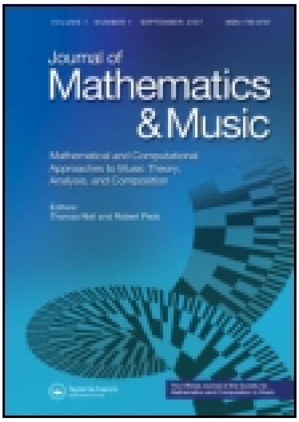

Journal of Mathematics and Music: Mathematical and Computational Approaches to Music Theory, Analysis, Composition and Performance
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tmam20

Mathematical and computational approaches to music: challenges in an interdisciplinary enterprise

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Published online: 06 Jul 2012.

Journal of Mathematics and Music
Vol. 6, No. 2, July 2012, 73–81

INTRODUCTION

Mathematical and computational approaches to music: challenges in an interdisciplinary enterprise

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This Special Issue is dedicated to explicate and discuss methodological issues in the interdisciplinary research field of mathematical and computational approaches to music. It arose from a lively panel discussion at the third International Conference on Mathematics and Computation in Music 2011 in Paris. We have organized this panel in order to initiate the much needed interdisciplinary dialogue on the How, Why, and What of our modelling of and theorizing about music in the wide field of science, humanities and cognitive approaches to music research. From the contributions of the three panelists to this Special Issue, we extract key topics that the interdisciplinary scientific community needs to address in order to enable the different disciplines to productively complement one another in achieving a comprehensive approach to music as a complex, yet fundamental human trait.

Keywords: mathematics; computation; musicology; interdisciplinary research

1. Motivation and background of this Special Issue

Both mathematical and computational approaches to music have thrived over the last decades, with new societies and conferences emerging (such as MCM [1] and ISMIR [2]). At the same time, gaps between different research directions within this multidisciplinary endeavour have been noticed, that might hamper the promising utilization of these new scientific methods for answering essential questions in music research [3]. For instance, Cook [4] states that we have been standing quite long at a *moment of opportunity* with respect to the relation between computational approaches and musicology, without reaching the full potential of the interdisciplinary enterprise. Likewise, Marsden [5] discusses possibilities to overcome the existing *gulf* between traditional music analysis and computational approaches to music analysis in order to prevent that this gulf impedes music research. Noll and Peck [6] acknowledge a gap between mathematical and computational approaches to music in the first issue of the *Journal of Mathematics and Music* (JMM) in 2007 and express the hope that the dialogue and collaboration between mathematical and computational approaches will be intensified.

In our own interdisciplinary work environment, employing both mathematics and computation, we have encountered questions such as ‘What does this abstract model tell me about this musical piece?’ or ‘Why is this musicological theory so vague?’ – which relate to basic methodological

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ISSN 1745-9737 print/ISSN 1745-9745 online

http://dx.doi.org/10.1080/17459737.2012.704154
http://www.tandfonline.com

issues in different disciplines. It seems that often a methodological disagreement or misunderstanding prevents us from recognizing and acknowledging the contribution of a specific research approach in a different area that might have otherwise fostered important insights for our own research. Different research interests and underlying methodologies are too often not communicated in such a way that a researcher from a different field might grasp the ideas quickly and relate them to his/her own research interest. As a consequence, the different areas involved in music research are not deeply connected in order to enable a comprehensive approach to music.

Therefore, in order to stimulate such an interdisciplinary dialogue, we have organized a panel discussion at the third International Conference on Mathematics and Computation in Music (MCM) 2011 at IRCAM[1] in Paris 'Bridging the Gap: Computational and Mathematical Approaches in Music Research' with panelists Alan Marsden, Guerino Mazzola, and Geraint Wiggins. Many of the conference participants reacted enthusiastically to the panel, acknowledging that fundamental questions have been raised that are important for the research community to address (for a review, see [7]). In order to convey the momentum to a broader audience, the editors of *JMM* suggested to dedicate this Special Issue to such an interdisciplinary discourse between the three panelists. We thank Thomas Noll for suggesting 'Musical Form, Forms And Formenlehre' [8] as a model that captures a dialogue within a succession of articles in order to continue the discussion of the three panelists in a written form.

2. Computation and mathematics in music

Mathematical and computational approaches to music have contributed to many different research areas of musicology,[2] spanning a wide range of topics. In the following, we highlight some areas of music research that have benefited from mathematics and computation, after which we indicate challenges that exist in the resulting interdisciplinary research, and reflect on the panel discussion on this topic at MCM 2011.

Computation and music

The first academic computational projects in music started in the 1960s. From the beginning, an important motivation of computational research was that the computer allows to process huge amounts of information that are difficult to oversee for a human being, such that entire new research areas might be explored [9]. Since then, computational approaches to music have led to increasing efforts towards the formalization and empirical testing of musicological concepts, which resulted in a greater visibility of musicology within scientific disciplines [10]. Computational approaches have contributed to music theory and analysis [11], musical performance research [12], historical musicology [13], ethnomusicology [14], and cognitive musicology [15] (for a more detailed overview, see [16]). Moreover, they have enabled new research areas such as Music Information Retrieval (MIR), which develops computational methods to retrieve music from large data-bases, as a response to the explosion in music digitization over the last decades [17].

Mathematics and music

The connection between music and mathematics is very old, going back at least to the Pythagorean school, which studied interval relations in terms of ratios of whole numbers; later the study of relations between intervals and numbers has become an important part of theories on tuning and temperament. Although 12-tone equal temperament became the standard temperament in the early Romantic period, temperament and tuning are still topics of research [18,19]. A topic related to this is the study of consonance and dissonance which does not only combine mathematics and music [20], but has also been studied from psycho-acoustic perspectives [21–23]. Musical set theory became well-known in the twentieth century, and provides methods and concepts that can be applied to tonal and atonal music [24–26]. Scale theory, a subdivision of musical set theory, highlights properties of the diatonic collection, such as maximal evenness [27], and well-formedness [28]. Transformational theory, developed in the 1980s, models musical transformations as elements of a mathematical group and has been used to analyse both tonal and atonal music [29,30]. A branch of transformational theory called Neo-Riemannian theory became popular in the 1990s [31]. Topos theory has also been applied to music, which resulted in, among others, topologies for rhythm, melody, and harmony, and has been used to deal with musical performances [32]. Mathematical music analysis covers various methods to analyse music using different mathematical theories [33]. Mathematical approaches to composition have also been made, for example by Kircher [34], Xenakis [35] and Mazzola [32, Chapter 50] and [36, Chapter 25].

Interdisciplinary challenges

While mathematical and computational approaches to music have successfully contributed to many different aspects of music research, the interconnections between the different areas are not very deep yet. For instance, both the potential of models developed in MIR to contribute to musicology and the potential of musicology to contribute to building these models have not yet been fully exploited, as argued in [16]. As of today, computational applications to musicology have not yet merged into a widespread discipline as envisioned, for instance, by Morehen and Bent [37]. With respect to the many different models developed for specific aspects of music (e.g. mathematical models in scale theory and transformational theory; computational models of melodic similarity, of metre and motivic analysis), it is not yet clear as to how far these specific models contribute to developing a more comprehensive approach to music. This would require substantial collaboration between researchers in the different fields, which presupposes a thorough reflection and discussion of the involved methodologies.

Panel discussion 'Bridging the Gap', MCM 2011

Alan Marsden, Guerino Mazzola, and Geraint Wiggins served as panelists in the MCM panel discussion 'Bridging the Gap: Computational and Mathematical Approaches in Music Research'. All three are working in the field of music while they take a different interdisciplinary approach on this. During the discussion, the panelists agreed on the benefits of the interdisciplinary research field and highlighted different challenges. For Marsden, the big challenge in the interdisciplinary discourse is not to erase the differences between disciplines, but to talk to each other. Mazzola argued that mathematics should not be misunderstood as the provider of a magic box that automatically creates 'interesting' music. Wiggins stressed the importance of a cognitive approach towards music research. The different viewpoints of the authors are elaborated on in this Special Issue.

3. Format and theme of papers in this issue

The following four questions were offered to the panelists, derived from questions during the panel discussion, to serve as the basis for their position papers on the topic of methodological issues in computational and mathematical approaches to music:

(1) **Benefits**: What key contributions did mathematical and computational approaches bring to the field of music research according to your point of view?

(2) **Failures**: What are examples of pitfalls that occurred within computational and/or mathematical approaches to music research in the past? What can we learn from them?
(3) **Challenges**: What challenges are we facing now within computational and/or mathematical approaches to music? What unexplored fields and questions have the potential to move our understanding of music forward with the help of computational and/or mathematical approaches? What steps need to be taken now and in the near future in order to fully unfold the potential of computational and/or mathematical approaches to music?
(4) **Interdisciplinary discourse**: How can we strengthen the connections between the three fields of mathematical, computational, and musicological approaches to music? Are there different ways of 'understanding' music in these three fields? In what context are the differences between the disciplines (mathematics, music research, computer science) a useful source for innovative research on music-related questions? When do the differences between the disciplines become a stumbling block for interdisciplinary research, and what needs to be done to overcome that?

After distributing the position papers to the fellow contributors, authors responded to each other, such that each article is commented by the other two authors. Wiggins chose to combine his two replies in one paper. Finally, each author elaborated his position taken in his initial text by reacting to the response papers of his fellow contributors in a final reply. To capture the dialogue form, the articles are divided into three sections corresponding to the three authors (Mazzola, Wiggins, Marsden), each section displaying first the position paper, followed by the response papers by the fellow contributors, finished by the final paper. Wiggins' response paper is placed in Mazzola's section.

4. Three methodological reflections

In his position paper, Mazzola answers our questions by discussing examples from his (and his collaborators') work on mathematical and computational approaches to rhythm, harmony, melody, performance, and gesture research. He shows how the developed models have contributed to a conceptual clarification of musicological concepts. Wiggins argues in his position paper that music theory is a Folk Psychology in Ravenscroft's terms. He concludes that we need to achieve a scientific meta-theory of music and discusses mathematics and computation in current music research and regarding their potential to contribute to the envisioned meta-theory. Marsden points in his position paper to different foci of interest within mathematical, computational, and musicological approaches, discussing examples such as paradigmatic analysis and the gap-fill principle. He advocates how the dialogue between researchers in both fields needs to address the different interests and methods in order to find productive ways of investigating music in an interdisciplinary manner. In the following, we list some of the answers provided by the three authors.

Benefits of mathematics and computation for music research:

- key contributions to music technology, e.g. to mp3-format (Mazzola, Marsden)
- conceptual clarification of music theoretic concepts (Mazzola)
- mathematics allows to understand existing music from its positioning within a broader framework of possible musics (Marsden, Mazzola)
- mathematics affords explanations of the consequences of tuning (Marsden)
- generalizations in mathematics allow addressing the evolution of music (Mazzola)
- computation allows rigorous testing of evolutionary hypotheses (Wiggins)

- computation is a hard test of the efficiency and precision of conceptualization and operationalization of musical thoughts (Mazzola)
- computation allows creation of educational music tools (Marsden, Wiggins)

Failures and pitfalls in combining mathematics, computation, and musicology:

- it is a failure to study the musical products without the musical processes (Wiggins)
- it is a failure to treat music theory not as fundamentally perceptual (Wiggins)
- it is a failure to use information-theory approaches to music (Mazzola)
- it is a failure to neglect the sign-theoretical shape of musical phenomena in favour of purely formal aspects (Mazzola)
- it is a failure that mathematicians concentrate too much on scale theories; musicologists are more interested in musical pieces (Marsden, Mazzola)
- it is a failure to reject higher math as a means to understand complex musical structures on the grounds that everybody understands music in some sense (Mazzola)

Challenges we are facing in computational and mathematical approaches to music:

- musicologists have not yet taken up the tools offered by mathematics and computation (Marsden)
- accounting for the ‘messiness’ of humans is a challenge for the preciseness in mathematics and computation (Marsden, Wiggins)
- classification of musical objects in mathematics alone is not enough, we need to apply them to musical pieces within computational experiments (Mazzola)
- to achieve a meta-theory of music, we need to rise above the study of particular examples of music tying all musical cultures together through time (Wiggins)
- we need to model musical behaviour rather than merely music products (Wiggins)
- mathematicians need to consider more empirical, statistical work for a better connection between rationalism and empiricism (Marsden)

In the **interdisciplinary discourse**, we need to consider:

- humility is essential for interdisciplinary work (Marsden)
- honesty about what is within scope and what not is important (Marsden)
- using the fuzzy concept of ‘musicality’ as an argument to dismiss mathematical or computational models of music is not appropriate (Mazzola)
- achieving mathematical theories that are computational and can be tested by comparison with humans requires substantial interdisciplinarity (Wiggins)

In the following section, we point to some key topics that arose from the discussion between the three authors that we consider to provide important challenges for the interdisciplinary scientific community.

4.1 The precision of mathematics and computation versus the imprecision of humans

Mazzola addresses in his position paper how the imprecision of musicological concepts was a starting point for his development of mathematical models and theories for music in order to achieve conceptual clarification. Marsden and Wiggins respond that this impreciseness, however, has its roots in the impreciseness or ‘messiness’ of humans[3] such that an imprecise model might be even more accurate than a precise one. Wiggins argues that human cognition is not precise and that statistics provides a means to deal with human messiness. Marsden demonstrates on Nattiez’s paradigmatic analysis that his imprecise concept of ‘paradigm’ is probably more accurate to describe the listeners' perspective than a precise mathematical formulation, since it allows the concept to adapt to the circumstances in hearing the piece rather than 'breaking down' at some point.

This discussion points us to one of the key achievements of mathematics and computation in music, namely their ability to indicate to us where exactly the messiness lies in the detail. The 'breaking down' of a formula or a model at a certain point provides an important moment of insight into the investigation of the phenomenon. For instance, the fact that computational algorithms find much more motivic patterns than humans do, as pointed out by Marsden, provides an explicit demonstration of Marsden's account of selectivity that is part of the human process of analysing a musical piece. The mismatch between the amount and nature of patterns detected by algorithms and humans also indicates how little we seem to know yet about those selection processes. However, too often we evaluate, for instance, the success of computational approaches only in terms of how well their results fit a certain given 'ground truth' (be it a musicological concept, be it empirical data gained from listening experiments, etc.). This points us to an as yet hardly sufficiently explored direction for the evaluation of our models: in analysing and exploring those cases where the results of a model do not fit or seem disappointing, possibly the greatest potential lies to understand something about our observed phenomenon, which allows further inquiry on the research subject.

4.2 What kind of theory is music theory—and what are the consequences for mathematical and computational approaches to music?

Wiggins discusses in his position paper the nature of music as a cognitive and cultural construct and argues that music theory is not a theory of 'music itself', but a descriptive model[4] of music perception in a shared musical culture. Hence, he considers it a failure not to treat music theory as fundamentally perceptual and concludes that mathematical models should not merely address abstract musical structures (e.g. scales, chords, intervals), but should also address perception.

Marsden and Mazzola respond that investigating perception is indeed important for music research, yet it cannot explain everything of interest. While Wiggins states that the semiotics of music is in the interpretation of the listener, Marsden and Mazzola point to important aspects of music that do not relate to the listener's perspective. Wiggins' characterization of music theory as being purely descriptive is challenged to some extent by Marsden and Mazzola, such as with the fact that music theory through the cycle of theory, practice, education, and perception is not only descriptive, but influences the phenomenon it is describing. They show that music theory has sometimes arisen from posteriori description, sometimes it has preceded the compositional practice.

Over the course of the discussion in the response papers, Wiggins' demand to investigate music in terms of cognition and perception is questioned by Mazzola's characterization of music theory as not being a branch of psychology since music cannot be reduced to psychological entities. In his final paper, Marsden argues that the points of view of his fellow contributors correspond to different positions in music research that are not necessarily mutually exclusive and shows how these positions correspond to different loci of 'music'. Furthermore, he argues that from the fundamental perspective of empiricism and rationalism, music research has benefited from both approaches, and suggests that a theory which is grounded in both perspectives is preferable to one 'which is otherwise similar but grounded on only one'.

Elaborating on this, we would like to add that mathematical and computational approaches that do not directly model psychological processes can nevertheless contribute in the end to the understanding of those, in the sense that Popper has suggested: 'In general, we may learn a great deal about behaviour and psychology from the study of the products' [39, p. 114]. For instance, in the context of research on oral transmission of Dutch folk songs in the WITCHCRAFT-project[5] at Utrecht University, the musical process of oral transmission could not be observed directly, yet the products of this process were accessible (field recordings of Dutch folk songs hosted by the Meertens Institute in Amsterdam). The successful computational approach to melodic similarity developed during the project [40] allows now to find related melodies in a corpus of 7000 Dutch folk songs, such that this computational tool assists the study of the process of oral transmission. In further research, this might deliver very interesting musical material from the cognitive point of view on how humans memorize melodies and hence might contribute to Wiggins' envisioned cognitive-based theory of music.

As Wiggins explicates in his final paper, mathematical models of music can serve to challenge psychologists, neuroscientists, and musicologists to explain how a specific process arises in 'cognitive, neural, evolutionary and historical terms'. Wiggins' vision to develop music theory into a more meaningful entity by moving more explicitly into the psychological domain hence poses yet unexplored challenges for mathematical and computational modelling.

4.3 The use of models and the issue of abstraction

All three authors agree that formalizations alone do not make theories better. Mazzola describes in his position paper the process of abstraction used by mathematicians, who are interested in precise concepts and take the freedom to abstract from some aspects of the phenomena investigated. Marsden responds that the idea of a model which abstracts from certain aspects is still difficult for musicologists and that they care most for the contribution of a model to explaining the musical phenomena. Therefore, the musicologist will hardly be convinced that a model is useful, unless its application has proven to provide important insights. Marsden also argues in his position paper that a mathematician should never think that the theory fully explains the musical phenomenon, since 'there is no single unequivocal set of entities which makes up a piece of music'.

Precisely formalized models in mathematics and computation allow empirical work in music research on a large scale through the application of these models to large corpora of digitized music in computational experiments. This enables data-rich approaches to music [41]. However, the evaluation of these models that are built on abstractions regarding their contribution to our understanding of music is yet a challenging issue. While standardized methods of model evaluation have been developed (and are further refined) for the application-driven approaches in MIR in the yearly MIREX-competitions,[6] we need to invest more interdisciplinary efforts into the evaluation of how the different models allow these important insights and how they might contribute to a broader theory of music.

5. Envisioned perspectives for interdisciplinarity

All three authors discuss how a widening of the perspectives in current research on music can be achieved. Marsden gives from the musicological point of view examples of what topics would benefit from mathematical and computational approaches. He argues that we understand things better if we see how they relate to other things we know, which relates to Mazzola's argument that mathematics gives us potential theories in which we can study the properties of existing music in relation to non-existing variants [42]. Wiggins considers perception and cognition as the wider context that surrounds music and suggests to study systematically the whole process from sound waves, ears, and neurons up to symphonies. He requests not to restrict music theory to the Western perspectives of music, but to take the wider context of other music cultures into consideration. Moreover we would like to add the importance of social aspects of human engagement with music that we need to bring into this wider perspective on music research.

However, how can such a bigger picture of music be achieved? Developing mathematical models, implementing efficient algorithms, carrying out experimental verifications, simulating music evolutionary processes computationally—all of this requires substantial research efforts which can hardly be carried out by individuals, nor by research groups located in just one discipline. Here, we come full circle to the importance of the interdisciplinary discourse, since such a bigger picture will hardly be achieved without interdisciplinary collaboration. That, however, requires to acknowledge research results as being intermediary from the perspective of the big picture, and to carve out their potential to contribute in the end towards it. For instance, developing a mathematical model might require much effort in itself, such that the empirical verification has to be carried out by someone else or simply at a later stage. Yet for achieving the bigger picture in an interdisciplinary enterprise, we need to develop virtues on how to build bridges (to come back to the topic of our panel discussion at MCM in Paris, 2011) between our different contributions to music research. Building these bridges requires considerable efforts to invest in the interdisciplinary dialogue, and we argue that from the broader picture of computational and mathematical approaches to music, we are in need of these efforts. We hope that this Special Issue contributes to strengthening our efforts in investing in the interdisciplinary dialogue in order to help our research to become more fruitful from a wide perspective on music as a complex, yet fundamental human trait that offers numerous challenges as a research subject.

Acknowledgments

We thank Thomas Noll and Robert Peck for giving us the opportunity to organize and edit this Special Issue, and for their advise on various issues. Many thanks go to the three authors, Alan Marsden, Guerino Mazzola, and Geraint Wiggins, for their contributions, interesting exchanges, and feedback. We thank Bas de Haas, Marcelo Rodríguez-López, Frans Wiering, and Remco Veltkamp for comments on an earlier draft of this text. Anja Volk is supported by the Netherlands Organization for Scientific Research, NWO-VIDI grant (276.35.001). Aline Honingh is supported by the Netherlands Organization for Scientific Research, partly through an NWO-VENI grant (639.021.126) and partly through an NWO-VICI grant to Rens Bod (277.70.006).

Notes

1. Institute for Research and Coordination in Acoustics and Music.
2. We use the term ‘musicology’ here in its broadest sense, hence comprising music theory, empirical, historical, systematic musicology, ethnomusicology, etc.
3. This is not meant in a pejorative manner.
4. Wiggins uses the term ‘descriptive’ as opposed to ‘explanatory’ according to Wiggins [38].
5. http://www.cs.uu.nl/research/projects/witchcraft
6. see ‘http://www.music-ir.org/mirex/wiki/MIREX_HOME’

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