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Shape Algorithmics

Remco C. Veltkamp[1]

1. Introduction. Shape algorithmics is about the algorithmic aspects of the processing of sets of points, curves, regions, and volumes that represent form information. This includes aspects such as shape representation, shape similarity measures, matching two (sets of) points, curves, regions, or volumes, skeletonization, shape decomposition, shape approximation, morphing, retrieval from a large collection, and perceptual aspects. For this special issue, papers were solicited addressing general methodological issues as well as significant case studies in this area, concerning aspects such as algorithm design, theoretical complexity, practical efficiency, or the algorithm's behavior.

In general, **algorithmics** is about the design and analysis of algorithms, proving properties such as correctness, invariance, complexity of running time and storage used, and optimality. It is good to realize that there is no universally agreed upon definition of what **shape** is. Impressions of shape can be conveyed by color or intensity patterns (texture), from which a geometrical representation can be derived. This is shown already in Plato's work Meno (380 B.C.). This is one of the so-called Socratic dialogues, where two persons discuss aspects of virtue; to honor and memorialize him, one of the two persons is called Socrates. In this particular dialogue, the other person is Meno. In the dialogue it is discussed how difficult it is to give precise definitions. Socrates describes shape as follows (the word "figure" is used for shape):

"figure is the only existing thing that is found always following color".

This does not satisfy Meno, after which Socrates gives a definition in "terms employed in geometrical problems":

"figure is limit of solid".

Indeed, shape is considered mainly geometrical. We use the term shape for a geometrical pattern, consisting of a set of points, curves, surfaces, solids, etc. This is commonly done, although "shape" is sometimes used in a more restricted sense for a geometrical pattern modulo some transformation group, in particular, similarity transformations (combinations of translations, rotations, and scalings).

Taking the notion of shape a little broader, it can be considered as a single visual entity, which is different from a single *geometric* entity. For example, a finite point set is not a single geometric entity, but can well be perceived as a single visual entity, a shape. Perceptual aspects are important when it comes to shape sensed by human vision. Apart

[1] Institute of Computing and Information Sciences, Utrecht University, Padualaan 14, 3584 CH Utrecht, The Netherlands. Remco.Veltkamp@cs.uu.nl.

from geometry, the notion of shape is also concerned with structure (connectivity, part–whole relations), attributes (color, texture, labels), semantics (functionality, features), and interaction with time (animation, deformation, morphing).

The big challenge in shape algorithmics for the next 10 years is the processing of shape information in a perceptually and semantically relevant manner, that is both guaranteed effective and efficient. This is becoming a real issue since ever more information is being conveyed in multidimensional and multimedia formats. Shape algorithmics plays a role in the encoding of shape such as in the MPEG-4 video standard, and the content description and retrieval as with the extensible standard MPEG-7. An even bigger role for shape algorithmics is in the so-called fourth wave in multimedia: three-dimensional models and scenes.

2. Contributions. The current issue contains 13 papers, selected from a total of 40 submissions. There are many ways in which these contributions can be categorized. The following one is certainly not the only reasonable one; note also that the chosen categories are not mutually exclusive.

Discrete Shape. Two papers deal with shape in the digital space: shapes described as sets of points in $\mathbb{Z}^2$ (pixels) or $\mathbb{Z}^3$ (voxels). The geometry on integer lattices goes under various names, such as digital geometry and arithmetic geometry, and has many algebraic aspects. One typical issue is the conversion from continuous to discrete shape representations and the reverse reconstruction.

Eckhardt and Reiter discuss the problem of decomposing the boundary of a digital planar object into concave and convex polygonal parts. The discrete nature of the shape makes the problem difficult. They propose an algorithm that runs in linear time.

Sivignon, Dupont, and Chassery also deal with the problem of conversion of a digital object. They consider three-dimensional objects, and want a conversion into a polyhedral model. As a step toward this goal, they have developed an algorithm to decompose the discrete volume into pieces of digital planes.

Shape Matching. The shape matching problem can be formulated in different ways. The computation problem is to compute how similar two objects are with respect to some chosen similarity measure. The optimization problem is to find a transformation that minimizes a chosen similarity measure.

Alt, Knauer, and Wenk consider a problem of the first type. They investigate and compare the properties of the Hausdorff distance and the Fréchet distance for measuring the similarity between two planar curves.

Gavrilov, Indyk, Motwani, and Venkatasubramanian have developed algorithms for matching point patterns in two and three dimensions. They take a geometric approach as well as a string matching approach. In addition to a theoretical analysis, they make an experimental comparison with other known algorithms.

Rosenhahn, Perwass, and Sommer look at pose estimation, which can be considered an instance of the optimization problem of matching, where the pose parameters must be found that give the best match between three-dimensional model data and two-dimensional image data. They use the framework of geometric (Clifford) algebras for the geometric analysis of the problem.

Approximation. Approximation is the process of representing a given shape differently, under certain constraints, typically with a small approximation error. Often the constraint is to approximate the shape with a smaller number of constituting elements such that the similarity with the original is still good, in terms of a chosen error measure. The following three papers demonstrate three different instantiations of the problem.

Chew and Kedem take as original shapes a collection of protein molecules, and find an approximation to all of them, a so-called consensus shape. This shape then provides a compact representation of the significant structural information of the whole collection.

Daescu takes as original shape a two- or three-dimensional polyline. Among other things, he presents an output sensitive algorithm for finding the approximating polyline with the minimum number of vertices, keeping the approximation error below a given threshold.

Efrat, Hoffmann, Knauer, Kriegel, Rote, and Wenk consider as input a set of spots (points). They present an algorithm that finds a set of ellipses that cover a set of required points and do not cover given forbidden points. The required point set is thus approximated with a small number of ellipses, close to the minimum number of ellipses.

Analysis. There is a plethora of properties of shapes that can be of interest. The next five papers all present different ways to interrogate shapes so as to provide various ways of analysis.

Bose and Morin consider the problem of assessing the roundness of two- or three-dimensional objects from a set of probed points. In this way the quality of disks and balls can be measured. They present an analysis of the problem, and an algorithm to compute the quality.

Dey and Zhao look into the problem of computing the medial axis of a three-dimensional shape, a skeletal structure often used to analyze shapes. In contrast with the two-dimensional case, the three-dimensional medial axis is difficult to compute. They present an algorithm to construct a skeleton similar to the medial axis, and prove that it converges to the medial axis.

Kazhdan, Chazelle, Dobkin, Funkhouser, and Rusinkiewicz present a shape descriptor that describes the extent to which the shape is reflective symmetric, and give algorithms to compute it. Apart from a theoretical analysis, they show experimental results about robustness with respect to noise and point sampling. It is shown how the descriptor performs when used for shape retrieval.

Mortara, Patanè, Spagnuolo, Falcidieno, and Rossignac present an analysis method for the characterization of the local behavior of polyhedral surfaces. They describe a multiresolution approach, looking at the surface neighborhood that falls inside balls of varying size. The way the curvature then varies indicates the type of surface.

Pascucci and Cole-McLaughlin, finally, introduce algorithms for the computation of the relations between the connected components of the level sets (iso-value surfaces) in a three-dimensional scalar field, such as the number of through-holes. Apart from a theoretical analysis of the time complexity, the performance of a parallel implementation is shown.

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