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Abstract

By using a nonmonotonic semantics one tries to extract more information from a theory than would be possible by classical means. Such a nonmonotonic semantics is called informative if it satisfies both supraclassicality (the nonmonotonic models are a subset of the set of classical models) and consistency preservation (nonmonotonic models exist whenever the theory is consistent). Most nonmonotonic semantics, however, satisfy supraclassicality but lack consistency preservation. In such cases we propose to apply theory revision in order to construct an informative (revised) semantics.

We present some postulates for such nonmonotonic theory revision and we will show that, unlike in classical theory revision, nonmonotonic theories have to be *expanded* instead of contracted in order to give them a satisfactory meaning. Finally, we state some conditions on the nonmonotonic semantics to be satisfied in order revise theories successfully.

1 Introduction and Motivation

One of the primary advantages nonmonotonic reasoning should have above classical reasoning is to allow one to draw stronger conclusions than can be obtained by classical means, i.e. the semantics should be *more informative* than the classical semantics. *More informative* here means that

- nonmonotonic reasoning, in general, should be *stronger* than classical reasoning. That is, every conclusion obtained by classical means should also be obtainable

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by nonmonotonic reasoning, i.e., nonmonotonic reasoning should be *supraclassical*;

- on the other hand, nonmonotonic reasoning should not collapse, if it is still possible to draw conclusions by classical means. That is, the (set of) conclusions obtained by nonmonotonic reasoning should be as least as *informative* as the conclusions obtained by classical reasoning.

  So nonmonotonic reasoning should not run into inconsistencies whenever the theory is classically consistent. This principle is also known as *consistency preservation.*

Comparing nonmonotonic reasoning with classical reasoning applied to the same theory T implies that we distinguish a nonmonotonic interpretation of T and a classical interpretation of T.

1.1 Classical and non-classical readings of a theory

It may not be immediately clear what we mean by the classical reading and the nonmonotonic models of a theory T. In a forthcoming paper ([14]) we will give a general treatment of nonmonotonic vss. first-order interpretations of a given theory. Here, we give some examples.
In our view, a nonmonotonic semantics gives rise to the selection of certain acceptable or preferred models of the theory instead of considering the total class of ordinary (classical) models of the theory.
This idea is most prominently present in the preferential model semantics developed by Shoham [12] and further analysed by Makinson [5, 6] and Kraus, Lehmann and Magidor [4]. Given a preference relation between models of a first order theory T, instead of taking into account every model of a first-order theory, only the most preferred models are chosen as the (nonmonotonic) models of T. Circumscription is a special case of such preferential semantics.
There are also approaches in which the model selection is guided by special interpretations of classical connectives, such as, for example, in logic programming. Here, the implication and negation connectives can be given a special meaning in order to select the nonmonotonic models of the theory. Since in these semantics every nonmonotonic model also is a classical model, i.e. respects the standard meaning of these connectives, these semantics again are supraclassical. Thus, we propose to use the ordinary meaning of the connectives to obtain the classical meaning of theories T that assign a non-standard interpretation to some of the connectives.
Finally, there is a class of nonmonotonic approaches, where a classical language is *extended* by the introduction of new symbols that have a special meaning. These symbols constitute the syntactical guides that may help us in finding the acceptable models among the set of all models. Here the problem to distinguish between a classical and non-classical reading is more involved.

In such cases we propose to translate these special symbols or formulas containing these special symbols back into first-order formulas. For example, in a default theory $\Delta = (W, D)$ over a first-order language $\mathcal{L}$, we translate every default rule

$$\delta = \frac{\alpha \,;\, M\beta_1, \ldots, M\beta_n}{\gamma}$$

into a first-order formula

$$\delta_{f.o} = \alpha \wedge \beta_1 \wedge \ldots \wedge \beta_n \rightarrow \gamma$$

Then the classical reading of Δ is given by the theory

$$\Delta_{f.o} = W \cup \{\delta_{f.o} \mid \delta \in D\}$$

It is not difficult to see that every model of an extension E of Δ also is a classical model of $\Delta_{f.o}$.
Likewise, if T is an auto-epistemic theory, its classical interpretation is given by the models of the theory $T_{f.o}$ derived from T by substituting (recursively) every formula of the form $L\phi$ by the formula ϕ. The nonmonotonic models of T are the models of the objective part of the auto-epistemic extensions of T. Again, by the conditions that every auto-epistemic extension E of T has to satisfy, it can be seen that every model of E is also a classical model of $T_{f.o}$.

1.2 The lack of consistency-preservation

While almost all nonmonotonic logics are supraclassical (cf. [6]), most of them do not satisfy consistency preservation. For example, the following well-known formalisms: *default logic* (with the exception of normal default logic), *auto-epistemic logic*, *nonmonotonic semantics of logic programming* and *preferential entailment semantics* all lack consistency preservation.

We consider this an unfortunate state of affairs. In particular, we consider the principle of consistency preservation as extremely useful in such applications as debugging and diagnosis. Here, we use nonmonotonic reasoning to draw conclusions about the expected behaviour of a system if everything goes well, i.e. the (normality) assumptions are not violated. But we would also like to draw conclusions in exceptional cases, where the normality assumptions do not hold and 'ordinary' nonmonotonic reasoning fails.
The reason for this failure is that in most nonmonotonic semantics, especially those used in logic programming, there is no possibility to recover from a conflict, if some assumptions made in reasoning nonmonotonically turn out to be responsible for violating a constraint.
Let us give a motivating example.

Example 1.1

Suppose you are at the airport knowing that

1. you are treated as a class A passenger iff you receive a special pass-through card,
2. everyone receiving a special pass-through card has direct access to the gates via a special port.
3. but, if there is no reason to receive a special card then you will pass through the normal port and
4. if you have access through the normal port and it can be assumed that you are not a class A passenger then you will be checked.
5. Every VIP is treated as a class A passenger and finally,
6. it happens that you are not checked.

The following program describes this situation:

$$\begin{array}{rcl} P: \ class\_A\_passenger & \leftarrow & receive\_special\_card. \\ receive\_special\_card & \leftarrow & class\_A\_passenger. \\ direct\_access\_by\_special\_port & \leftarrow & receive\_special\_card. \\ access\_by\_normal\_port & \leftarrow & \neg receive\_special\_card. \\ checked & \leftarrow & access\_by\_normal\_port, \neg class\_A\_passenger. \\ class\_\_A\_passenger & \leftarrow & VIP. \\ \bot & \leftarrow & checked. \end{array}$$

Note that P is classically consistent: it has four classical models where the first two models are

$$M_1 = \{cAp, rsc, dasp, \neg anp, \neg ch, \neg VIP\}$$

and

$$M_2 = \{cAp, rsc, dasp, \neg anp, \neg ch, VIP\},$$

while M_3 and M_4 are obtained by making anp true in M_1 and M_2, respectively.
If you would use the stable model semantics as your intended semantics, however, P is nonmonotonically inconsistent: $Stable(P) = \emptyset$. The reason why, should be clear: there is no reason to assume that you should receive a special card, but this assumption is directly responsible for the violation of the constraint 6.
So the program is not classically inconsistent, but the problem is that none of the classical models is a stable model of the program. This means that stability as a criterion to select acceptable models from the set of classical models fails and that we have to select other models.

The problem is, which models we choose. Clearly, if you add the fact

$$receive\_special\_card \leftarrow$$

or the fact

$$VIP \leftarrow$$

to P, both M_1 and M_2 will occur as a stable model of an expanded version of P. But it is also possible to add the rule

$$class\_A\_passenger \leftarrow \neg access\_by\_normal\_port$$

to P and obtain M_1 as a stable model of the resulting expansion.
Note that in the first and the last case, we add some information that is classically derivable from the program. ■

This example suggests that consistency preservation may be obtained by changing our program P to a related theory P', and to use the intended semantics of P' to give P a suitable meaning. That is, we may find intended models of the original theory by applying *theory revision.*
Basically, this is the idea applied in the dominant classical AGM theory revision framework (see [2]). Here, a classically inconsistent theory T is transformed into a classical consistent theory T' and the models of T' are used to give a meaning to T.
We will generalize this idea to a *classically consistent, but nonmonotonically inconsistent* theory T. We will apply theory revision to T and transform T to another theory T' that does have intended models. Then we use the intended models of T' as the intended models for T. In this way we will construct a semantics that is consistency-preserving.

The problem then is how to characterize *suitable* theory transformations. We will formulate some fairly simple postulates for nonmonotonic theory revision and then we will show that, unlike classical theory revision, nonmonotonic theory revision has to be performed by *expanding* the original theory instead of contracting it.

2 Restoring consistency preservation by theory revision

As stated in the introduction, we would like to have a nonmonotonic logic to satisfy the principles of supraclassicality and consistency preservation.
To state these principles in a more precise way, we will assume that, given a not necessarily closed theory T specified in some first-order language $\mathcal{L}$, our nonmonotonic logic is characterized by a set $Sem(T)$ of *nonmonotonic* models for T. We will denote the set of *classical* models of T by $Mod(T)$. Then both principles can be formulated as follows:

1. **Supra**(classicality):
   For every theory T, $Sem(T) \subseteq Mod(T)$;

2. **Cons**(istency preservation):
   For every theory T, $Sem(T) \neq \emptyset$, whenever $Mod(T) \neq \emptyset$.

It has been observed that almost every nonmonotonic semantics satisfies **Supra**, while few satisfy **Cons** ([6]). So let us assume that we have a theory T and a semantics Sem such that **Supra** is satisfied, but **Cons** is not. We will look for a semantics Sem^* revising Sem that satisfies both principles.

The problem we are confronted with closely resembles the problem of *theory revision* for classical theories: there we are forced to revise our interpretation of a theory T if T turns out to be *classically inconsistent*, i.e. $Mod(T) = \emptyset$, while here we have to revise our nonmonotonic interpretation of the theory if $Sem(T) = \emptyset$, while still $Mod(T)$ may be nonempty.
In the well-known AGM-approach to theory revision (cf. [2]), revision is accomplished by *theory transformation*: the current inconsistent theory T is replaced by a transformation $R(T)$ of T and the (classical) meaning of $R(T)$ is used to provide a suitable meaning for T.
Since we are aiming at restoring consistency preservation, we will not deal with the problem what to do if $Sem(T) = Mod(T) = \emptyset$, but we will also apply this idea of theory revision by theory transformation. That is, if $Sem(T) = \emptyset$, we propose to derive the proposed meaning $Sem^*(T)$ of T by

(i) transforming T to some theory $T' = R(T)$,

(ii) applying the original semantics Sem to T', and

(iii) requiring that $Sem^*(T) = Sem(R(T))$.

Since we want to deal with nonmonotonic revision in classically consistent theories, we will assume that there is some class $\mathcal{T}$of classically consistent theories, i.e. for every $T \in \mathcal{T}$, $Mod(T) \neq \emptyset$. We call a pair $(\mathcal{T}, Sem)$ a *nonmonotonic semantics*. The nonmonotonic semantics we want to have are *informative* semantics:

Definition 2.1 (Informative semantics)
A nonmonotonic semantics $(\mathcal{T}, Sem)$ *is called* informative *if it satisfies both* **Supra** *and* **Cons**.

A theory transformation R is a computable mapping from $\mathcal{T}$ to $\mathcal{T}$. We would like to know which transformations are suitable and which are not. In order to give such a characterization, we will present some postulates for the triple $(\mathcal{T}, Sem, R)$, called a *revision framework*, where $(\mathcal{T}, Sem)$ is a nonmonotonic semantics and R is a theory-transformation $R : \mathcal{T} \rightarrow \mathcal{T}$. Furthermore, we assume that Sem is supraclassical with respect to $\mathcal{T}$.

We propose to use the following simple postulates:

P1. $Sem(R(T)) \neq \emptyset$ whenever $Mod(T) \neq \emptyset$.

This postulate specifies that revision should be *successful*: we should find a transformation R such that $Sem(T)$ exists if T is consistent.

P2. $R(T) = T$, whenever $Sem(T) \neq \emptyset$.

We should be careful in extending our semantics: only in those cases in which Sem does not provide a meaning for T, it is allowed to change T.

P3. $Mod(R(T)) = Mod(T)$.

This postulate stipulates that theory transformation should be classically neutral: we should not change the class of models from which the subset of models has to be chosen that we (nonmonotonically) prefer.

Note that the revised semantics Sem^* based on Sem is meant to satisfy both **Supra** and **Cons**.

Definition 2.2 (Successful revision frameworks)
A revision framework $(\mathcal{T}, Sem, R)$ *is called* successful *if*

1. *it satisfies the postulates P1-P3 and*
2. *the revised nonmonotonic semantics* $(\mathcal{T}, Sem^*)$*, where* $Sem^*(T) = Sem(R(T))$ *is informative.*

Let us first check that indeed, if $(\mathcal{T}, Sem, R)$ satisfies the postulates **P1-3** and **Supra**, then Sem^* will satisfy both **Supra** and **Cons**.

Observation 2.3 (the postulates guarantee success)
If

1. $(\mathcal{T}, Sem, R)$ *satisfies P1-P3 and*

2. Sem *satisfies* **Supra**

then $(\mathcal{T}, Sem^*)$ *is informative.*

PROOF By P1, it immediately follows that Sem^* satisfies **Cons**.
Since Sem satisfies **Supra**, we have $Sem^*(T) = Sem(R(T)) \subseteq Mod(R(T))$. Hence, by P3, $Sem^*(T) \subseteq Mod(T)$. So Sem^* satisfies **Supra**, too. ∎

Given these postulates P1-P3, we would like to investigate the following problems:

1. What can we say about the nature of revision functions for nonmonotonic theories and how do they compare to revision functions used in classical theory revision?
2. What will be needed for *minimal* revision?
3. Which conditions have to be satisfied by a semantics $(\mathcal{T}, Sem)$ in order to find a successful revision framework $(\mathcal{T}, Sem, R)$?

We will try to formulate some general answers to this question.

3 What revision functions should be used

In the standard (AGM-inspired) theory revision literature ([2]), *retraction* is the only appropriate theory revision operator to give a suitable meaning to an inconsistent theory: From the current inconsistent theory T some parts are retracted and the (consistent) remaining part T' is used to give T its meaning.
It turns out that revision by retraction, at least for a large part of nonmonotonic semantics, is *not* suitable[1].

3.1 Reasonable semantics: Weak Confirmation of Evidence

Given that a nonmonotonic semantics $(\mathcal{T}, Sem)$ obeys the principle of supra-classicality, what should be reasonable to expect from it? Let us define the following consequence operator $\mid\!\sim_{Sem}$:

$$T \mid\!\sim_{Sem} x \quad \text{iff} \quad \exists M \in Sem(T) \text{ s.t. } M \models x$$

So $T \mid\!\sim_{Sem} x$ holds iff according to an acceptable model M of T, x is true. Then the least thing we might expect is that there is still some acceptable model for the theory

[1] In fact, we are not able to come up with a reasonable nonmonotonic semantics for which it can be proven that retraction is an option in nonmonotonic theory revision.

T' obtained from T and x, that is, nonmonotonic reasoning should not collapse just by adding a (brave) nonmonotonic consequence of the theory.
Slightly generalizing, we introduce the following *Weak Confirmation of Evidence* (abbreviated by **WCE**)[2] principle:

Definition 3.1 (WCE)
For every $T \in \mathcal{T}$ and $\Phi \subseteq WFF(\mathcal{L})$, if $T \mid\!\sim_{Sem} \Phi$, then there exists some $\Psi \subseteq WFF(\mathcal{L})$, such that $T + \Phi \mid\!\sim_{Sem} \Psi$.

As far as we know, this principle holds for every nonmonotonic semantics currently known.

Remark. Note that this principle can be seen as a weakening of a brave form of *cautious monotony*:

$$T \mid\!\sim_{Sem} x, T \mid\!\sim_{Sem} y \text{ implies } T + x \mid\!\sim_{Sem} y$$

■

The following proposition is an almost direct consequence of the definition of $\mid\!\sim_{Sem}$ and is useful in proving properties of WCE:

Proposition 3.2 *$T \mid\!\sim_{Sem} \Phi$ iff $Sem(T) \cap Mod(\Phi) \neq \emptyset$.*

PROOF By definition $T \mid\!\sim_{Sem} \Phi$ iff there is an $M \in Sem(T)$ such that $M \models \Phi$ iff there is an $M \in Sem(T)$ such that $M \in Mod(\Phi)$ iff $M \in Sem(T) \cap Mod(\Phi)$. ■

It is interesting to note that **WCE** is satisfied by every informative semantics:

Proposition 3.3
If Sem is an informative semantics, then Sem satisfies **WCE**.

PROOF Suppose that $T \mid\!\sim_{Sem} \Phi$ for some set Φ. Since **Supra** is satisfied, this implies that there is a model $M \in Mod(T)$ satisfying Φ. Therefore, $Mod(T) \cap Mod(\Phi) = Mod(T + \Phi) \neq \emptyset$. Then by **Cons**, it follows that $Sem(T + \Phi) \neq \emptyset$. Hence, there exists a Ψ such that $T + \Phi \mid\!\sim_{Sem} \Psi$ and, therefore, **WCE** is satisfied. ■

Since **Cons** is not implied by **Supra+WCE**, so **WCE** is a weaker property than **Cons** in the presence of **Supra**.

[2]This principle is a weaker variant of the Confirmation of Evidence principle, introduced by Reiter. We are grateful to Wiktor Marek for giving the reference.

3.2 Retraction is not suitable for revision

Note that in the AGM approach, whenever a theory is classically inconsistent we have to apply theory contraction to give it a meaning. If the theory is consistent, we can leave the theory unchanged. So let us define the following notion of a *pure* retraction function:

Definition 3.4 (Pure retraction)
Let $\mathcal{T}$ be a class of theories and $R : \mathcal{T} \to \mathcal{T}$ be a theory-transformation.
Then R is a pure *retraction function iff $\forall T \in \mathcal{T}.\ R(T) \subseteq T$ and there exists some $T \in \mathcal{T}$ such that $R(T) \neq T$.*

It might be that pure retraction functions are not useful for nonmonotonic theory revision, while for some (but not all) theories a mixture of adding some information and retraction is more appropriate. Therefore, we will allow for such cases and define the following notion of *weak* retraction functions:

Definition 3.5 (Weak retraction)
Let $\mathcal{T}$ be a class of theories and $R : \mathcal{T} \to \mathcal{T}$ be a theory-transformation.
Then R is a weak *retraction function iff $\exists T \in \mathcal{T}.\ R(T) \subset T$ and $T \neq R(T)$.*

Note that the class of pure expansion functions is contained in the class of weak expansion functions.
We will now prove that even weak retraction is not a suitable option for nonmonotonic theory-revision if the postulates stated above are satisfied and the nonmonotonic semantics satisfies **WCE** and **Supra**.

Theorem 3.6
Let $(\mathcal{T}, Sem)$ be a nonmonotonic semantics satisfying **Supra+WCE**, *but not* **Cons**. *Then the revision framework $(\mathcal{T}, Sem, R)$ cannot be successful if R is a weak retraction function.*

PROOF Suppose, on the contrary, that R is a weak retraction function in the successful framework $(\mathcal{T}, Sem, R)$, where $(\mathcal{T}, Sem)$ satisfies **Supra+WCE**.
Since R is a weak contraction function, there is a theory $T \in \mathcal{T}$ such that $R(T) = T' \subset T$ and $T \neq T'$, while $Mod(T) \neq \emptyset$.
Since the revision framework is assumed to be successful, the postulates P1-P3 have to be satisfied. By P2, we have

$$Sem(T) = \emptyset$$

and by P1, it follows that $Sem(R(T)) \neq \emptyset$. Hence,

$$Sem(T') = Sem(R(T)) \neq \emptyset.$$

So, let M be an arbitrary model in $Sem(T')$.

By P3 and **Supra**, we have

$$\emptyset \subset Sem(T') = Sem(R(T)) \subseteq Mod(T).$$

Hence, $Sem(T') \cap Mod(T) \neq \emptyset$, so by Proposition 3.2

$$T' \mid\!\sim_{Sem} T$$

and therefore, by **WCE**,

$$T + T' \mid\!\sim_{Sem} \Psi$$

for some Ψ. But then, since $T' \subset T$, it follows that

$$Sem(T) = Sem(T + T') \neq \emptyset,$$

contradicting the fact that $Sem(T) = \emptyset$. Therefore, R cannot be a weak retraction function if the framework is successful. ■

Since pure and weak retraction functions now can be excluded, we can take a look at *mixed transformations* and *pure expansion functions.*

Definition 3.7 (Pure expansion)
A pure expansion *function is a theory transformation R such that for all* $T \in \mathcal{T}$, $R(T) \supseteq T$.

Definition 3.8 (Mixed transformation)
We call a revision function R a mixed *revision function if for some* $T \in \mathcal{T}$, $R(T) - T \neq \emptyset$ *and* $T - R(T) \neq \emptyset$.

It turns out that the class of mixed transformations is obsolete in the following sense: we can show that the class of mixed functions can be represented by the class of pure expansion functions, i.e. for every mixed revision function R satisfying the postulates, there exists a pure expansion function R' such that $R'(T) = R(T) + T$ and R' also satisfies the postulates.

Lemma 3.9
Let $(\mathcal{T}, Sem, R)$ *be a successful revision framework, where* Sem *satisfies* **Supra** *and* **WCE** *(but not* **Cons***) and* R *is a mixed transformation.*
Then the revision framework $(\mathcal{T}, Sem, R')$*, where* R' *is a pure expansion function defined as*[3] $R'(T) = R(T) + T$*, is also successful.*

PROOF Let the pure expansion function R' be defined as $R'(T) = R(T) + T$. We have to prove that $(\mathcal{T}, Sem, R')$ satisfies the postulates P1-P3.

[3]Note that $R'(T)$ and $R(T)$ are classically equivalent, i.e. $Mod(R'(T)) = Mod(R(T))$

Since $R(T) = T$ whenever $Sem(T) \neq \emptyset$, it follows immediately that R' satisfies Postulate P2.
Since

$$Mod(R'(T)) = Mod(R(T) + T) = Mod(R(T)) \cap Mod(T)$$

and, by P3,

$$Mod(R(T)) = Mod(T),$$

it follows that

$$Mod(R'(T)) = Mod(T),$$

hence, R' satisfies Postulate P3.

Finally, we have to show that R' is successful, i.e. P1 is satisfied.
Assume that $Mod(T) \neq \emptyset$. Since postulate P1 is satisfied for R, $Sem(R(T)) \neq \emptyset$. So take a model $M \in Sem(R(T))$.
By **Supra**, it follows that $M \in Mod(R(T))$ and since R satisfies P3, $M \in Mod(T)$.
Hence,

$$M \in Sem(R(T)) \cap Mod(T).$$

But then, by Proposition 3.2 and **WCE** it follows that

$$Sem(R(T) + T) = Sem(R'(T)) \neq \emptyset,$$

so P1 is satisfied for R'. ∎

3.3 Minimal revision and pure expansion

While in the previous section we showed that revision functions can be represented by pure expansion functions, in this section we will show that in order to perform minimal revision, only pure expansion functions should be applied.

Definition 3.10 (Minimal revision)
We say that $(\mathcal{T}, Sem, R)$ is a minimal *revision system if R satisfies the postulates P1-P3 and the following minimality postulate:*

> *P4 For every $R' \neq R$ satisfying the postulates P1-P3, if $(R(T) \ominus T) \subseteq (R'(T) \ominus T)$ then $R(T) = R'(T)$.*

Here, $\ominus$ is the symmetrical set-difference operator.

This postulate expresses that successful revisions should minimize the additions to and retractions from the original theory.
As an almost direct consequence of the preceding lemma we have:

Theorem 3.11
If $(\mathcal{T}, Sem, R)$ is a minimal revision system satisfying the postulates P1-P4, then R is a pure expansion function.

Example 3.12
Let $T = \{Lp\}$ be an auto-epistemic theory. This theory is classically consistent, having a classical model $M = \{p\}$. T, however, does not have an auto-epistemic extension. Consider the revision $T' = R(T) = T + \{p\}$. It satisfies the postulates and now M is the model of the objective part of T''s only auto-epistemic extension.

4 Successful revision frameworks

In the preceding sections we assumed that the nonmonotonic semantics $(\mathcal{T}, Sem)$ satisfies **Supra** and **WCE** and then we proved some properties of the theory transformation R and the resulting revised semantics.
Let us now turn to the other side and let us assume that we have a successful revision framework $(\mathcal{T}, Sem, R)$. Then we would like to know which properties we could derive for Sem and R to hold.
Our first result states that indeed supraclassicality is a necessary condition for a nonmonotonic semantics in order for the framework to be applicable:

Proposition 4.1
If $(\mathcal{T}, Sem, R)$ is a successful revision framework, $(\mathcal{T}, Sem)$ must satisfy **Supra**.

PROOF Let $T \in \mathcal{T}$. We prove that $Sem(T) \subseteq Mod(T)$. If $Sem(T) = \emptyset$ we are done, so assume $Sem(T) \neq \emptyset$. Then, by P2, $R(T) = T$, hence $Sem(T) = Sem(R(T))$. By P3, it follows that $Sem(T) = Sem(R(T) \subseteq Mod(T)$. ∎

Without **WCE**, we have a simple necessary and sufficient condition for a successful revision framework. Essentially, it states that in every subclass of classically-equivalent theories, there exists at least one theory T such that $Sem(T) \neq \emptyset$.

Lemma 4.2
Let $(\mathcal{T}, Sem)$ satisfy **Supra**. *Then there exists a successful framework $(\mathcal{T}, Sem, R)$ iff for every $T \in \mathcal{T}$ there is a $T' \in \mathcal{T}$ such that $Mod(T) = Mod(T')$ and $Sem(T') \neq \emptyset$.*

PROOF Trivial, by the definition of successful revision frameworks. ∎

If, however, we add **WCE**, we have a stronger result:

Lemma 4.3
Let $(\mathcal{T}, Sem)$ *satisfy* **Supra + WCE**.
Then there exists a successful framework $(\mathcal{T}, Sem, R)$ *iff for every* $T \in \mathcal{T}$, $Sem(Cn(T)) \neq \emptyset$.

PROOF Assume that Sem satisfies **Supra + WCE** and that there exists a successful framework $(\mathcal{T}, Sem, R)$ for some R. So, take an arbitrary $T \in \mathcal{T}$. Then $Sem(R(T)) \neq \emptyset$. We have to prove that $Sem(Cn(T)) \neq \emptyset$. Note that, by P3, $Mod(R(T)) = Mod(T)$. Since $Mod(T) = Mod(Cn(T)$, it follows that

$$Sem(R(T)) \cap Mod(Cn(T)) \neq \emptyset,$$

so by Proposition 3.2 and **WCE** it follows that $R(T) + Cn(T) \mid\!\sim_{Sem} \Phi$, for some Φ. Since $Mod(R(T)) = Mod(Cn(T))$, it follows that $R(T) \subseteq Cn(T)$. Hence, by **WCE**, $Sem(R(T) \cup Cn(T)) = Sem(Cn(T)) \neq \emptyset$.
Conversely, let Sem satisfy **Supra + WCE** and assume that for every $T \in \mathcal{T}$, $Sem(Cn(T)) \neq \emptyset$. Then define $R(T)$ as $R(T) = T$ if $Sem(T) \neq \emptyset$ and $R(T) = Cn(T)$ otherwise. It is not difficult to see that $(\mathcal{T}, Sem, R)$ is a successful revision framework. ■

Sometimes we have a semantics Sem that is not consistency-preserving, but we can use another semantics Sem' that is consistency-preserving, if, by syntactical manipulation of (parts of) a theory, Sem can be reduced to Sem':

Definition 4.4 (Reducibility)
Let Sem, Sem' *be two semantics for a class* $\mathcal{T}$ *of theories. We say that* Sem *is* classically reducible *to* Sem' *iff for all* $T \in \mathcal{T}$, *there is a theory* $T' \in T$ *such that* $T \models T'$ *and* $Sem(T') = Sem'(T')$.

Theorem 4.5
If $\mathcal{T}$ *is class of theories,* Sem *is classically reducible to* Sem' *in* $\mathcal{T}$ *and* Sem' *satisfies* **Supra** *and* **Cons**, *then there is a successful revision framework* $(\mathcal{T}, Sem, R)$ *for* $(\mathcal{T}, Sem)$.

PROOF For every $T \in \mathcal{T}$, define $R(T)$ as follows: $R(T) = T$ if $Sem(T) \neq \emptyset$ and $R(T) = T + T'$ else.
We show that in $(\mathcal{T}, Sem, R)$, the postulates P1-P3 are satisfied.
Let $Mod(T) \neq \emptyset$. Let T' be such that $Sem(T') = Sem'(T')$. Since T is consistent, T' is also consistent. Since Sem' is consistency-preserving, it follows that $Sem'(T') = Sem(T') \neq \emptyset$. Let $M \in Sem(T') \subseteq Mod(T') \subseteq Mod(T)$. Then, by **WCE**, it follows that $Sem(T + T') = Sem(R(T)) \neq \emptyset$. So P1 is satisfied.
Postulate P2 is satisfied by construction of R. Finally, P3 is satisfied, since

$$Mod(R(T)) = Mod(T + T') = Mod(T) \cap Mod(T') = Mod(T).$$

■

An example of such a class of theories is the class of normal logic programs with constraints, where for each program P always an equivalent program P' can be found such that $MinMod(P') = Stable(P')$, where $MinMod$ is the minimal model semantics and *Stable* the stable model semantics.

5 Discussion

We have presented some postulates for revision of nonmonotonic theories and we have shown that given some fairly weak conditions on the nonmonotonic semantics, revision of such theories should be accomplished by expansion instead of contraction.
At first sight, the idea of revision by retraction might be strange. It can be explained as follows. In nonmonotonic reasoning we reason by making assumptions concerning the truth or falsehood of certain statements and derive conclusions from them. However, as soon as we detect some violation of constraints or are not able to derive any conclusion from a theory, we realize that we must have assumed to much: some of these assumptions may not be compatible. Now the only way to get out, is to state explicitly that one or more assumptions should not be made, i.e. to expand the original theory.
The idea of revision by expansion in nonmonotonic theory revision has been discussed before. In Auto-Epistemic Logic (AEL) for example, Morris ([8]) has suggested something like theory expansion for auto-epistemic theories that do not have an AE-extension. The simple idea is: if there is no AE-extension for a set of premises S, then a set-inclusion minimal set of ordinary (i.e. modal-operator-free) premises is added to S such that an AE-extension exists.
In logic programming, the work of Pereira et al. ([9]) on Contradiction Removal Semantics can be seen as a special expansion method, allowing for revision of assumptions.

In truth maintenance, belief revision has been performed by a pure expansion technique, called *dependency-directed backtracking (ddb)* ([1, 10, 11]). As these methods mainly have been stated informally and in a procedural way, there were little or no formal results. Recently, in [13], we have shown that ddb is not suitable for the stable model semantics and only can be complete if the semantics is as weak as a positivistic or supported model semantics.

Recently, Inoue and Sakama in [3] proposed a very general approach to revision of nonmonotonic theories by proposing to revise a theory T by a minimal set of additions I and removals O such that $T + I - O$ has an acceptable model. Our results show that in most cases, when T is classically consistent, removal of formulas in the form of retraction is not necessary.

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