryce*. The argument will be that *Mason* and *Bryce* share several factors (both for plaintiff and defendant), and so, since *Bryce* was decided for the plaintiff, so too should *Mason* be decided, provided that any differences between them can be argued away by substitution and cancellation.

In the next section we will present the argumentation schemes built from this language.

3.4 CATO style Argumentation Schemes

In this section we specify the defeasible inference rules \mathcal{R}_d of our ASPIC+ argumentation system. For readability we will not specify them with the rule symbol \Rightarrow but as *argumentation schemes*, i.e., with a double horizontal inference bar. Rule schemes will be named by expressions $Name(x_1, \dots, x_n)$ where the predicate $Name$ stands for the informal name of the rule and x_1, \dots, x_n are all free variables occurring in the scheme. These variables are replaced by ground terms for each instance of the scheme, resulting in closed formulas that are the names of the scheme instances according to the function n mentioned just before Definition 2.3.

In this section we will always suppose that we wish to argue the *curr* for the plaintiff. Arguments for the defendant are similar, except that the strengthening and weakening factor partitions are reversed as discussed above. The argument is that the *curr* should be decided for the plaintiff because the common p factors were preferred to the common d factors in the *prec*.⁶

CS1(*cur*, *prec*, p , d):

$$\begin{array}{l} commonPfactors(curr, prec) = p, \\ commonDfactors(curr, prec) = d, \\ \underline{\underline{preferred(p, d)}} \\ outcome(curr) = Plaintiff \end{array}$$

Instantiating CS1, where our *curr* is *Mason* and our *prec* is *Bryce*, we have the following argument, indicated with **Mason(Bryce)A1**:

Mason(Bryce)A1

$$\begin{array}{l} commonPfactors(Mason, Bryce) = \{F6, F21\}, \\ commonDfactors(Mason, Bryce) = \{F1\}, \\ \underline{\underline{preferred(\{F6, F21\}, \{F1\})}} \\ outcome(Mason) = Plaintiff \end{array}$$

Note that strictly speaking some of these premises are derived from $\mathcal{K}_n \cup \mathcal{K}_p$. However, to keep the arguments reasonably readable we will leave strict derivations from the knowledge base implicit.

We will assume at this point that the information about cases in our KB is correct, or at least beyond dispute; this is relaxed in section 4.2. In ASPIC+ terms this makes the case facts *axioms* and so the first two premises cannot be questioned. The third, however, needs to be established, and this will be done using CS2, which we will describe after considering undercutters to CS1.

⁶Note that CS1 uses only a subset of the factors from the precedent: this is because CS2 also encapsulates the rule broadening move as discussed in [?], which is necessary to adapt the *prec* so as to match the *curr*.

There may, of course, be rebuttals, using a variety of argumentation schemes, but we need to recognise that even if such a preference has been established in the *prec*, it may not be applicable to the *curr*, because the defendant has arguments in the *curr* that were not available in the *prec*. We therefore have the undercutting attack for arguments using CS1.

U1.1(*cur*, *prec*, *p*, *d*):

$$\frac{f \in \text{currDfactors}(\text{curr}, \text{prec})}{\neg \text{CS1}(\text{cur}, \text{prec}, p, d)}$$

Instantiating U1.1 with *Mason*, *Bryce* and the relevant sets, we have an undercutter argument:

Mason(Bryce)A2:

$$\frac{F16 \in \text{currDfactors}(\text{Mason}, \text{Bryce})}{\neg \text{CS1}(\text{Mason}, \text{Bryce}, \{F6, F21\}, \{F1\})}$$

While this presents a challenge to the plaintiff, the argument for the plaintiff can be defended if the distinctions between the cases can be *downplayed*. The undercutting move of U1.1 is one way of distinguishing the two cases, and in CATO the abstract factor hierarchy allows us to downplay distinctions. This downplaying can be done in two ways, *substitution* or *cancellation*, corresponding to the two different kinds of extra strength the *curr* may have. Accordingly we introduce two schemes that can be used to provide undercutters of U1.1:

U1.1.1(*curr*, *prec*, *f*₁, *f*₂, *p*, *d*):

$$\frac{f_1 \in \text{currDfactors}(\text{curr}, \text{prec}), \\ f_2 \in \text{dFactors}(\text{prec}), \\ \text{substitutes}(f_1, f_2)}{\neg \text{U1.1}(\text{cur}, \text{prec}, p, d)}$$

U1.1.2(*curr*, *prec*, *f*₁, *f*₂, *p*, *d*):

$$\frac{f_1 \in \text{currDfactors}(\text{curr}, \text{prec}), \\ f_2 \in \text{pFactors}(\text{curr}), \\ \text{cancels}(f_1, f_2)}{\neg \text{U1.1}(\text{cur}, \text{prec}, p, d)}$$

The idea here is that as the undercutting factor in the *curr* has the same parent as a factor in *prec*, we can substitute for the undercutting factor, where the point is that the abstract factor can be seen to have been applied also in the *prec*; alternatively, the undercutting factor in the *curr* is cancelled out by some other factor in *curr*, so that the abstract factor does not apply. Instantiating U1.1.2 with our running example and given that we previously determined that *cancels*(*F15*, *F16*), we can form the following argument:

Mason(Bryce)A3:

$$\frac{F16 \in \text{currDfactors}(\text{Mason}, \text{Bryce}), \\ F15 \in \text{pFactors}(\text{Mason}), \\ \text{cancels}(F15, F16)}{\neg \text{U1.1}(\text{cur}, \text{prec}, p, d)}$$

We now turn to the argumentation scheme to establish the preference between two sets of factors, required to justify the third premise of CS1.

CS2(*cur*, *prec*, *p*, *d*):

$$\begin{aligned} & \text{commonPfactors}(\text{curr}, \text{prec}) = p, \\ & \text{commonDfactors}(\text{curr}, \text{prec}) = d, \\ & \frac{\text{outcome}(\text{prec}) = \text{Plaintiff}}{\text{preferred}(p, d)} \end{aligned}$$

Note that CS2 establishes a preference between two particular sets. It might seem natural to add that from $\text{preferred}(p, d)$ we should be able to derive $\text{preferred}(p', d)$ where $p' \supset p$ and $\text{preferred}(p, d')$ where $d' \subset d$, as in, for example [?]. This, however, would be to go beyond CATO. Moreover it would arguably go against the spirit of CATO-style reasoning, which insists that all claims about preferences are based on a *specific* precedent. CATO always argues using a particular precedent, never with a set of precedents. If the current case does in fact contain additional pro-plaintiff factors or fewer pro-defendant factors, these are made use of in different arguments employing the argumentation schemes CS3 and CS4 discussed below.

Instantiating CS2, we have an argument for the preference, as mentioned above:

Mason(Bryce)A4:

$$\begin{aligned} & \text{commonPfactors}(\text{Mason}, \text{Bryce}) = \{F6, F21\} \\ & \text{commonDfactors}(\text{Mason}, \text{Bryce}) = \{F16\} \\ & \frac{\text{outcome}(\text{Bryce}) = \text{Plaintiff}}{\text{preferred}(\{F6, F21\}, \{F16\})} \end{aligned}$$

All of the premises of CS2 are taken from our database, or straightforward set operations on such data and so represent ASPIC+ axioms which cannot be questioned. It is, however, possible to both rebut and to undercut the argument.

R2.1($\text{cur}, \text{prec}, \text{prec2}, p, d$):

$$\begin{aligned} & p \subseteq \text{commonPfactors}(\text{curr}, \text{prec2}), \\ & d \subseteq \text{commonDfactors}(\text{curr}, \text{prec2}), \\ & \frac{\text{outcome}(\text{prec}) = \text{Defendant}}{\neg \text{preferred}(p, d)} \end{aligned}$$

Attacks made using R2.1 offer counter examples in which the same comparison was available in a case decided for the defendant, suggesting that the preference is opposite, and so providing a rebuttal. We do not consider such rebuttals further in this paper, but in arguing a case they would be subject to attacks using the schemes introduced in this paper, just like CS2. The following scheme can be used to undercut CS2.

U2.1($\text{curr}, \text{prec}, p, d$):

$$\frac{f \in \text{precPfactors}(\text{curr}, \text{prec})}{\neg \text{CS2}(\text{curr}, \text{prec}, p, d)}$$

Instantiating U2.1 with *Mason* and *Bryce*, we have two arguments, one for each factor in *precP*factors:

Mason(Bryce)A5:

$$\frac{F4 \in \text{precPfactors}(\text{Mason}, \text{Bryce})}{\neg \text{CS2}(\text{Mason}, \text{Bryce}, \{F6, F21\}, \{F1\})}$$

Mason(Bryce)A5':

$$\frac{F18 \in \text{precPfactors}(\text{Mason}, \text{Bryce})}{\neg \text{CS2}(\text{Mason}, \text{Bryce}, \{F6, F21\}, \{F1\})}$$

U2.1 undercuts the argument by suggesting that it may have been the additional plaintiff factors available in the *prec* that tipped the balance, and so distinguishing the *curr* and the *prec*. Like U1.1, U2.1 can be undercut if we can downplay the distinction.

U2.1.1($\text{curr}, \text{prec}, f_1, f_2, p, d$):

$$\begin{array}{l}
f_1 \in \text{precPfactors}(\text{curr}, \text{prec}), \\
f_2 \in \text{pFactors}(\text{curr}), \\
\text{substitutes}(f_1, f_2) \\
\hline
\neg \mathbf{U2.1}(\text{curr}, \text{prec}, p, d)
\end{array}$$

U2.1.2(*curr*, *prec*, f_1 , f_2 , p , d):

$$\begin{array}{l}
f_1 \in \text{precPfactors}(\text{curr}, \text{prec}), \\
f_2 \in \text{dFactors}(\text{curr}), \\
\text{cancels}(f_1, f_2) \\
\hline
\mathbf{U2.1}(\text{curr}, \text{prec}, p, d)
\end{array}$$

We instantiate U2.1.1, which undercuts U2.1:

Mason(Bryce)A6:

$$\begin{array}{l}
F4 \in \text{precPfactors}(\text{Mason}, \text{Bryce}), \\
F6 \in \text{pFactors}(\text{Mason}), \\
\text{substitutes}(F4, F6) \\
\hline
\neg \mathbf{U2.1}(\text{Mason}, \text{Bryce}, \{F6, F21\}, \{F1\})
\end{array}$$

At this point we have: the main argument for the plaintiff based on a particular *prec*, comprising an application of a preference and an argument for the preference; undercutters of these two subarguments; and undercutters of some of these undercutting arguments. We may still, however, have some strengths of the *curr* unused, and so we can add some supplementary arguments.

CS3(*curr*, *prec*, f_1 , p , d):

$$\begin{array}{l}
\text{commonPfactors}(\text{curr}, \text{prec}) = p, \\
\text{commonDfactors}(\text{curr}, \text{prec}) = d, \\
\text{preferred}(p, d), \\
f_1 \in \text{currPfactors}(\text{curr}, \text{prec}), \\
\neg \exists f_2 (f_2 \in \text{dFactors}(\text{curr}) \wedge \text{cancels}(f_2, f_1)), \\
\neg \exists f_3 (f_3 \in \text{pFactors}(\text{prec}) \wedge \text{substitutes}(f_1, f_3)) \\
\hline
\text{outcome}(\text{curr}) = \text{Plaintiff}
\end{array}$$

CS4(*curr*, *prec*, f_1 , p , d):

$$\begin{array}{l}
\text{commonPfactors}(\text{curr}, \text{prec}) = p, \\
\text{commonDfactors}(\text{curr}, \text{prec}) = d, \\
\text{preferred}(p, d), \\
f_1 \in \text{precDfactors}(\text{curr}, \text{prec}), \\
\neg \exists f_2 (f_2 \in \text{currPFactors}(\text{curr}) \wedge \text{cancels}(f_2, f_1)), \\
\neg \exists f_3 (f_3 \in \text{dFactors}(\text{curr}) \wedge \text{substitutes}(f_1, f_3)) \\
\hline
\text{outcome}(\text{curr}) = \text{Plaintiff}
\end{array}$$

These arguments make use of the factors not used to substitute or cancel factors cited to undercut the arguments for the plaintiff based on the *prec*. Thus CS3 points to additional plaintiff factors in the *curr* that were not used to cancel or substitute for factors otherwise used. CS4 does the same thing in terms of factors that made the defendant's case stronger in the *prec*. Note that both require the *preferred(P, D)* as a premise, and so must use CS2 to establish this. This seems, from a logical point of view, somewhat odd, since the premises of CS1 are a subset of CS3 and the conclusion is the same. Traditionally in work on computational argumentation, arguments are defined so that the premises should be a minimal subset from which the conclusion may be derived [?]. Yet these are presented as arguments in CATO, and so we need schemes for them if we are to reconstruct CATO. Essentially these arguments, which appear in CATO as the move *emphasise strengths*, are intended to have a kind of rhetorical force, rather than a logical force.

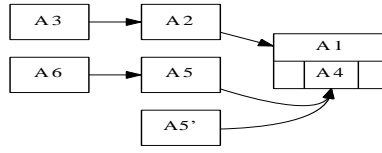


Figure 2: Mason(Bryce) partial Argument Graph

From a logical point of view, the case is already won, but in order to stress how superior the plaintiff's position is, his advocate adds that not only has the preference been established, but there remains all this unused ammunition which could have countered stronger arguments against the position. The idea seems to be to reassure the judge deciding for the plaintiff that the decision is not a close one, but quite clear and convincing.

3.5 Running Example Result

We have the following defeat relations between arguments, which are represented in Figure 2, where we indicate that **Mason (Bryce) A4** is a subargument of **Mason (Bryce) A1**:

- $\text{defeat}(\text{Mason}(\text{Bryce})\text{A2}, \text{Mason}(\text{Bryce})\text{A1})$
- $\text{defeat}(\text{Mason}(\text{Bryce})\text{A3}, \text{Mason}(\text{Bryce})\text{A2})$
- $\text{defeat}(\text{Mason}(\text{Bryce})\text{A5}, \text{Mason}(\text{Bryce})\text{A4})$
- $\text{defeat}(\text{Mason}(\text{Bryce})\text{A5}', \text{Mason}(\text{Bryce})\text{A4})$
- $\text{defeat}(\text{Mason}(\text{Bryce})\text{A6}, \text{Mason}(\text{Bryce})\text{A5})$

Following [?] and the assumption in ASPIC+ that an attack on a subargument is an attack on the argument, there is a unique extension, containing $\{\text{Mason}(\text{Bryce})\text{A6}, \text{Mason}(\text{Bryce})\text{A5}' \text{ and } \text{Mason}(\text{Bryce})\text{A3}\}$. In particular, **Mason(Bryce)A1** does not appear in any extension as its subargument is defeated by the unattacked **Mason(Bryce)A5'**. While the cases have common factors, *Bryce* was decided in favour of the plaintiff, and the preference for the decision holds, we have not succeeded in eliminating all significant distinctions; in particular, we have not found a substitution for *F18*. Were we to have found such a substitution, then we would have a successful attack on **Mason(Bryce)A5'**, in which case the extension would contain $\{\text{Mason}(\text{Bryce})\text{A6}, \text{Mason}(\text{Bryce})\text{A4}, \text{Mason}(\text{Bryce})\text{A3}, \text{Mason}(\text{Bryce})\text{A1}\}$ but not **Mason(Bryce)A2**, **Mason(Bryce)A5** and **Mason(Bryce)A5'**, so in this case *Bryce* would have been a good precedent for *Mason* as informally discussed previously.

Though this might appear to be a negative result, we can transform it into a positive result by finding an argument against **Mason(Bryce)A5'**, which requires that we substitute or cancel *F18* based on comparable factors in the factor hierarchy. We might argue that *F18 Identical-Products* holds in both cases, but was too obvious to be explicitly mentioned in *Mason*, and so was omitted from the initial analysis performed for CATO. Alternatively, we could argue that *F18* should be seen as providing too weak a factor to distinguish the cases. As another possibility, we can argue that **Mason(Bryce)A6** should rest on resolving the relative strength of *F4* and *F6*, if that becomes an issue. In all three instances, we would need to argue about the factors themselves, which is the subject of the next section.

3.6 Rationality postulates

To prove that our argumentation theory satisfies the rationality postulates of consistency and strict closure, the following properties need to be proven ([? ?]):

- \mathcal{R}_s is closed under contraposition or transposition.

- Strict consequence is *c-classical* in that if $S \vdash \varphi, \neg\varphi$, then any maximal subset of S strictly implies the negation of the remaining element. (Here $S \vdash \varphi$ means that there exists a strict argument for φ with all premises taken from S .)
- AT is well-formed in that if φ is a contrary of ψ then $\psi \notin Kn$ and ψ is not the consequent of a strict rule.
- The argument ordering is *reasonable* as defined in [? ?].
- The closure of \mathcal{K}_n under strict rules is consistent.

The first three properties are immediate from the fact that \mathcal{L} is a first-order language and \vdash in our case corresponds to first-order consequence. Above we used an argument ordering in which all strict-and-firm arguments are preferred over all other arguments and all non-strict-and-firm arguments have equal strength: given this it is easy to show that the argument ordering is reasonable. It remains to show that the closure of \mathcal{K}_n under strict rules is consistent. Since in our case \mathcal{L} is a first-order language and \mathcal{R}_s and \vdash correspond to first-order consequence, this can be proven by specifying a first-order model in which all our axioms are true.

Proposition 3.1 The closure of \mathcal{K}_n under strict rules of the argumentation theory specified above is consistent.

Proof:

We construct a model and then verify that all elements of \mathcal{K}_n are true in the model. Then completeness of first-order logic implies the proposition. The model contains just one factor, one precedent and one current case, where both cases share the factor as a pro-plaintiff factor and both cases are won by the plaintiff⁷.

For ease of notation, we equate below the various model elements with the language elements that denote them, letting the context disambiguate. Capital I stands as usual for the interpretation function of the language in the model.

- The sorts, relations and functions of the model are those corresponding to, respectively, the sorts, predicates and function symbols of \mathcal{L} .
- Individuals:
 - $Curr$ of sort $curr$ and $Prec$ of sort $prec$ (recall that both are subsorts of the sort $case$)
 - $Plaintiff$ and $Defendant$ of sort $parties$
 - $F1$ of sort $factors$
 - \emptyset and $\{F1\}$ of sort $sets$
- Interpretation of predicates (those not listed are empty):
 - $I(pFactor) = \{F1\}$
 - $I(outcome) = \{Plaintiff\}$
 - $I(hasPfactor) = \{(Curr, F1), (Prec, F1)\}$
 - $I(hasFactor) = \{(Curr, F1), (Prec, F1)\}$
 - The interpretation of \subseteq and \in is obvious and left implicit.
- Interpretation of functions:
 - $Factors(Curr) = \{F1\}$
 - $pFactors(Curr) = \{F1\}$

⁷Although this example is minimal it is not unrealistic. It would fit, for example, the representation of *Pierson v Post* in [? ?].

- $Factors(Prec) = \{F1\}$
- $pFactors(Prec) = \{F1\}$
- $dFactors(Curr) = \emptyset$
- $dFactors(Prec) = \emptyset$
- $commonPFactors(Curr, Prec) = \{F1\}$
- $commonDFactors(Curr, Prec) = \emptyset$
- $currPFactors(Curr, Prec) = \emptyset$
- $currDFactors(Curr, Prec) = \emptyset$
- $precPFactors(Curr, Prec) = \emptyset$
- $precDFactors(Curr, Prec) = \emptyset$
- $outcome(Curr) = Plaintiff$
- $outcome(Prec) = Plaintiff$
- The interpretation of $\{\}$, \cup , \cap and \setminus is obvious and left implicit.

We now verify that all axioms are true in this model.

- Axiom 1 is clearly true for the following three cases: $I(s) = I(s') = \emptyset$; $I(s) = I(s') = \{F1\}$; $I(s) = \emptyset$ while $I(s') = \{F1\}$. Since these are all cases that can arise, Axiom 1 is universally true. In the same way it is easy to verify that Axioms 2 - 4 are true.
- For axiom 5 note first that this axiom is in fact a scheme for a set of axioms and that in our case we only need to consider the version with two variables s and x_1 . Then two cases have to be considered, in both of which we have $I(x_1) = F1$. The first case is when $I(s) = \emptyset$. Then the left-hand side of the equivalence equals to $\emptyset = \{F1\}$, which is false in the model. In this case the right-hand side reduces to $F1 \in \emptyset$, which is also false in the model, so the equivalence is true. The second case is when if $I(s) = \{F1\}$. Then the left-hand side of the equivalence equals to $\{F1\} = \{F1\}$, which is true in the model. In this case the right-hand side reduces to $F1 \in \{F1\}$, which is also true in the model, so the equivalence is again true.
- Axiom 6 is true since $I(dFactor) = \emptyset$.
- Axiom 7 is true since there is only one individual of sort *factor*, namely $F1$, and we have that $I(pFactor) = \{F1\}$ and $I(hasPfactor) = I(hasFactor) = \{(Curr, F1), (Prec, F1)\}$. Likewise, Axiom 8 is true since $I(dFactor) = I(hasPfactor) = \emptyset$.
- Axiom 9 is true since our model contains only one factor and two cases. For the first case we have $Factors(Curr) = \{F1\}$ so $F1 \in Factors(Curr)$ and we have $(Curr, F1) \in I(hasFactor)$. The second case (with $Prec$) is identical. So both sides of the equivalence are true for all f and *case*. Axiom 10 can be verified in the same way. Axiom 11 is true since the model has no pro-defendant factors.
- For axioms 12-17 note that all three variables can be instantiated in only one way. Then axiom 12 is true since $commonPFactors(Curr, Prec) = \{F1\}$, $pFactors(Curr) = pFactors(Prec) = \{F1\}$ and $\{F1\} \cap \{F1\} = \{F1\}$. Similarly, axiom 13 is true since $commonDFactors(Curr, Prec) = \emptyset$, $dFactors(Curr) = dFactors(Prec) = \emptyset$ and $\emptyset \cap \emptyset = \emptyset$. Axiom 14 is true since $currPFactors(Curr, Prec) = \emptyset$, $pFactors(Curr) = \{F1\}$, $commonPFactors(Curr, Prec) = \{F1\}$ and $\{F1\} \setminus \{F1\} = \emptyset$. Axioms 15, 16 and 17 can be verified in the same way.
- Axioms 18-20 are true since the interpretations of the predicates *ancestor*, *parentFactor*, *substitutes* and *cancel*s are all empty.
- Finally, the domain-closure and unique-names axioms must be verified. The relevant domain closure axioms are:

- $\forall case, factor. hasFactor(case, factor) \equiv ((case = Curr \wedge factor = F1) \vee (case = Prec \wedge factor = F1))$
- $\forall case. case = Curr \vee case = Prec$
- $Curr \neq Prec$
- $\forall factor. factor = F1$

It is straightforward to show that these are true in the model.

Corollary 3.2 The Dung-style argumentation framework corresponding to the argumentation theory defined above satisfies all four rationality postulates as formulated in [?].

4 Beyond Factor-Based Reasoning

Thus far we have considered reasoning from cases represented as sets of factors to their outcomes. This has been the focus of most work on case-based reasoning in AI and Law, and the understanding of this aspect of reasoning with cases is quite mature. This has enabled us to propose a set of argumentation schemes to capture this reasoning with some confidence. There is, however, a lot more to reasoning with legal cases than this: cases do not arrive neatly packaged as bundles of factors, but as rather messy collections of facts.

Once the facts of a case have been established - and this may not be straightforward since the move from evidence to facts is often itself the subject of debate, legal reasoning can be seen, following Ross [?] and more recently [?], as a two stage process, first from the established facts to intermediate predicates, and then from these intermediate predicates to legal consequences. CATO has been explicitly identified with the second of these steps (e.g. [?]). Finding these intermediate predicates is by no means simple, and different intermediate concepts require different strategies. Some can be given by listing facts, which supply sufficient, and possibly collectively necessary, conditions but others require consideration of a range of facts, none of which supply sufficient or necessary conditions. However, as argued in [?], which factors hold of a case or which side is favoured by a particular fact may be the whole point. It is even sometimes necessary to argue about what factors there are. To tell the whole story of reasoning with cases, therefore, it is necessary to consider the step from facts to factors. In this section we will propose some initial argumentation schemes for this step of the process. Since there is no well understood model to work with, our proposals will be more tentative than was the case in the previous section. None the less, we hope that they will provide useful pointers as to the way forward. Argumentation schemes for reasoning about what factors hold in a case relative to the facts of a case are introduced in section 4.2. Further schemes for reasoning about exclusory relations between factors are discussed in section 4.3. Finally, section 4.4 presents schemes for reasoning about factors along dimensions.

4.1 Dimensions in Legal Case-Based Reasoning

Dimensions, rather than discrete factors, were used in Rissland and Ashley's HYPO [?], the system from which CATO was developed. Since factors as in CATO predominate in the literature [? ? ?], some background discussion on and justification for dimensions is warranted. Dimensions have an *extent* and values along the extent. In contrast to factors, which are either simply present or absent, a dimension, if present, may favour the plaintiff or defendant *to a particular degree*. Dimensions encompass a range of values, with the extreme pro-plaintiff value at one end and the extreme pro-defendant value at the other. Thus, at some unspecified point along the range the dimension will cease to favour the plaintiff and start to favour the defendant. Dimensions and factors are, however, related.

In one relationship, while a dimension may be continuous, factors are intervals along the dimension and ordered with respect to one another; in other words as in [?], factors can be taken as the *values* positioned along a dimension.⁸ For example, one dimension in HYPO is *Secrets-Voluntarily-Disclosed*, and ranges from 0 to 10,000,000 disclosees, 0 being the pro-plaintiff direction. In CATO, this dimension is expressed

⁸This is related to a general phenomena in cognition of *categorical perception* [?].

as factors that are ranked in strength. There is a pro-defendant factor *Secrets-Disclosed- Outsiders*, which is present if any disclosure at all had been made, effectively stating that the dimension favours the defendant rather than the plaintiff if a single person is disclosed to, and after that no further force is given to the defendant if there are a million disclosures. In this respect it is a relatively weak factor for the defendant. In addition, there is a *Disclosure-In-Public-Forum* factor, which is intended to cover extensive non-specific disclosure. This is a stronger factor for the defendant. If the latter, stronger, factor applies, then the former does not. Thus, we must reason not only with respect to the factors that hold of a case, but also with the *relative strength* of the factors one to the other. A number of HYPO dimensions are Boolean and counted as present only for one end of the range (e.g. *Common-Employee-Sole-Developer*), and these map straightforwardly to a single CATO factor.

Some other dimensions found in HYPO and used as the basis for CATO factors are not related by a strength ordering relative to some measurable parameter. Most interesting is the HYPO dimension *Security-Measures-Adopted*, which has a range (from pro-defendant to pro-plaintiff):

- Minimal measures, Access to premises controlled, Restrictions on Entry by Visitors, Restrictions on Entry by Employees, Product Marked Confidential, Employee Trade Secrets Program Exists, Restrictions on hardcopy release, Employee non-disclosure agreements.

In CATO, this translates into several factors without reference to relative strength or position along some continuous parameter: *No- Security-Measures*, *Security- Measures*, *Outsider-Disclosures- Restricted*, and *Agreed-Not-To-Disclose*. The increasing support for the plaintiff's cause is indicated by additional factors, in effect, a cumulative reading, rather than by an ordering of the factors according to strength, and which associates them with a point along a HYPO dimension.

Although factors dominated thinking about this style of reasoning in AI and Law for some time (e.g. [? ? ?]), the need for dimensions was argued for in [?]. Chief amongst the reasons was that the key issue of the case may be about where along the dimension a factor falls and, having situated it, whether the factor favours the plaintiff or defendant. The classic *Pierson v Post* is an example: the dispute turns on when pursuit can be counted as justifying possession, for which different degrees of progress towards bodily possession need to be recognised.⁹ Contrast this with the representation based on factors in, for example [?], where the case is assigned the factor *caught*, and Post is then left without an argument¹⁰

We can further illustrate the issues using *Pierson v Post* as basis for further reasoning about the factors and schemes that apply in cases. We will take Post as the plaintiff, as in the original action. In [?], the only factors present are *notCaught* and *OpenLand*, both of which are pro-defendant. Thus any case found for the defendant where the incident had taken place on open land and the plaintiff had not caught the animal would serve as a precedent; the plaintiff had nothing on which to base the plaintiff's case, all additional factors in the chosen precedent strengthening the defendant's case. In fact the argument put forward for the plaintiff was that Post was sufficiently close to, and sufficiently certain of, taking bodily possession of the fox that it should be *counted as caught*.

Essentially this is an argument against the presence of a factor favouring the defendant, and an argument in favour of the presence of a factor favouring the plaintiff. What this means in ASPIC+ terms is that the status of the factors attributed to the case cease to be axioms and become instead premises requiring justification. What form might this justification take?

In *Pierson v Post*, the defendant's argument was in terms of particular authorities.¹¹ Tompkins, arguing for the defendant, cites Justinian, Fleca, and Bracton, all of whom seem to say that actual bodily possession is required, and Puttendorf and Barbeyrac, who seem to allow some latitude, but still require mortal wounding. Livingston, arguing for the plaintiff, claims that certain capture would also be enough for Barbeyrac, but also says that it should be so found in this case for the teleological purpose of encouraging the destruction of vermin. Neither of these lines of argument are case based or reasoning on the basis of the relationship of facts and factors per se, but make use of generic argumentation schemes such as *Argument from Authority* and *Sufficient Condition Scheme for Practical Reasoning* found in works such as [?].

⁹In brief the facts were these. Post was chasing a fox with horse and hounds and had cornered it when Pierson intervened and killed it with a fence pole. Post sued Pierson for taking his fox. On appeal, Pierson won on the grounds that only by mortally wounding or seizing the animal can one acquire possession of it, not simply by pursuing it.

¹⁰For a more recent attempt to represent *Pierson v Post* with factors see [?].

¹¹For a detailed reconstruction of the arguments in this case see [?].

4.2 Reasoning from Facts to Factors

There is, however, one argumentation scheme for case-based reasoning from facts to factors that can be articulated. Suppose an undecided case concerning capturing a wild animal is being argued where the plaintiff claims that the animal was *caught* on the basis of hot pursuit and inevitable capture. Moreover, we take *Pierson v Post* as a *precedent*. Yet, though the argument was put forward in the precedent that the plaintiff should be counted as having caught the animal, this was not sustained, but rather the precedent establishes that the factor *not-Caught* applies to the benefit of the defendant. To argue thus, even if unsuccessfully, requires that cases are not only represented in terms of summarising factors (e.g. *not-Caught*) but also to the underlying *facts* which, in effect, support the factor (e.g. *hot pursuit* and *inevitable capture*). In addition, it indicates that we must reason about *schemes* for relationships between case facts and factors; that is, in *Pierson v Post*, it is argued that *hot pursuit* and *inevitable capture* do not imply *caught*, whereas in our hypothetical case, it is argued that they do.

In HYPO there are some procedures that determine whether a dimension applies in terms of facts stored about the case, in effect, providing schemes to reason about the factor category in the case. These schemes are not provided with any justification and no source is given, but are simply hard-coded into the dimension frames. We could therefore expect our KB also to contain schemes relating factors to facts. To our previous notation, we add a binary relation *hasFact* between a case and a fact:

- *hasFact(case, fact)*; e.g. *hasFact(PiersonVPost, HotPursuit)*

We also add a corresponding sort for *facts* to \mathcal{L} .

We also introduce a five-place rule relation:

- *rule(rulename, fact, factor, justifier, justificationtype)*,
where *fact* is a fact, a *justifier* is some (or none) *judicial authority or case decision*, and the *justification type* is from *Authority, Definition, or Contention*.

Accordingly, we add to \mathcal{L} sorts for *rules, justifiers* and *justificationtypes*.

We have a sample of five rules, which are discussed further below.

- *rule(Rule1, NoBodilyPossession, NotCaught, Justinian, Authority)*
- *rule(Rule2, MortallyWounded, Caught, Puttendorf, Authority)*
- *rule(Rule3, NoPursuit, NotCaught, None, Definition)*
- *rule(Rule4, HotPursuit, NotCaught, None, Contention)*
- *rule(Rule5, HotPursuit, Caught, None, Contention)*

These legal rules will be included as facts in \mathcal{K}_p . The last argument of the *rule* predicate stands for the legal justification type of the rule. *Rule3* needs no further justification: it is true simply in virtue of the standard English meaning of the words. The other rules, however, do need some justification. In [?] three types of justification were suggested: authority, legal practice and precedent cases. Of the above *Rule1* and *Rule2* are justified by authorities, Justinian and Puttendorf respectively; if we accept Justinian (Puttendorf) as an authority we accept *Rule1* (*Rule2*). Rules *Rule4* and *Rule5* are not justified by authority or legal practice. At the time of *Pierson*, there were no precedent cases to provide justification, and the dispute was whether *Rule4* or *Rule5* should hold. Following the decision of *Pierson v Post*, we assume that *Rule4* holds, though Livingston argued for *Rule5*. This, however, remains an interpretation of the analyst: all talk of factors, and the attribution of factors to cases comes from the analyst, not the judge.

The analyst would therefore need to record this interpretation in the knowledge base based on a family of argumentation schemes:

CS5(*rulename, fact, factor, justifier, justificationtype, curr, prec*):

$$\frac{\text{hasFact}(\text{curr}, \text{fact}) \quad \text{rule}(\text{rulename}, \text{fact}, \text{factor}, \text{justifier}, \text{justificationtype})}{\text{hasFactor}(\text{curr}, \text{factor})}$$

Note that CS5 can be instantiated using both *Rule4* and *Rule5*, with the resulting arguments rebutting one another. Undercutters would be, for example, arguments to the effect that the justifier was not an acceptable authority. In *Pierson v Post Livingston* argues that Justinian cannot be considered a legitimate authority as he is too ancient, and society has changed too much since he was current.

A technical complication is that since now $hasFactor(curr, factor)$ expressions are defeasibly derivable, the method with predicate completion does not work any more. An alternative (taken from [?]) is to replace the predicate completion with the following defeasible rule and to give this rule lower priority than any other rule:

- $\Rightarrow \neg hasFactor(curr, factor)$

The unique-names and domain-closure axioms are retained and are now also formulated for rules and facts. Then any argument for an unnegated $hasFactor$ conclusion will strictly defeat any argument using this defeasible rule. With this method any argument for, for instance,

$$(*) \quad \forall factor. hasFactor(Mason, factor) \equiv \\ factor = F1 \vee F6 \vee factor = F15 \vee F16 \vee factor = F21$$

is justified if and only if the formulas $hasFactor(Mason, F1)$, $hasFactor(Mason, F6)$, $hasFactor(Mason, F15)$, $hasFactor(Mason, F16)$ and $hasFactor(Mason, F21)$ have justified arguments. This is since (*) is strictly derived from these five formulas and the unique-names and domain-closure axioms, so any defeater of an argument for (*) defeats an argument for any of these five formulas.

Reasoning about facts, factor sets, and factors requires this additional level of representation, which is reflected in differences between HYPO and CATO. HYPO stores case facts and calculates the applicability of dimensions on the basis of these. HYPO deterministically assigns dimensions on the basis of facts and does not support argument about them. CATO does not store case facts, and provides a descriptive rather than a computational characterisation of factors, relying on an analyst to assign them to cases; it does not support either argument or justification. We need make no further assumptions here about what additional information the knowledge bases contains, but there may be a list of possible facts along particular dimensions as in, e.g. [?] and [?], or, perhaps, some more complicated ontology representing the domain. For arguments from authority and teleological arguments for the presence of a factor, we rely on the standard argumentation schemes. As noted above, there are some similarities with the validity rules of [?], but whereas there the validity in question was a legal rule, here it is the qualification of facts as factors used in rules, which are used by the analyst and are not themselves legal rules. Note that these arguments for and against the presence of factors conflict through rebuttal, and so preferences, among sources, purposes, precedents, or a combination of these, may be required to determine which argument is accepted, in the light of a specific audience [?].

4.3 Reasoning about Factor Incompatibility

In section 3.4, we provided rules for arguing about substituting or cancelling factors in relation to the factor hierarchy. In this section, we discuss other ways to reason about factors. In CATO, when analysing cases the analyst is required to respect the fact that some pairs of factors are incompatible, so that the presence of one factor in a case provides an argument against the presence of another factor in that case. This is obvious in the case of clearly dichotomous factors such as *caught* and *notCaught*, but it is much more widespread than this in CATO. In [?] each factor has a textual explanation of when it does and does not apply. Often the latter includes circumstances where some other factor does apply. In [?] we have:

F20 InfoKnownToCompetitors (d)

Description: Plaintiff's information was known to competitors.

This factor shows that plaintiff's information was known in the industry or available from sources outside plaintiff's business.

The factor applies if: The information plaintiff claims as its trade secret is general knowledge in the industry or trade.

The factor does not apply if: Competitor's knowledge of plaintiff's information results solely from disclosures made by plaintiff. (In this situation, *F10* applies.) Or if the information could

be compiled from publicly available sources, but there was no evidence that competitors had actually done so. (In this situation, $F2_4$ applies.)

$F10$ is *Secrets-Disclosed-Outsiders* and $F2_4$ is *Info-Obtainable-Elsewhere*. Thus $F10$ and $F2_4$ are incompatible with $F20$ and with one another.

To express the relationship of exclusion, we need an additional predicate:

- $excludes(factor1, factor2)$

$excludes$ is a symmetric relation, so we add the following axiom to \mathcal{K}_n :

$$21. \forall factor1, factor2. excludes(factor1, factor2) \implies excludes(factor2, factor1)$$

Now if one side argues that $F2$ is present, but the other believes that $F20$ more appropriately describes the facts, it is important to ensure that both are not taken as present. This gives rise to an additional axiom:

$$22. \forall case, factor1, factor2. hasFactor(case, factor1) \wedge excludes(factor1, factor2) \implies \neg hasFactor(case, factor2)$$

There are other argumentation schemes that can be used to establish that a factor is or is not in a case, such as those based on authority or purpose. Also note that if there is an argument for $factor1$ excluding $factor2$, there will also be a rebutting argument for $factor2$ excluding $factor1$. Which will be accepted will depend on which is supported by the stronger arguments.

In section 4.1, we recognised that factors may favour their party to different extents. In light of this, we need to reconsider our notions of cancellation and substitution. Arguments based on substitution and cancellation were used to undercut arguments distinguishing cases. Given that undercutters always win, these are powerful arguments. But suppose the factors in question were $F20$ and $F2_4$, as defined above. It is clear from the description that $F20$ is intended to be more pro-defendant than $F2_4$, for $F20$ represents an actual rather than a merely possible state of affairs. Thus if we have $F2_4$ in a *curr* and $F20$ in a *prec*, there is no problem in substitution: when considering the two under the common abstract factor the plaintiff's case is stronger, because the factor for the defendant is weaker in the *curr*. But if plaintiff attempts to argue that $F2_4$ in a *prec* substitutes for $F20$ in a *curr*, undercutting an instance of U1.1, the issue is less clear. The defendant can at least argue that $F2_4$ is not strong enough to be substituted for $F20$.

This has been handled in different ways in different applications: CATO indicated different degrees of influence by distinguishing *thin* and *fat* links in the factor hierarchy. IBP [?], which developed from CATO, introduced the idea of knock-out factors, which could be neither substituted for nor cancelled - indeed were entirely decisive with regards to a particular issue. The most quantitative approach can be found in [?], in which a limited number of dimensions, essentially corresponding to IBP issues, were each considered to have twenty slots, ten pro-plaintiff and ten pro-defendant, and every factor was assigned one of these slots. This enabled the difference in importance as well as the ordering to be considered. The positions were then mapped into weights in several ways, so that differences in the relative importance of the dimensions could also be considered. These issues require us to reason with dimensions, which is the topic of the next section.

4.4 Reasoning along Dimensions

Turning to dimensions, we want to introduce arguments about *substitution* and *cancellation*. As substitution and cancellation were defined using axioms and always held of factors sharing the same abstract factor, we cannot attack statements that one factor substitutes or cancels another. While we might recast the original axioms as defeasible rules, we prefer to leave the language established originally untouched as far as possible, instead proposing undercutters of the rules U1.1.1 and U1.1.2.

First we introduce some additional language and sorts to relate to dimensions. We have new sorts for dimensions and numbers and we add a function symbol *Position* which takes two arguments: a dimension name and a factor name, and returns the position of that dimension occupied by the factor. How the position is expressed needs some serious consideration, but here we simply adopt the approach of [?] and use a

number between ten and minus ten. Ten is the extreme plaintiff position, minus ten is the extreme defendant position and zero indicates that the point is in the centre of the dimension, and so could be used to indicated the absence of any factor on the dimension.

- $Position(dimension, factor) = position$

For example:

- $Position(OutsideDisclosure, F20) = -6$
- $Position(OutsideDisclosure, F24) = -4$

Using the *Position* function symbol we now introduce the notion of one factor being stronger than another. For cancellation, we are interested in comparing the pro-plaintiff degree of a factor with the pro-defendant degree of another factor. Therefore we will be interested in the *absolute* value of the positions along the dimension. Accordingly, the following formula is added to \mathcal{K}_n .

23. $\forall factor1, factor2, dimension, degree.$
 $stronger(factor1, factor2, dimension, degree) \equiv$
 $(degree = |Position(dimension, factor1)| - |Position(dimension, factor2)|$
 $\wedge degree > 0)$

Definitions of the arithmetic notation can be added to \mathcal{K}_n as it was done above in section 3.3 for set-theoretic notation.

Now we can undercut arguments made using U1.1.1 and U1.1.2.

U7(*curr, prec, f₁, f₂, p, d, dimension, degree*):

$$\frac{stronger(f_1, f_2, dimension, degree)}{\neg \mathbf{U1.1.1}(curr, prec, f_1, f_2, p, d)}$$

U8(*curr, prec, f₁, f₂, p, d, degree*):

$$\frac{stronger(f_1, f_2, dimension, degree)}{\neg \mathbf{U1.1.2}(curr, prec, f_1, f_2, p, d)}$$

This, however, means that any amount of additional strength, however small, is sufficient to prevent substitution and cancellation: that is, only *a fortiori* arguments will succeed. This rather conservative point of view may be what we want, but we may equally wish to allow the possibility of a small difference being discounted. We therefore need a notion of a significant difference. We design the predicate to allow that this may vary according to the dimension under consideration.

- $significantDifference(dimension, threshold)$

This allows for an undercutter to arguments made using **U7**.

U7.1(*curr, prec, f₁, f₂, p, d, dimension, threshold*):

$$\frac{significantDifference(dimension, threshold)}{\frac{degree < threshold}{\neg \mathbf{U7}(curr, prec, f_1, f_2, p, d, dimension, degree)}}$$

Suppose we are considering the number of disclosures. It would be possible to regard each different number of disclosures as a factor on this dimension. Now the difference between 3 and 4 disclosures is not significant, but the difference between 3 and 40 is. So suppose the threshold is 20. Then while 4 disclosures is stronger for *d* than 3 disclosures the difference is not significant, and so we would not want to differentiate between cases with these different numbers of disclosures. We need, however, to be able to differentiate between 3 and 23 (or more). But while 20 disclosures is good for this dimension, 20 days

would not be significant for the dimension decreased development time. There the threshold might be 60 days. Thus the threshold is relative to the dimension, and the degree of difference needs to be compared to the relevant threshold. This perhaps holds true even when the numbers are normalised: if factors are assigned to 10 slots as in [?], for some dimensions we might regard any gap as significant, while for others we would need two or three slots between factors to make the difference.

Where do the thresholds come from? In most cases they will need to be simply assumed or be determined by some feel for legal practice or even rely on personal preference [?]. For example, we can try to rescue *Bryce* as a precedent for *Mason* by setting the threshold above the degree of *F18*, so undercutting *A5'*. Disagreement here is quite natural. It would be possible to use a case based argument to set bounds on the threshold, provided one could find a pair of precedent cases in which, using the argumentation schemes defined here, it was possible to show that a substitution for, or cancellation of, a stronger factor had been made successfully. This is, in itself, quite a complex argument, and we will not attempt to reduce it to an argumentation scheme here, but rather take the threshold as given.

5 Discussion

By articulating the process of reasoning with precedent cases in this way, we can see it as a sequence of stages in a dialogue between **P**laintiff and **D**efendant, which are as follows, where every option is available:

1. **P**: Assert that the decision should be in favour of the plaintiff since factors favouring the plaintiff are preferred to factors favouring the defendant;
 - (a) **D**: Cite additional points in favour of the defendant
 - (b) **P**: Substitute for, dismiss, or cancel these additional strengths
 - (c) **D**: Dispute strength of substituting or cancelling factors
2. **P**: Identify a precedent case that justifies a preference applicable to the current case;
 - (a) **D**: Cite additional points in favouring the plaintiff in the precedent
 - (b) **P**: Substitute for, dismiss, or cancel these additional strengths
 - (c) **D**: Dispute strength of substituting or cancelling factors
 - (d) **D**: Identify a precedent case that justifies a preference for the defendant applicable to the current case
 - i. **P**: Cite additional points in favouring the defendant in the precedent:
 - ii. **D**: Substitute for, dismiss, or cancel these additional strengths
 - iii. **P**: Dispute strength of substituting or cancelling factors
3. **D**: Dispute which factors are present in the current case
 - (a) **P**: Defend original factors

Different systems will support more or fewer of these stages. At one extreme we have a neural network style system such as that described in [?] in which the system acts as a black box taking factors (or facts) as an input and expressing a preference based on its internalisation of the set of precedents. Such a system supports only step 1. *CATO*, from which our discussion began, supports the identification of the preference in 2, the distinguishing moves in 1a and 2a (although it does not discriminate between them), and the counter example move of 2d. *CATO* also supports the downplaying of 1b and 2b, but does not distinguish between substitution and cancellation. *HYPO* links facts and dimensions, and so can explain 3, but not support argument about it. Hypothetical arguments in *HYPO* were intended to explore the issues raised in 1c, 2c, and 2d(iii), but this aspect of *HYPO* was never fully developed in [?]. These considerations are also used internally in the most advanced version of Chorley's *AGATHA* [?], although the resulting arguments are not transparent to the user.

Our representation in terms of *ASPIC+* identifies the underlying knowledge base required by each stage. Given such knowledge in the KB, the specification of the argumentation schemes in this paper would

permit straightforward implementation, using a defeasible reasoner to instantiate the schemes from the KB, identifying the attack relations, and then evaluating them as in a Dungian argumentation framework. The first of these steps, identifying the arguments, is achieved for stages 1, 1a, 1b 2, 2a and 2b, using Prolog as the defeasible reasoner in a program described in [?].

Another benefit from representing these arguments in terms of ASPIC+ is that we can regard cases described under factors as but one source of arguments. At the top level, stage 1, there may be arguments for the defendant rebutting our case based argument for the plaintiff and these arguments may be based on cases, authority, purpose, or whatever other kind of argument our opponent wishes to advance. Similarly, the premises of our arguments often require other, generic, argumentation schemes, such as authority and purpose, to justify them. By providing a framework in which all kinds of argument can be represented equally, we can readily provide a framework in which reasoning of many different kinds can be deployed. Note that this is done without recasting the various distinctive case based aspects of CATO style arguments uniformly as ordinary rules, as was the case in e.g. [?].

A final important insight is gained by recognising that the above indicates at which points choice is possible, and at which points the judgement is constrained. Let us relate this to the steps above. At step 1 we may get arguments, constructed with a variety of schemes, for and against deciding for the plaintiff, which conflict through rebuttal and so can be decided through preferences. The attack of 1a, however, cannot be rejected on the grounds of preference, but can only be defeated by 1b, which in turn can only be defeated by an argument from 1c. Arguments in stage 1c itself, however, may be resolved on grounds of preference. Similarly although the rebuttals arising at 2d may be decided by preferences, 2a can only be defended by 2b, and 2b by 2c, at which stage preferences may be used to resolve competing arguments. When considering 2d, only at 2d(iii) do preferences play a role. Thus although we may think of case based reasoning as involving a choice between the plaintiff and the defendant arguments, in fact, choice operates at a number of quite specific, fine-grained points in the debate.

6 Concluding Remarks

In this paper, we have clarified a range of aspects of legal case-based reasoning with factors using formal defeasible arguments modelled within the ASPIC+ framework. The choice of ASPIC+ has made it possible to prove consistency and closure results for our formalisation by exploiting the metatheory of ASPIC+. Our formalisation has also illustrated the potential of the ASPIC+ framework for formalising reasoning with argumentation schemes. The schemes reconstructing CATO are proposed as definitive, but those in Section 4 are rather more tentative and will require further work to validate and refine them. Still, we feel that we have made a contribution, given that this aspect of legal reasoning has not yet been investigated with any real success. In future work, we will look to further extend this approach, integrating additional aspects and examples of legal case based reasoning, such as the *issues* of IBP. In particular, we will consider how to argue comparatively about precedents to find the most on-point cases using the claim lattice of HYPO and CATO, which we would reconstruct as a tree of arguments in attack relations.

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