Journal of Mathematics and Music
Vol. 00, No. 2, July 2008, 1–17

RESEARCH ARTICLE

Persistence and Change: Local and Global Components of Meter Induction Using Inner Metric Analysis

Anja Volk (née Fleischer)*

Utrecht University, Department of Information and Computing Sciences, P.O. Box 80.089, NL-3508 TB Utrecht

(*Preprint; Final version published by the Journal of Mathematics and Computation in Music. Download final version under http://dx.doi.org/10.1080/17459730802312399*)

This paper compares local and global perspectives on the metric structure of musical compositions using the computational model of *Inner Metric Analysis*. The comparison addresses different hierarchical levels of compositions with respect to metric stability and change. Inner Metric Analysis generates metric hierarchies that are persistent from the global perspective and sensitive to changes from the local perspective. While the global perspective indicates the predominant metric characteristic of a piece or fragment, the local perspective distinguishes between regions of different metric characteristics. Like studies of harmony that investigate tonality as a hierarchic web of relations between chords, phrases and large sections, this article suggests that the time organization of a piece consists of different hierarchical levels. The description of local and global metric characteristics of Webern's Op. 27 using Inner Metric Analysis complements and extends arguments given by David Lewin about a metrical problem observed in this piece.

Keywords: metric structure; meter hierarchy; local and global analytic perspectives.

1. Introduction

While the time signature of a piece in Western classical music often suggests that the meter is persistent throughout the piece, metric hierarchies can shift, change and dissolve considerably within a piece [1–3]. Hence, musical compositions may be characterized by changes in metric state between large sections or within short sections, leading to shifts in perspective not unlike the changing hierarchical relationships between chords, phrases and large sections of tonal compositions (e.g. [4, 5]). This article uses the computational model of *Inner Metric Analysis* [6–9] to explore local sensitivity to global metric changes (and vice versa) in order to characterize different metrical states in a musical composition.

Inner Metric Analysis provides a structural description of the inner metric structure of musical pieces by inducing a *metric weight* for each note. The metric weight is a quantification of the note's metric importance within the piece. Metric weight profiles often show layers such that a metric hierarchy is generated solely from information about the onsets of notes (not from their durations or harmonic values). As is often noted (e.g. [1]), a discrepancy between meter implied by the actual placement of a piece's notes and that implied by the placement of the bar lines

* Email: volk@cs.uu.nl

ISSN: 1745-9737 print/ISSN 1745-9745 online

DOI: 10.1080/1745973YYxxxxxxxx
http://www.informaworld.com

often creates appealing conflicts that should be considered an important ingredient of the metric structure. Thus I refer to the *inner* metric structure evoked by the rhythmic configuration of the notes as opposed to the *outer* metric structure implied by the bar lines and time signature.

Elsewhere I have used the metric weight approach to introduce a concept of *metric coherence* that denotes cases where the inner and outer metric structures concur—that is, where the metric structure evoked by the notes is synchronous with the normative metric state given by the bar lines [6]. My application of Inner Metric Analysis to compositions by Dufay, Ockeghem, Wilbye, Morley, Händel, Bach, Mozart, Beethoven, Schubert, Schumann, and others in [6] demonstrates that metric coherence is usual in a number of works noted for their strict metricity. In contrast, my study of Brahms' symphonies [10] has generated metric weight profiles that agree with scholars' perceptions of perceived meters in conflict with the notated bar lines. The application of Inner Metric Analysis to the temporal patterns used by Povel & Essens [11] reported in [6] and the application of Inner Metric Analysis within listening tests in [12] have both demonstrated that structural description using metric weights is consistent with existing understandings of metric perception.

Nestke & Noll [8] introduce a number of extensions to Inner Metric Analysis, such as the definition of *spectral weight* as a modification of metric weight. The present article shows that metric and spectral weights convey different perspectives on the metric structure of a piece with respect to local and global information. The musical analyses show how these perspectives contribute to the description of the meter of a work in terms of the poles of persistence and change. Section 2 defines the two kinds of weight, Section 3 studies how these weights mediate between different perspectives on the metric structure, and Section 4 applies these perspectives to a metrical problem in Webern's Piano Variations Op. 27. Thus the article demonstrates the use of a computational tool in the analyst's engagement with the musical score.

2. Introduction to the Model of Inner Metric Analysis

2.1. *Background*

The model of *Inner Metric Analysis* goes back to the notion of *local meters* and *metric weights* as defined by Mazzola & Zahorka [13]. Mazzola introduced a geometrical approach to describing the global structure of a musical piece by decomposing it into local objects and studying their mutual relations in terms of *simplicial complexes* in [14].[1] This approach has been applied to different analytical problems, such as motivic analysis in [16], and it underlies Inner Metric Analysis as well. The local objects in the metric model are pulses (or groups of note onsets), which are therefore called *local meters*.[2]

2.2. *Pulses*

In Inner Metric Analysis, a pulse consists of a series of at least three events that are equally spaced in time. Although human analysts often elide the distinctions

[1] The term "simplicial complex" refers to a particular topological space, consisting of points, lines, triangles, tetrahedrons, and so forth, that are glued together. For an introduction to algebraic topology see [15].
[2] This article does not discuss the geometric motivation underlying the model; for the description of the *metric complex* as a simplicial complex see [6, 7].

between individual beat and pulse as well as between pulse and metrical hierarchy, the multiplicity of pulses detectable by this computational model demands that we observe the distinctions here. Inner Metric Analysis is based on pulse descriptions. In this respect the model follows music-theoretic approaches to meter such as [1–3, 17–19], which are based on the detection of pulses (or "layers of motion" or "pulse streams"). Computational approaches to meter such as [11, 20–22] are often based on pulse detection as well.

The model defines the pulses of a piece of music on the basis of note onsets. It does not take into account features often said to be important for the induction of meter such as changes in the harmonic and melodic domains, dynamic accents, density accents, and registral accents. Perhaps surprisingly, the time organization given by note onsets often provides highly differentiated metric information. For instance, in [23] periodicity in the onset location is the most important cue for determining meter. Inner Metric Analysis, then, treats notes as the primary indicator of metric structure.

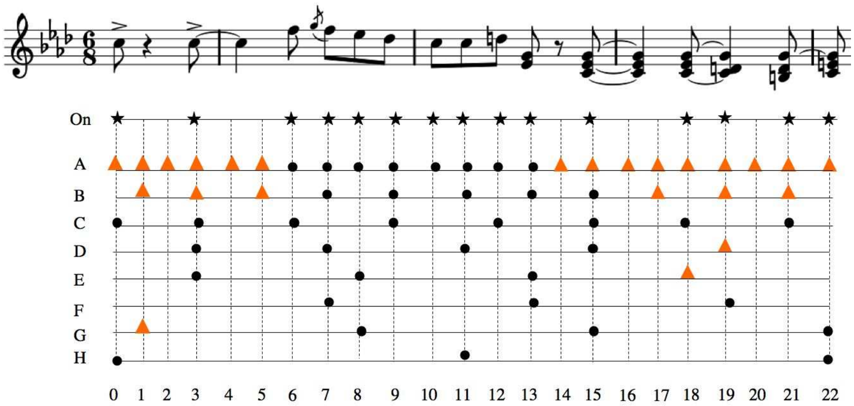

Figure 1. The Set *On* ⋆ and all local meters • and their extensions ▲ in Chopin's Ballade no. 3, Op. 47 (bars 69-73 as discussed by Riemann [24], p. 101 and Krebs [1], p. 10.)

An example from Chopin's Ballade No. 3, Op. 47 illustrates this description of pulse. The passage is given by Krebs [1] as a situation of metrical conflict, and Riemann [24] suggested rebarring it. First we project the notes onto the set On of all onsets of notes or attack points. We enumerate the onsets of this example in terms of eighth notes starting with 0 for the first onset. In Figure 1 each element of this set[1] is given in the first row below the notes as a star $\star$ (disregarding the grace note). Hence, the set On consists of the elements $0, 3, 6, 7, 8, 9, 10, 11, 12, 13, 15, 18, 19, 21,$ and 22. Within this set all subsets $m \subset On$ of equally spaced onsets are candidates for the pulses which are called *local meters*. We consider a subset m as a local meter if it contains at least three onsets and is maximal, that is, not a subset of any other subset m' consisting of equally distanced onsets. Figure 1 shows all local meters enumerated as $A, B, C, \ldots, H$. Dark circles • indicate elements of the local meter, and triangles ▲ illustrate the extension of the local meter throughout the entire piece (to be discussed in Section 2.4). The condition of maximality can be easily illustrated by means of the local meter A. This local meter consists of the onsets $6, 7, 8, 9, 10, 11, 12,$ and 13. It contains, for instance, the subsets $A' = \{6, 8, 10, 12\}$, $A'' = \{6, 7, 8\}$, and $A''' = \{7, 10, 13\}$ of equally distanced onsets which are therefore not maximal and hence not considered as local meters. However,

[1]In this example all notes of a chord are projected onto the set On. A different application of the model would distinguish between different voices, for instance between the melody and the accompanying voices. For this example these two different approaches lead to similar analytical results.

$B = \{7, 9, 11, 13, 15\}$ is maximal because, although it begins with a subset of A, it continues on to include 15.

Each local meter can be identified by three parameters: the starting point or first onset s; the distance d between the consecutive onsets of the local meter (the period); and the number of repetitions k of the period, which equals the number of onsets in the local meter minus 1, called the length. Hence, we denote any local meter as $m_{s,d,k} = \{s+id, i = 0, 1, \ldots, k\}$. In Table 1 all local meters of the example are listed with their corresponding parameters s, d and k.

local meter m	A	B	C	D	E	F	G	H
start onset s	6	7	0	3	3	7	8	0
period d	1	2	3	4	5	6	7	11
length k	7	4	7	3	2	2	2	2

Table 1. List of all local meters of the example in Figure 1 with their corresponding parameters.

2.3. *Metric weight*

After detecting all the local meters in a given piece, we may define a *metric weight* for each onset that reflects the number of local meters that coincide at this onset. Onsets where many pulse layers coincide get a greater weight than onsets where few pulse layers coincide. Moreover, longer repetitions contribute more weight than shorter ones. That is, the more stably established in a chain of successive and regular events, the more significant the onset.

We first define the weight w_p of a local meter $m_{s,d,k}$ as the power function $w_p(m_{s,d,k}) = k^p$ with p as a variable parameter. This enables us, instead of just considering the length k as the contribution of each local meter, to test different amounts of influence of local meters depending on their length. The higher the value of p, the greater the weight of longer local meters in comparison to shorter local meters.

The metric weight for each onset o is calculated as the sum of weights $w_p(m_{s,d,k})$ of those local meters that contain the onset o. For example, the very first onset 0 in the example of Figure 1 participates in the local meters C and H. Hence, the sum of weights of the local meters w_p in this case equals $7^p + 2^p$. The contribution of local meter H, $w_p(m_{s,d,k}) = 2^p$, is relatively small, and one may wish to exclude such short local meters altogether. Thus we introduce a further variable parameter ℓ which regulates the minimum length of the local meters. (Since a local meter must contain at least three onsets, ℓ cannot be smaller than 2.) Varying ℓ and varying p both make the study of the influence of the short versus long local meters possible, in different ways.[1] If we let $M(\ell)$ be the set of all local meters of the piece of length at least ℓ, that is to say, $M(\ell) = \{m_{s,d,k} : k \geq \ell\}$, the general metric weight of an onset, $o \in On$, is as follows:

$$W_{\ell,p}(o) = \sum_{\{m \in M(\ell) : o \in m\}} k^p. \tag{1}$$

2.4. *Spectral weight*

While raising the value of p or ℓ, then, affords a larger-scale view of the meter of a passage or composition, metric weight still offers a local perspective in comparison

[1] In this paper different values are considered only for p; on the variation of ℓ see [25].

to the global perspective offered by a measurement called *spectral weight*. Spectral weight [8] is a further refinement of metric weight and is based on the extension of each local meter throughout a given piece or fragment, denoted as $ext(m_{s,d,k}) = \{s + id, \forall i\}$ with i as integers. The additional elements of each local meter in the extensions in our example of Figure 1 are indicated as triangles such that an extension $ext(m_{s,d,k})$ consists of all the circles • and triangles ▲ in a given row. In the calculation of spectral weight each local meter contributes a weight to all events in its extension. For example, the contribution $w_p(A)$ of the first local meter A is now added to the weight of all time points of the score on a grid of eighth notes, because the extension of a local meter with period $d = 1$ meets all time points of this grid. Hence, the spectral weight allows the assignment of weights to silences as well, in contrast to the metric weight that is defined only on note onsets.

The extension of the local meters in the spectral weight approach corresponds to an idea discussed by Krebs [1]. He argues that pulses do not only affect the metric structure in regions where they are active. Even after a pulse has stopped we may continue to count along this pulse in the following section. Yet in the calculation of spectral weight, local meters are also extended to the beginning of the piece. This extension to the beginning of the piece models the effect of reinterpreting the past of a given event retrospectively based on new incoming events. The difference between the spectral and metric weights for the very first onset in Figure 1 is as follows. The metric weight of this onset depends on the contributions w_p of the local meters C and H. The spectral weight depends, in addition to these local meters, on the contributions of the local meter A, since its extension coincides at this first onset as well.

The spectral weight of a given onset or silence event t is the weighted sum of the length of all those local meters whose extensions coincide at t and have a length of at least ℓ. In mathematical terms, the spectral weight is defined as:

$$SW_{\ell,p}(t) = \sum_{\{m \in M(\ell): t \in ext(m)\}} k^p. \tag{2}$$

2.5. *Example of the Model's Application: Chopin's Ballade No. 3, Op. 47*

To illustrate the use of the model, let us see how it treats the metric conflict identified by Riemann and Krebs in the Chopin example of Figure 1. Figure 2 shows the metric and spectral weight profiles using $\ell = 2$ and values for p varying from 2 to 4. The x-axis of a weight profile represents time and the y-axis the metric or spectral weight. The light-dark boundaries of the grayscale background assist visualization of the graphs by indicating bar lines.

The metric weight profile of Figure 2(a) shows the greatest weights in the first and second bars, namely on the third and sixth eighth notes of the first full bar and the third eighth note of the second bar. The corresponding spectral weight profile of Figure 2(b) reveals that these onsets form two layers in the spectral weight profile, namely the highest layer that is visible as high peaks on the last eighth notes of all bars, and the secondary layer seen in the lower peaks on the third eighth notes of all bars. The weights of all other onsets form the lowest layer. Remarkably, then, a graph that originates in calculating the strengths of the pulses to which each onset belongs generates a metric hierarchy. However, this hierarchy is in conflict with the notated bar lines, since the first and fourth eighth notes should form the highest metric layer. Thus Figures 2(c) and 2(d) show the same weight profiles with bar lines shifted towards the greatest weights. According to the spectral weight profile,

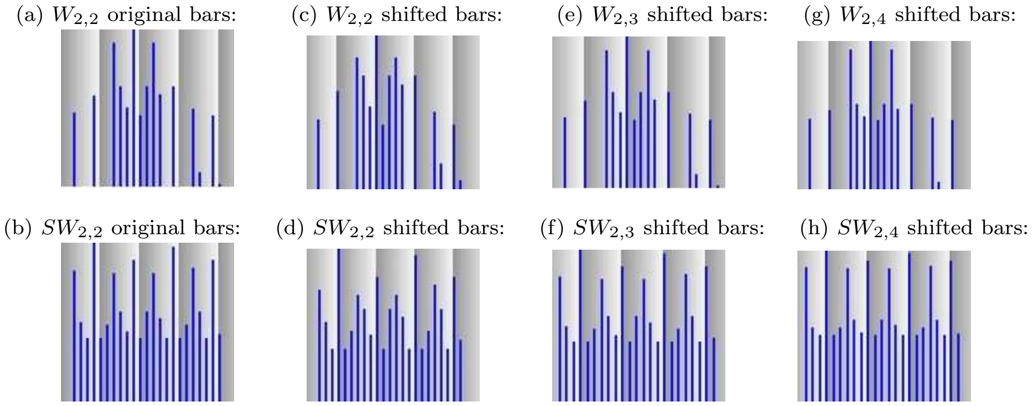

Figure 2. Above: Metric weight profiles for the example in Figure 1 with different values for p. The background indicates the bar lines. Below: Corresponding spectral weight profiles.

the onsets of this passage create a metric hierarchy typical for a $\frac{6}{8}$ meter with great weights on the first onsets followed by secondary weights on the fourth onsets. Hence, Inner Metric Analysis illustrates the metric conflict noted by Riemann and Krebs by suggesting a way to see how onsets in the right hand are shifted one eighth-note ahead of the notated $\frac{6}{8}$ barlines.[1]

In addition, the spectral weight profile in Figure 2(d) shows yet another layer across the bars. The weights of the first onsets of the first full bar and the third bar build a higher layer than the weights of the first onsets of the second and fourth bars. Hence, a hypermeter is created. However, increasing the influence of the longer local meters by increasing the value of p to 3 and then to 4 as shown in Figures 2(e) – (h) tends to even these differences out. Increased values of p often lead to less differentiated weight profiles by evening out subtle differences caused by shorter local meters. In this case, shorter local meters are responsible for creating the hypermeter of two bars.

As shown in [6], the parameter $p = 2$ tends to be high enough to allow stable weight layers on the one hand and to reveal subtle variations within these layers on the other. Henceforth all weights discussed are based on the parameters $p = \ell = 2$, if not indicated otherwise. For a systematic study of variation in these parameters see [25].

3. Comparison of Local and Global Perspectives

As we have seen, metric weight considers only pulses active during a given onset while spectral weight also considers pulses from other regions of the composition. Therefore, metric weight provides a more local perspective on metric structure, in contrast to the global perspective provided by spectral weight. The comparison of these local and global perspectives often offers a way to understand how local metric changes are contextualized within a passage's overall metric organization. In some cases the global spectral profile may be distinguished from the local by the emergence of layers.

[1] Riemann [24] suggested a different rebarring of this example starting with the quarter rest as the first onset of the first bar and compressing the rhythm of the chords in the last bars into triplets.

3.1. *The sensitivity of the metric weights concerning changes of the metric state*

In comparison to the metric weight profiles, spectral weight profiles are more stable and hence less sensitive to local changes of the inner metric structure. Significant changes in the inner metric structure may be reflected by the metric weight but not by the spectral weight. Let us see how Inner Metric Analysis handles a change in the grouping structure in the melody of Brahms' Symphony No. 3, third movement. The motive that introduces the change is characterized by a rhythmic displacement in which the third eighth note of the $\frac{3}{8}$ bar is tied to the first note of the next bar. As Figure 3 shows, bar 26 contains such a tie, while bar line 27/28 is the last bar line not elided by a tie. Starting at bar line 28/29, the grouping structure systematically changes towards the third eighth note as the beginning of the group.

Figure 3. Melody of mm. 20-37 of Brahms' Symphony No. 3, 3rd mvmt.

The metric weight profile of the melody in the upper graph of Figure 4 shows a pronounced change starting at bar 26. In the first half of Figure 4 the greatest metric weights are located on the first beats of the bars. Bars 26 and 27 start to assign greater weights on the third beats. From bar 28, the metric weights of the third beats build a very distinct layer (This region is indicated by the heavy horizontal line beneath the profile.) Notably, this highest layer starts at the third beat of bar 28 where the grouping structure changes systematically. This example thus demonstrates an interaction between the rhythmic grouping and the inner metric structure by assigning significantly heavy weights on the first beats in the first part and on the third beats in the second part.[1]

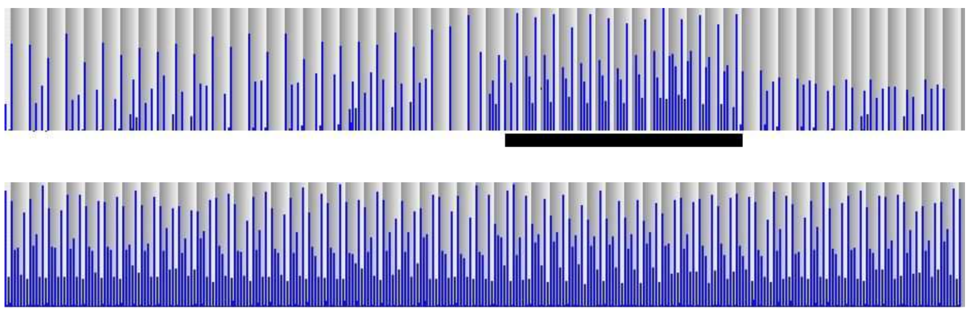

Figure 4. Above: Metric weight profile of the melody of Brahms' Symphony No. 3, 3rd mvmt., mm. 1-53. The heavy horizontal line begins at bar 28. Below: Spectral weight profile of the same melody.

The corresponding spectral weight profile in the lower graph of Figure 4 mixes these different characteristics by assigning great weights on the first and third beats through the entire segment. Thus the spectral weight illustrates the competing role of the first and third beats throughout the first part of the movement. The metric weight assigns these competing roles to different sections.

Spectral weight may, hence, obliterate differing metric characteristics of adjacent segments in cases where the inner metric structure changes significantly. It is able

[1]For a discussion of the gradual reorganization of the metric weight layers starting at the beginning of the second part of the profile see [26].

to derive a dominant metric characteristic of a piece or segment by ignoring local changes. Metric weight is sensitive to sudden changes and can be used to distinguish the metric characteristics of different segments.

3.2. *Amplification of Layers in Spectral Weight Profiles*

A more distinct emergence of layers often distinguishes spectral from metric weight profiles. Thus the metric weight of the example from Chopin's ballade in section 2.5 above does not exhibit distinct weight layers over the entire passage, while the spectral weight does. Such spectral layering is a feature of metric change in the first main section of the 3rd movement (Presto) of Haydn's Piano Sonata Op. 39 shown in Figure 5.

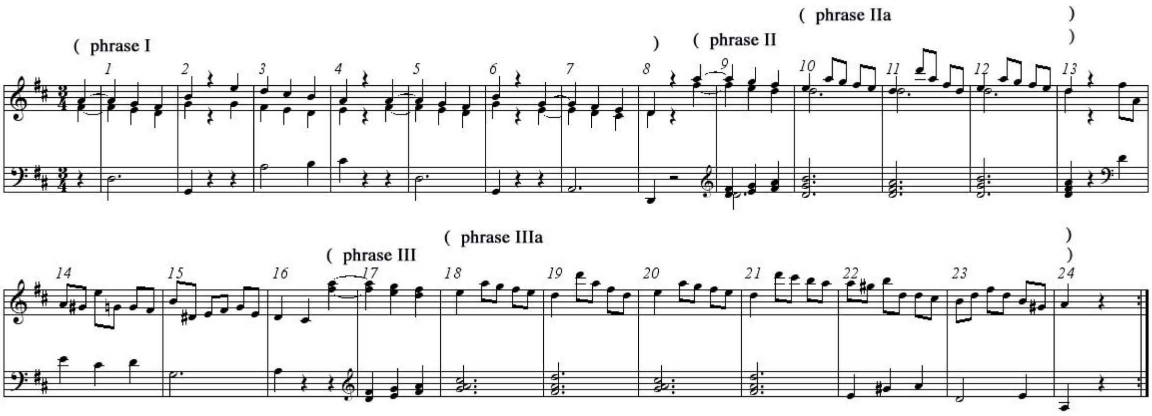

Figure 5. Phrases analyzed in the first main section of Joseph Haydn's Sonata Op. 39, 3rd mvmt.

In this excerpt the grouping structure of the right hand changes noticeably. For example, the first eight bars suggest units of six quarter notes, beginning on the last quarter note of each bar. Starting with bar 10, each bar begins a three-quarter-note unit. The last quarter note in bar 13 again initiates an upbeat structure and the following chain of eighth notes divides into units of two beats each separated by the melodic high points of f♯ (bar 13), e (bar 14) and b (bar 15). The following analysis compares the metric and spectral weights of the phrases bracketed in Figure 5, which contain different groupings in the right hand.

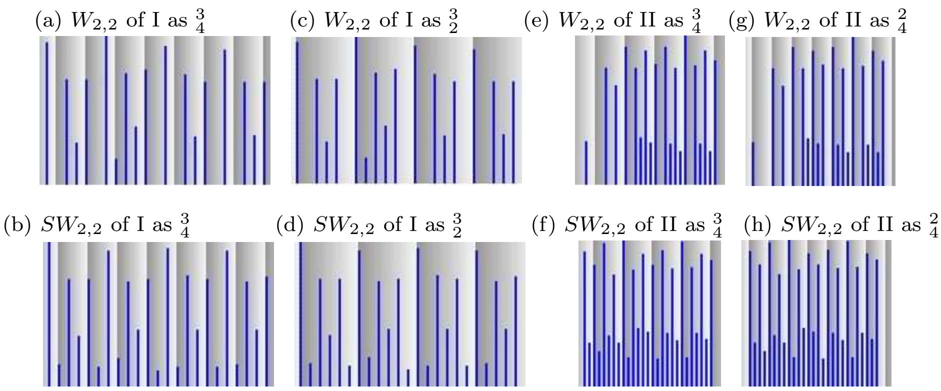

Figure 6. Metric and spectral weight profiles of phrases I and II

The layers of the metric and spectral weight profiles of phrase I shown in Figures 6(a) and (b) are not aligned with the metric structure implied by the bar lines. The greatest weights are located on the last quarter note of every other bar suggesting a six-quarter unit. Interpreting the same weight profiles according to a shifted $\frac{3}{2}$ in Figures 6(c) and (d) reveals an inner metric structure whose layers are quite consistent with $\frac{3}{2}$ meter. The spectral weight extends low weights present in the metric weight profile to a low layer throughout the entire phrase.

Phrase II contains a shift from the upbeat figure to the downbeat figure at bar 10. The layers of the metric and spectral weight profiles of this phrase shown in Figures 6(e) and (f) are not aligned with the notated $\frac{3}{4}$. Reinterpreting these weight profiles as in Figures 6(g) and (h) reveals that the weight layers are aligned with a $\frac{2}{4}$ meter. While the very first onset of the passage does not participate in the highest layer in the metric weight profile, the spectral weight profile extends this layer throughout the entire passage including the first onset.

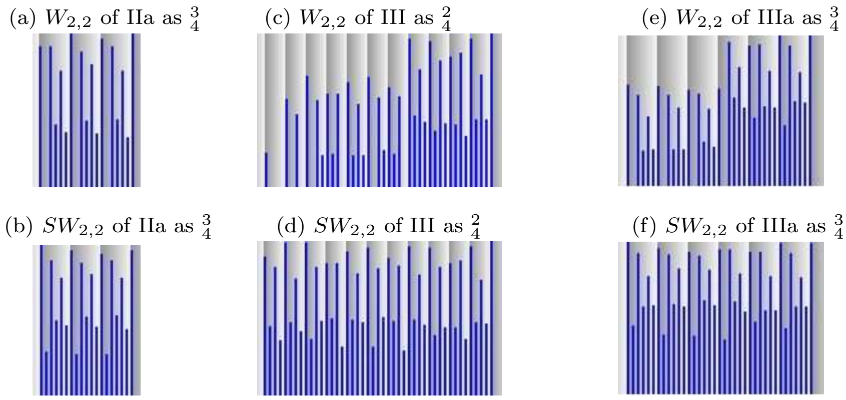

Figure 7. Metric and spectral weight profiles of phrases IIa, III and IIIa

This surprising $\frac{2}{4}$ meter is the result of the rhythmic displacement that begins phrase II. The layers of the metric and spectral weight profiles of phrase IIa in Figures 7(a) and (b) are aligned with the notated $\frac{3}{4}$. The first onset within the metric weight profile in Figure 7(a) does not participate in the highest layer built upon the first beat of all bars, but the spectral weight in Figure 7(b) extends this layer throughout the entire passage including the first onset. Since phrase IIa omits the rhythmically displaced beginning of phrase II it is clear that this displacement is responsible for the emergence of $\frac{2}{4}$ layers within the weight profiles of phrase II. That is, the structure of bars 10-13 unambiguously mediates the $\frac{3}{4}$ structure, but the rhythmic displacement at the beginning of phrase II results in the emergence of $\frac{2}{4}$ for the entire phrase.

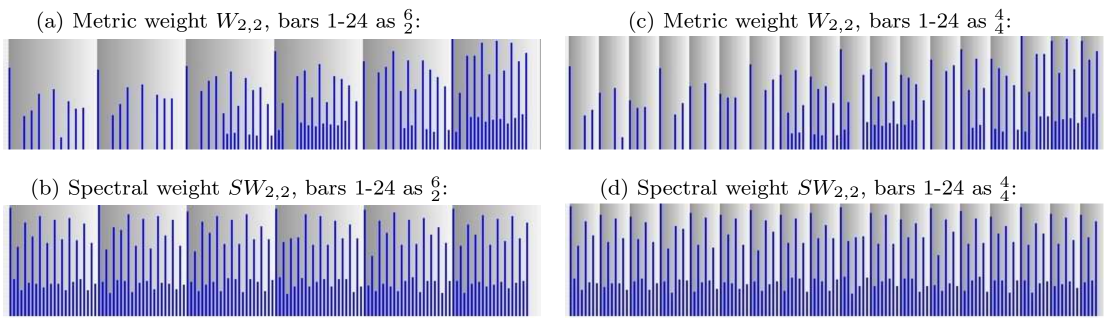

Figure 8. Metric and spectral weight profiles of bars 1-24 using $p = 2$

The rhythmic displacement at the beginning of phrase III has a similar influence. The layers of the metric and spectral weight profiles of phrase IIIa in Figures

7(e) and (f) are aligned with the $\frac{3}{4}$. While the metric weight profile in Figure 7(e) contains two regions of different overall levels in the weights, the spectral weight profile in Figure 7(f) unites these two regions to global layers throughout the entire passage. However, the weights of the second beats equal the weights of the first beats in the fourth and fifth bars of the figure (Haydn's bars 21 and 22), corresponding to the beginning of the higher level in the corresponding metric weight profile. The layers of the metric and spectral weight profiles of phrase III are not in synchronization with a $\frac{3}{4}$ and are interpreted in Figures 7(c) and (d) as $\frac{2}{4}$. Hence, the rhythmic displacement prevents the emergence of $\frac{3}{4}$ layers in this passage as it does in phrase II. However, the influence of the displacement is diminished in comparison to the influence in phrase II, as can be seen in the fact that phrase III's layers are also less distinct in the $\frac{2}{4}$ interpretation of Figure 7(d), where not all first beats have great weights.

Thus we have seen that the inner metric structure does not remain constant throughout this section and that it is often not aligned with the outer metric structure. The changing local metric weight profile prevents the emergence of consistent layers throughout the passage.

In the spectral weight profile of the entire section displayed in Figure 8(b), the interaction of the different metric characteristics of the phrases leads to a layering in which the first onset of every six-half-note duration has the greatest spectral weight. Hence, in terms of Haydn's notation, this global view suggests units of four bars starting on the first upbeat. The metric weight profile exhibits these great weights on the first onset of every six-half-note duration as well, but it lacks the distinct layering based on half notes. Hence, the $\frac{6}{2}$ interpretation is not convincing from the local perspective. Likewise, interpreting the same weight profiles as $\frac{4}{4}$ in Figures 8(c) and (d) shows that from the global perspective of the spectral weight a $\frac{4}{4}$ is plausible, though not from the local perspective of the metric weight.

Increasing the influence of long local meters by increasing the value of p to 3 and 4 in Figure 9 converges the local and global perspectives. Figure 9's interpretation of the spectral weight profile as $\frac{6}{4}$ brings out the most dominant metric characteristic from both the local and global perspectives. The spectral weight induces layers of four and two $\frac{3}{4}$ bars starting at the first upbeat. The metric weight profile shows these layers as well as a layer of single bars clearly in the first half of the weight profile, less obviously in the second half.

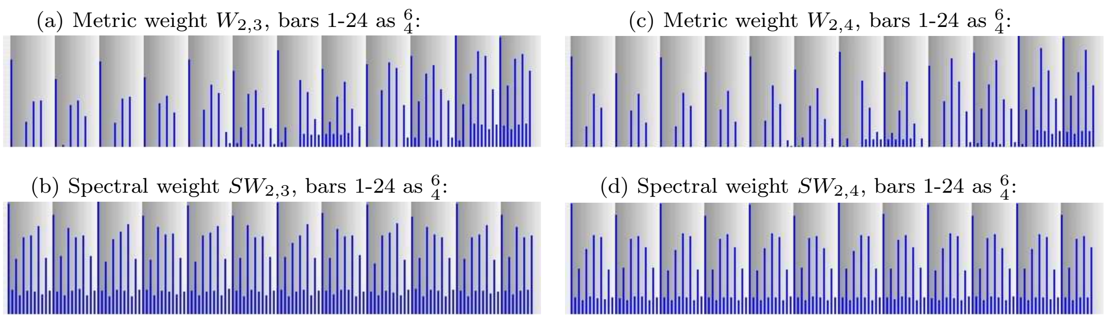

Figure 9. Metric and spectral weight profiles of bars 1-24 using $p = 3$ and $p = 4$

In contrast to the right-hand part, the left hand establishes the meter according to the notated bar lines unambiguously, as the metric and spectral weight profiles of the left hand in Figure 10 show. In both profiles the greatest weights are located on the first beats of the bars.

In these analyses of the phrases of the right hand as well as of the right- and left-hand parts of the entire section, the spectral weight profile often amplifies weight

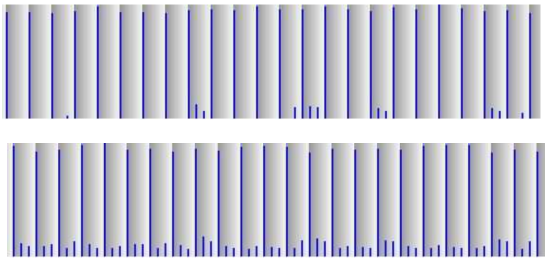

Figure 10. Metric (above) and spectral (below) weight profiles of the left hand of bars 1-24

layers that are less distinct in the metric weight profile. Thus, the spectral weight allows a consistent layering of the metric hierarchy throughout the entire passage. However, the spectral weight with $p = 2$ implies layers that are not convincing for the metric weight of the right hand in the first section. Only when the influence of short local meters is diminished do the local and global perspectives tend to converge. Hence, the short local meters that typically introduce more local differentiations prevent the emergence of distinct layers for the metric weight profile. From the local perspective, the changes within the inner metric structure interfere with a consistent metric interpretation of the entire section. That is, from measure to measure it is difficult to find a consistent underlying meter; but when the passage is viewed as a whole, a consistent meter—and indeed, a hypermeter—emerges.

4. A metrical problem in Webern's Piano Variations Op. 27

Investigating the interplay of local and global perspectives upon the second movement of Webern's Piano Variations Op. 27 (whose opening is given in Figure 11) helps illuminate a metrical problem in this piece observed by Lewin [27][1]: Webern notates the piece as $\frac{2}{4}$, but due to rhythmic pattern that repeats at intervals of three quavers we tend to perceive the piece in $\frac{3}{8}$. In response, Lewin studies different dimensions of the musical structure (such as the chord and pitch structures and the number of bars between structural downbeats) in order to find reasons why the notation in $\frac{2}{4}$ nevertheless reflects some intrinsic characteristics of the music.

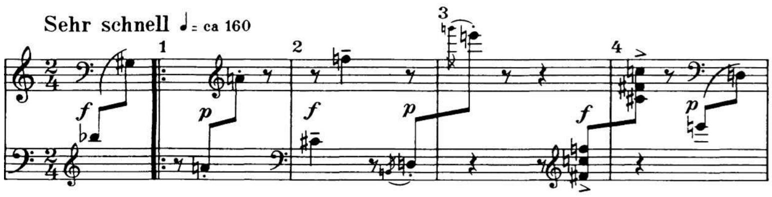

Figure 11. Opening of Webern's Piano Variations, 2nd mvmt.

Neither the metric nor the spectral weight profiles of the entire piece (including repetitions) in Figure 12 exhibit layers that correspond to $\frac{2}{4}$. The interpretation of the metric weight according to $\frac{3}{8}$ in Figure 13 reveals great weights on the first beats of bars starting at bar 8 of the figure (Webern's bar 6), indicated with an accent mark in Figure 13. This metric structure corresponds to Lewin's observation of a perceived $\frac{3}{8}$. However, the second half of the same metric weight graph (whose beginning is indicated with an asterisk) does not reveal any distinct layers. This

[1]Lewin's discussion of this piece begins in [28].

part corresponds to the second main section of the movement (bars 12-22 and repetition). Thus, judged by metric weight the two main sections of this movement exhibit different metric characteristics.

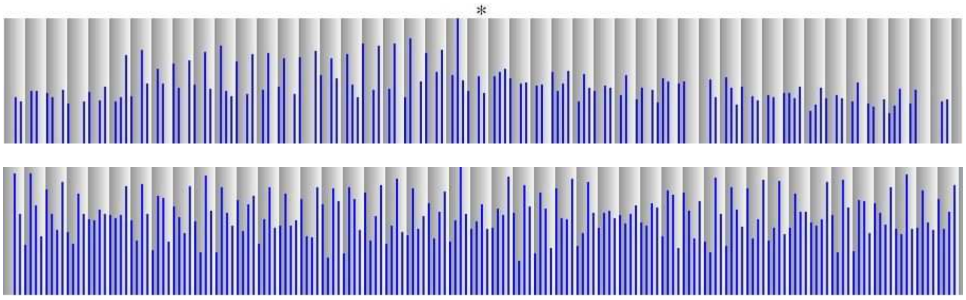

Figure 12. Metric (above) and spectral (below) weight profiles of the 2nd movement of Webern's Op. 27, interpreted as $\frac{2}{4}$. The asterisk indicates the beginning of the second main section after the repeat sign.

The spectral weight profile of the entire piece, shown in Figure 13, reveals that the weight layers corresponding to $\frac{3}{8}$ represent the most dominant metric characteristic throughout. In most of the bars the greatest weight is located on the first onset. However, because this does not apply to all bars, it appears that even from the global perspective of the spectral weight profile, the $\frac{3}{8}$ structure is disturbed by different local meters.

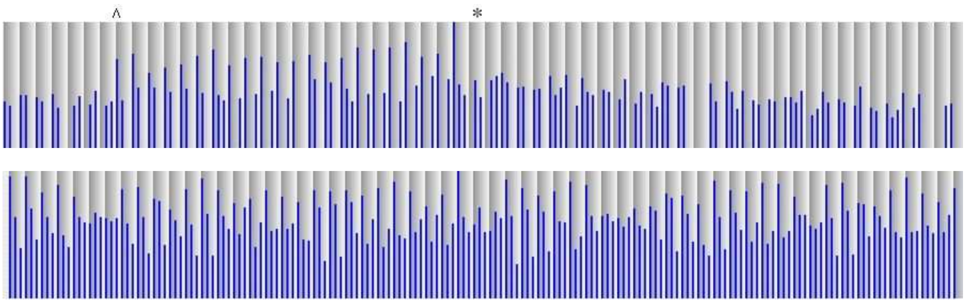

Figure 13. Metric (above) and spectral (below) weight profiles of the 2nd movement of Webern's Op. 27, interpreted as $\frac{3}{8}$. The asterisk indicates the beginning of the second main section after the repeat sign.

The metric weight profile of the entire piece suggests a division of the piece into two main sections corresponding to bars 1-11 and bars 12-22 in Webern's notation. This division corresponds with Webern's repeat sign after bar 11. In the following the metric structures of these two sections are therefore investigated independently of each other.

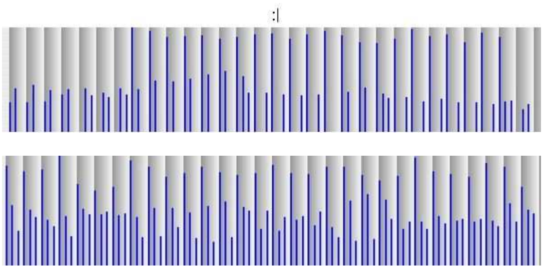

Figure 14. Metric (above) and spectral (below) weight profiles of the first main section (bars 1-11 with repetition), interpreted as $\frac{3}{8}$. The sign above the figure indicates the repeat sign in the score.

Without the influence of the second main section, the spectral weight profile of the first main section in Figure 14 reveals a highest layer built upon the weight of each $\frac{3}{8}$ bar. Within the metric weight profile this layer starts at bar 8. The isolated analysis of the first main section in Figure 14 confirms even more distinctly the metric characteristics observed in the metric weight profile of Figure 13.

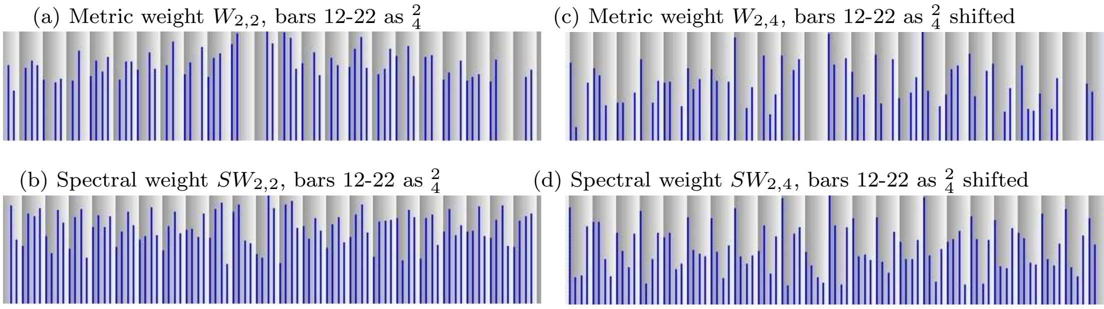

Figure 15. Metric and spectral weights of the second main section (12-22 with repetition). Left: $p = 2$ and original bar lines of $\frac{2}{4}$. Right: $p = 4$ and bar lines of $\frac{2}{4} + 1/4$ shift. Webern's repeat sign is located after the first two notes following the rest (recognizable as a gap) in the middle of (a) and (c).

The internal structure of the second main section differs from that of the first main section in that the metric and spectral weight profiles of the second main section, shown in Figures 15(a) and (b), do not show weight layers whether interpreted as $\frac{3}{8}$ or as $\frac{2}{4}$. However, increasing the influence of longer local meters causes the spectral weight to show great weights on many of the first beats of the displayed $\frac{2}{4}$ bars. Figures 15(c) and (d) show an example for the value $p = 4$. (The barlines in Figures 15(c) and (d) are shifted from the original notated bars by half a bar. Hence, the observed great metric weights are located in the middle of the notated bars.) The second main section therefore exhibits the potential of a structure in $\frac{2}{4}$. However, this can only be observed within the spectral weight when the influence of the longer local meter is increased. Moreover, the metric weight does not exhibit this structure. In sum, the influence of local meters supporting a $\frac{2}{4}$ in the second main part is present but not very strong.

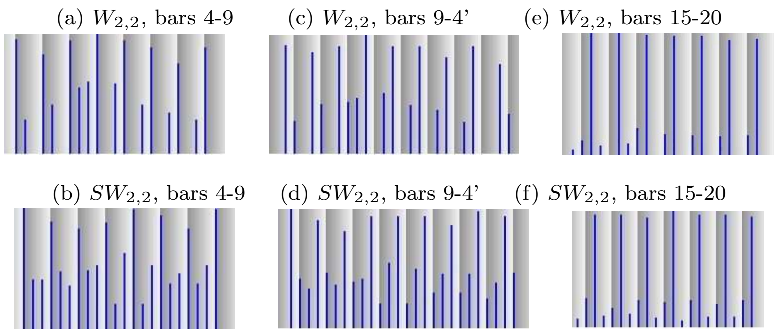

Figure 16. Metric and spectral weight profiles of bars 4-9, bars 9-4' and bars 15-20 interpreted as $\frac{3}{8}$

The analyses of the entire piece and of the isolated main sections therefore suggest that the first main part is characterized by a $\frac{3}{8}$ structure, while within the second main section local meters supporting $\frac{2}{4}$ have some influence. However, Lewin suggests a finer subdivision of the piece according to "large structural downbeats" (marked by three-tone chords) that occur at the bar lines 3/4, 8/9, 19/20 and between the first and second eighth notes of bar 15. According to Lewin's analysis the main structural phrases of the piece are therefore bars 4-9, 9-4' (bar 4 after the repetition), 4'-9', 9'-15, 15-20 and 20-15'. Lewin further observes that the phrases of bars 4-9, 9-4' and 15-20 are rhythmically very similar in that they follow a "rhythmic talea." Figure 17 shows his alignment of bars 4-9 with 9-4' and Figure 18 his alignment of bars 4-9 with 15-20. He characterizes the remaining two phrases outlined by the structural downbeats, those of bars 9'-15 and 20-15', as rhythmically more complex.

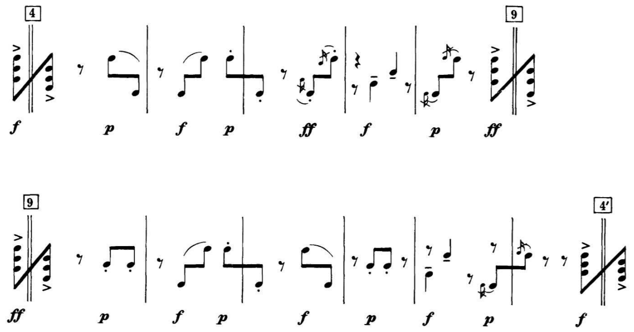

Figure 17. The rhythmic talea of bars 4-9 aligned with the rhythmic talea of bars 9-4', reproduced from [27], Figure 1, p. 346, with permission of Blackwell Publishing.

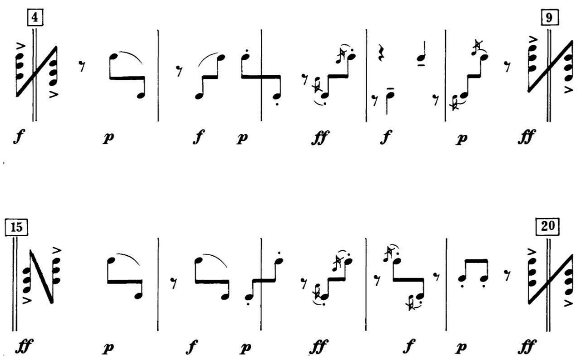

Figure 18. The rhythmic talea of bars 4-9 aligned with the rhythmic talea of bars 15-20, reproduced from [27], Figure 2, p. 347, with permission of Blackwell Publishing.

The metric and spectral weight profiles of the "talea" of bars 4-9 (shown in Figures 16(a) and (b)) and of its re-occurrence in bars 9'-4 (Figures 16(c) and (d)) and in bars 15-20 (Figures 16(e) and (f)) exhibit distinct weight layers indicating a $\frac{3}{8}$ meter. In contrast to Lewin's argument that mainly the introductory passage *before* the first structural downbeat at bar 4 is responsible for the effect of $\frac{3}{8}$, Inner Metric Analysis reveals that the phrases considered to be "rhythmic taleas" by Lewin are internally characterized by a $\frac{3}{8}$ structure. On the other hand, the metric and spectral weight profiles of the "rhythmically more complex" phrases shown in Figure 19 (bars 9'-15 and 20-15') do not exhibit distinct weight layers, and thus indeed elude a simple metric interpretation. In bars 9'-15 the local meters of periods of two quarter notes and three eighth notes respectively compete with each other in such a way that from the superposition of these local meters no clear winner emerges. Figures 20(a) and (b) show the metric and spectral weight profiles of this section after excluding local meters of period 3/8. The resulting weight profiles show great metric weights on the first beat of all bars (according to the notated $\frac{2}{4}$). Excluding local meters of period 2/4 in the metric and spectral weights (Figures 20(c) and (d)) reveals a hidden $\frac{3}{8}$ structure.

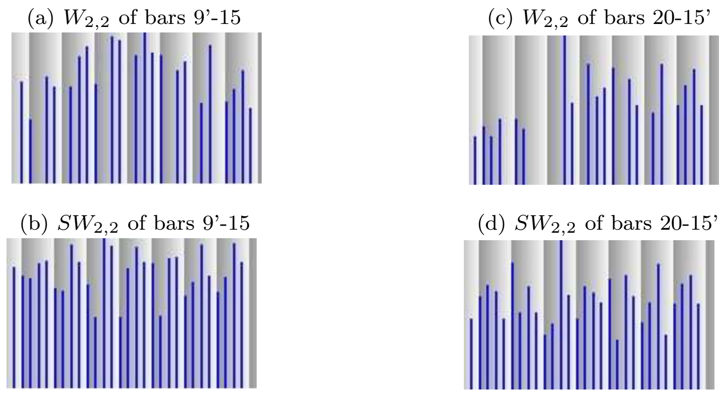

Figure 19. Metric and spectral weight profiles of bars 9'-15 and 20-15', original bar lines of $\frac{2}{4}$

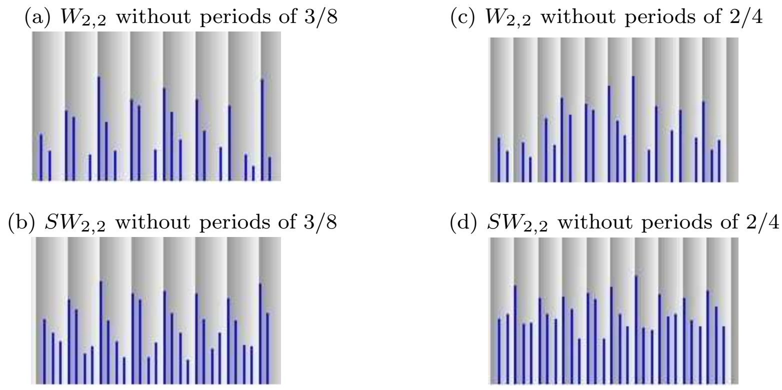

Figure 20. Metric (above) and spectral (below) weight profiles of bars 9'-15. Left: excluding local meters of period 3/8, with bar lines as $\frac{2}{4}$. Right: excluding local meters of period 2/4, with bar lines as $\frac{3}{8}$.

Interestingly, the third instance of the talea (bars 15-20) is surrounded by the "more complex" passages of bars 9'-15 and 20-15'. Even though the isolated analysis of bars 15-20 reveals a correspondence to $\frac{3}{8}$, the second main section viewed as a whole is not characterized by such a correspondence. (See Figure 15, above.) Hence the context of the rhythmically more complex passages of bars 9'-15 and 20-15' seems to prevent the emergence of $\frac{3}{8}$ layers in this part of the composition.

Therefore, it is worth examining the metric structure of this final talea in bars 15-20 when extended by the context of bars 9'-15, as an example of how this context influences the talea's metric interpretation. The metric and spectral weight profiles of bars 9'-20 in Figures 21(a) and (b) indicate a strong influence of local meters supporting $\frac{2}{4}$, which becomes more obvious when increasing the value of p to 3 (Figures 21(c) and (d)). That is, the metric and spectral weights using $p = 3$ reveal a $\frac{2}{4}$ meter underlying this section of the piece. This section combines the metrically ambiguous phrase of 9'-15 with that of the "talea" of bars 15-20. Hence the union of these two phrases is the first subsection found that clearly reveals an inner metric structure supporting the notated $\frac{2}{4}$.

To summarize, the comparison of local and global perspectives on the metric organization of the second movement of Webern's Op. 27 reveals that an intrinsic $\frac{3}{8}$ is much more dominant than an interpretation of the piece in $\frac{2}{4}$. However, local meters supporting $\frac{2}{4}$ also have a strong influence that sometimes prevents the emergence of weight layers at all or leads to weight layers supporting a $\frac{2}{4}$. Hence Inner Metric Analysis is consistent with Lewin's observation that both $\frac{3}{8}$ and $\frac{2}{4}$ are immanent in the structure of the piece. The local perspective provided by the metric weight suggests a division of the piece into two main sections exhibiting

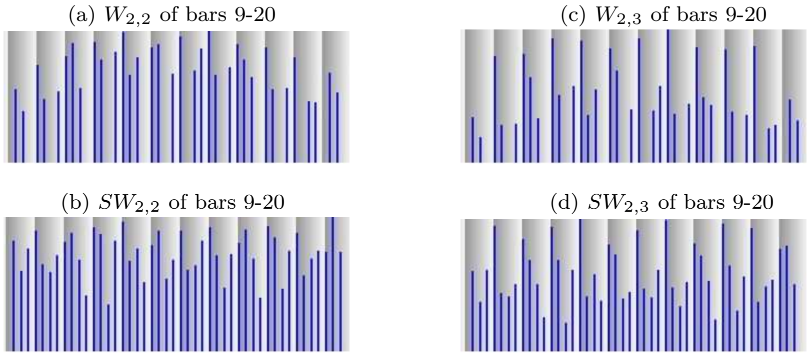

Figure 21. Metric and spectral weight profiles of bars 9-20: $p = 2$ (left) and $p = 3$ (right). Bar lines as $\frac{2}{4}$.

different metric characteristics, in agreement with the main sections of the score. The global perspective of the spectral weight reveals $\frac{3}{8}$ as the most dominant meter in this piece, but it also shows the existence of local meters supporting other weight layers such as $\frac{2}{4}$. Furthermore, Lewin's "rhythmic taleas" imply a $\frac{3}{8}$ meter, while his "rhythmically more complex" passages prevent the emergence of distinct weight layers. Nevertheless, from the combination of the two which straddles the repeat sign dividing the piece, a $\frac{2}{4}$ meter emerges that is consistent with Webern's notated meter. The application of Inner Metric Analysis to Webern's Op. 27 is consistent with Lewin's observation that the piece is likely to be perceived as $\frac{3}{8}$ but also contains structural elements that support the notated $\frac{2}{4}$.

5. Conclusion

Inner Metric Analysis realizes an objective and quantitative method of creating metric accents by assigning to each note a metric weight based on the superposition of specific pulses evoked by the onsets of all notes. The model has shown that the onsets of a musical piece contribute important information about the metric structure. Through computation, the model allows the testing of underlying assumptions about the metric structure of musical pieces on a large scale.

Yet interpreting the musical significance of the metric weight profiles generated by the model requires a degree of abstraction, since the weight assigned to a given onset is not primarily a description of that note's perception within the listening process, but rather a measure of the strength of the metric pulses to which it belongs.[1] To this end, this article shows how two different weight types describe different levels of metric organization with respect to local and global information. Spectral weight describes the predominant or global metric structure of a piece or segment. Metric weight grasps immediate or local changes in the metric structure and is able to distinguish between different metric states within a single analyzed passage. The musical analyses discussed show the benefits of comparing both kinds of weights in order to study how the meter of a work falls between the poles of persistence and change.

[1] [26] discusses a use of the model that focuses on the description of metric states at particular moments, which is closer to the modeling of the listening process.

6. Acknowledgments

This research has been funded by the University of Southern California Women in Science and Engineering Postdoctoral Fellowship Award. It made use of Integrated Media Systems Center Shared Facilities supported by the National Science Foundation under Cooperative Agreement No. EEC-9529152. In addition, part of this research has been funded by the WITCHCRAFT-project NWO 640-003-501 at Utrecht University. All analyses discussed make use of the software JMetro developed within the Research Group for Mathematical Music Theory KIT-MaMuTh at the Technical University Berlin, financed by the Volkswagen-Stiftung.

References

[1] H. Krebs *Fantasy Pieces: Metrical Dissonance in the Music of Robert Schumann*, Oxford University Press, 1999.
[2] J. Roeder, *Interacting Pulse Streams in Schoenberg's Atonal Polyphony*, Music Theory Spectrum 16 (1994), pp. 231–249.
[3] R. Cohn, *Complex Hemiolas, Ski-Hill Graphs and Metric Spaces*, Music Analysis 20 (2001), pp. 295–326.
[4] F. Lerdahl *Tonal Pitch Space*, Oxford University Press, 2001.
[5] F. Lerdahl and C.L. Krumhansl, *Modeling Tonal Tension*, Music Perception 24 (2007), pp. 329–366.
[6] A. Fleischer (Volk) *Die analytische Interpretation. Schritte zur Erschließung eines Forschungsfeldes am Beispiel der Metrik*, dissertation.de - Verlag im Internet Gmbh, Berlin, 2003.
[7] A. Fleischer (Volk), G. Mazzola, and T. Noll, *Computergestützte Musiktheorie*, Musiktheorie 4 (2000), pp. 314–325.
[8] A. Nestke and T. Noll, *Inner Metric Analysis*, Tatra Mountains Publications (2001), pp. 91–111.
[9] G. Mazzola *The Topos of Music*, Birkhäuser, Basel, 2002.
[10] A. Volk, *Metric Investigations in Brahms' Symphonies*, G. Mazzola, T. Noll and E. Lluis-Puebla, eds., epOs Music, Osnabrück, 2004, pp. 300–329.
[11] D.J. Povel and P. Essens, *Perception of Temporal Patterns*, Music Perception 2 (1985), pp. 411–440.
[12] A. Volk, *The Empirical Evaluation of a Mathematical Model for Inner Metric Analysis*, in *Proceedings of the 5th Triennal ESCOM Conference*, Hanover, 2003.
[13] G. Mazzola and O. Zahorka *Geometry and Logic of Musical Performance I-III*, SNSF research reports, Zürich, 1993-1995.
[14] G. Mazzola *Gruppen und Kategorien in der Musik*, Heldermann, Berlin, 1985.
[15] E. Spanier *Algebraic Topology*, McGraw-Hill, New York, 1966.
[16] A. Nestke, *Paradigmatic Motivic Analysis*, Perspectives of Mathematical and Computational Music Theory (2004), pp. 343–365.
[17] M. Yeston *The Stratification of Musical Rhythm*, Yale University Press, New Haven and London, 1976.
[18] F. Lerdahl and R. Jackendoff *A Generative Theory of Tonal Music*, Cambridge: MIT Press, 1983.
[19] C.F. Hasty *Meter as Rhythm*, Oxford University Press, 1997.
[20] R. Parncutt, *A Perceptual Model of Pulse Salience and Metrical Accent in Musical Rhythms*, Music Perception 11 (1994), pp. 409–464.
[21] E.W. Large, *On synchronizing movements to music*, Human Movement Science 19 (2000), pp. 527–566.
[22] P. Toiviainen and J.S. Snyder, *Tapping to Bach: Resonance-Based Modeling of Pulse*, Music Perception 21 (2003), pp. 43–80.
[23] P. Toiviainen and T. Eerola, *The Role of Accent Periodicities in Meter Induction: A Classification Study*, in *Proceedings of the 8th International Conference on Music Perception and Cognition*, Evanston, Illinois, 2004, pp. 422–425.
[24] H. Riemann *System der Musikalischen Rhythmik und Metrik*, Breitkopf und Härtel, Leipzig, 1903.
[25] A. Fleischer (Volk), *A Model of Metric Coherence*, in *Proceedings of the 2nd Conference Understanding and Creating Music*, Caserta, 2002.
[26] A. Volk, *Modeling a Processive Perspective on Meter in Music*, in *Proceedings of the International Computer Music Conference*, Barcelona, 2005, pp. 251–254.
[27] D. Lewin, *A Metrical Problem in Webern's Op. 27*, Music Analysis 12 (1993), pp. 343–354.
[28] D. Lewin, *A Metrical Problem in Webern's Op. 27*, Journal of Music Theory 6 (1962), pp. 124–132.