The Study of Syncopation using Inner Metric Analysis: Linking Theoretical and Experimental Analysis of Metre in Music

Anja Volk

Utrecht University

Department of Information and Computing Sciences

Postal address: PO Box 80089, 3508 TB Utrecht, The Netherlands

TEL +31(20) 4628507

FAX +31(20) 4628555

http://people.cs.uu.nl/volk/

EMAIL volk@cs.uu.nl

Draft version of the article published as following

Anja Volk. The Study of Syncopation using Inner Metric Analysis: Linking Theoretical and Experimental Analysis of Metre in Music. Journal of New Music Research, 37(4):259-273, 2008.
Download article: http://dx.doi.org/10.1080/09298210802680758

Abstract

This paper investigates the influence of syncopation on the metric structure of musical pieces using the computational model of Inner Metric Analysis. Inner Metric Analysis generates metric hierarchies evoked by the note onsets of a piece. Syncopation in the rhythmic structure influences these hierarchies in different ways. The comparison of local and global perspectives on the metric structure allows to distinguish between different amounts of syncopation present in a musical piece. This paper shows that the study of syncopation using Inner Metric Analysis contributes to explaining tapping performances of listeners. Hence comparing the structural descriptions generated by the model to results of a listening experiment helps to link music theoretic and perceptual studies.

1 Introduction

The complexity of the metric structure of music is a challenging research topic in both music theory and investigations concerning music perception and cognition. While music theorists investigate large-scale interrelations within the compositional architecture, such as the induction of hypermetre (Cohn, 1992) or the contribution of metre to the phrase structure of a work (Horlacher, 1992), empirical studies focus on the metric structure experienced while listening to the music. Though the perception of metric phenomena is obviously linked to the structures addressed by music theorists, the methods and results of these two research directions differ to a great extent. For instance, the in-depth structural analyses in music theory, which are often motivated by a personal listening experience, are hardly accompanied by empirical tests. Rather, music theory is concerned with finding structures in pieces which might enrich our experience of these pieces (Temperley, 2001). On the other hand, cognitive studies focus on more general perceptual questions, for instance, on how people tap to music (Snyder & Krumhansl, 2001), perceive the tempo (McAuly & Semple, 1999) or infer a metric interpretation while listening to the piece (Longuet-Higgins & Lee, 1984). However, these perceptual investigations often do not reach the complexity of metric descriptions provided in music theory, such as in how far the metric structure of a work contributes to the overall architecture of a musical piece.

Computational approaches to metre in music often address perceptual questions, such as the induction of the most salient beat within the listening process (Povel & Essens, 1985; Parncutt, 1994; Toiviainen & Snyder, 2003). Furthermore, a large number of models aims at the automatic extraction of metric information in the context of music information retrieval tasks, such as the determination of the beat, the determination of the tempo or the transcription of rhythm (Scheirer, 1998; Goto & Muraoka, 1999; Eck, 2001; Dixon, 2001; Tzanetakis, Essl, & Cook, 2002; McKinney & Moelants, 2004; Klapuri, Eronen, & Astola, 2006; Ellis, 2007; Davies & Plumbley, 2007; McKinney, Moelants, Davies, & Klapuri, 2007). However, the modelling of metric information in terms of a metric hierarchy going beyond the determination of the most salient beat, has received less attention (Steedman, 1977; Longuet-Higgins & Lee, 1984; Large & Kolen, 1994; Temperley, 2001; Volk, 2008b). Yet the induction of metric hierarchies by musical compositions serves as an important characterization of their time organization, since the metre of a work often cannot be reduced to the information provided by the time signature, as music theorists have shown (e.g. Krebs, 1999). Similarly, the tonal structure of a piece is not sufficiently described by the key signature, which does not contain information about modulations into other tonal regions within the piece. Hence, computational models have been developed to determine regions of stable key and regions of modulation (e.g. Chew, 2006) in order to capture the tonal organization of a piece of music.

Similarly, computational models inducing metric hierarchies contribute to the structural description of the time organization of musical pieces going beyond the most salient beat as often investigated by perceptual models. For instance, Steedman (1977), Longuet-Higgins and Lee (1984) and Temperley (2001) assign a metric interpretation to a piece that includes several hierarchical levels. While Steedman (1977) and Longuet-Higgins and Lee (1984) model the metric interpretation of monophonic pieces, the metric model in Temperley (2001) allows the processing of polyphonic pieces; it has mostly been applied to pieces of the common-practice era (Bach to Brahms). For a given piece the model generates five different metric levels (the tactus and two higher and lower levels respectively). In contrast to this, the computational model of Inner Metric Analysis (Fleischer et al., 2000; Mazzola, 2002; Volk, 2008b) does not premise how many levels have to be induced for a given piece. This enables the model to distinguish between pieces that are characterized by a strict metricity (hence induce several metric levels) and pieces that do not follow a strict metricity and hence lack stable metric levels.

Inner Metric Analysis generates metric weight profiles for a musical piece.[1] These weight profiles exhibit several layers for metrically strict pieces (such as dance pieces, see Fleischer 2003). However, the lack of layers in the metric weight profile serves as an adequate description of pieces that avoid a regular temporal organization, as often found in pieces outside of the common-practice era, such as pieces by Stravinsky, Webern or Xennakis (Volk, 2004a; Volk, 2007; Volk, 2008b). Furthermore, within a piece the weight layers can considerably change or dissolve, such that Inner Metric Analysis allows the description of the metre of musical pieces between the poles of persistence and change. Hence, Volk and Chew (2008) used Inner Metric Analysis to determine sections within a piece of stable metric organization. These sections correspond to structural units of the piece as observed by Lewin (1981). The comparison of the outcome of the model to music theoretic analyses in Fleischer (2003), Volk (2004b), Volk (2008b) and Volk and Chew (2008) demonstrated the model's potential to address complex metric situations studied in music theory.

The weight profiles induced by Inner Metric Analysis hence serve as a fine-grain structural description of the time organization of a musical piece. This paper investigates the influence of rhythmic syncopation on the metric structure of musical pieces.[2] Syncopation has been studied in music theory as a phenomenon that creates complex rhythmic structures that contribute to the overall formal design of a composition (Morrison, 1992; Temperley, 1999; Roeder, 2004). Cognitive studies investigated, for instance, in how far syncopated rhythms influence listeners' ability to perceive and produce short rhythmic patterns (Fitch & Rosenfeld, 2007). In this paper, syncopation is studied within an analytic approach that characterizes the effect of syncopation on the induction of metric hierarchies. In a second step, the model is applied to ragtimes that have been used for

[1]A piece is given in its most general form, the score, in order to abstract from performance information that any musician would add to the information provided by the score. This allows to investigate the metre of pieces on a general structural level.

[2]Huron (2006) discusses the phenomenon of syncopation in other domains, such as dynamic or harmonic syncopation. However, in this paper the use of the term 'syncopation' always refers to rhythmic syncopation.

a tapping experiment in Snyder and Krumhansl (2001). The comparison of the model's output to these tapping performances allows to link the structural description provided by Inner Metric Analysis to questions of perceptual studies.

The paper is organized as follows. Section 2 introduces the model, Section 3 investigates different analytic perspectives on how syncopation influences the induction of metric hierarchies, Section 4 applies these analytic perspectives to ragtimes used in an experiment by Snyder and Krumhansl (2001). Basic terms of the model are listed in the glossary at the end of this paper.

2 The Model of Inner Metric Analysis

Inner Metric Analysis provides a structural description of metre of musical pieces by inducing two types of weights (*metric* and *spectral* weights) for each note that are based on notes' onsets. These weights serve as a quantification of the note's metric importance within the piece, disregarding the given time signature and bar lines. Hence, the metric structure described by Inner Metric Analysis is independent of the bar lines. Although perceptual studies on metre often compare the output of the computational model to the location of bar lines within the piece[3], this may be counter to music theorists' insights that the bar lines often do not serve as an adequate reference point of the metric structure of a piece. For instance, Krebs (1999) has noted that a discrepancy between the metre of a piece's notes and that implied by the placement of the bar lines often creates appealing conflicts that should be considered an important ingredient of the metric structure. Therefore we distinguish between the *inner* metric structure evoked by the rhythmic configuration of the notes and the *outer* metric structure implied by the bar lines and time signature.

Inner Metric Analysis is based on score events that are equally distanced and form therefore a pulse. The methods of Inner Metric Analysis are rooted in Mathematical Music Theory. Mazzola (1985) introduced a geometrical description of the global structure of a musical piece by decomposing it into local objects. The mutual relations of the local objects are studied in terms of *simplicial complexes*[4]. This method has been applied to different analytic perspectives, such as to motivic analysis (Nestke, 2004) and underlies Inner Metric Analysis as well. The local objects in the metric model are the pulses, which are therefore called *local metres*.

[3]For instance, Desain and Honing (1999) evaluated the perceptual model in Longuet-Higgins and Lee (1984) by testing whether the model assigned the bar lines of national anthems "correctly" as given in the score.

[4]The term 'simplicial complex' refers to a particular topological space, consisting of points, lines, triangles, tetrahedons, and so forth, that are glued together. For an introduction into the terms of algebraic topology see Spanier (1966).

2.1 Pulses

In Inner Metric Analysis, a pulse consists of a series of at least three events that are equally spaced in time. Pulses have been used as the base for metric models both in experimental studies (Parncutt, 1987; Parncutt, 1994; Large, 2000; Large and Palmer, 2002; Toiviainen & Snyder, 2003) and in music theoretic approaches (Yeston, 1976; Lerdahl & Jackendoff, 1983; Krebs, 1999). A pulse (or *pulse stream* or *layer of motion*) consists of equally spaced events; for example, the most salient beat of a piece is one pulse present in the piece.[5] The superposition of pulses with the same phase (*nested* pulses) and with periods that are multiples of the periods of finer pulses as shown in Figure 1 has been used to explain the typical accent pattern of alternating strong and weak elements associated with a given time signature. Strong beats are those at which many pulses coincide, while at weak beats only few pulses coincide. However, music theorists have shown that often the pulses are not nested or do not have periods that are multiples of periods at the finer levels.

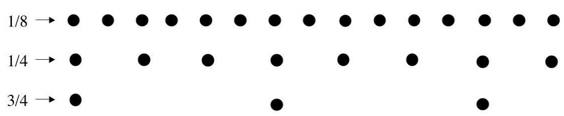

Figure 1: Superposition of three pulses in a $\frac{3}{4}$ metre.

One example of the latter situation in a $\frac{3}{4}$ metre occurs in Beethoven's *Eroica* shown in Figure 2. Krebs (1999) assigns to this example three different perceivable pulses indicated by numbers above and below the notes denoting the period of the pulse (as multiple of quarter notes). Two pulses have the period 3 with different phases and one pulse has the period 2.

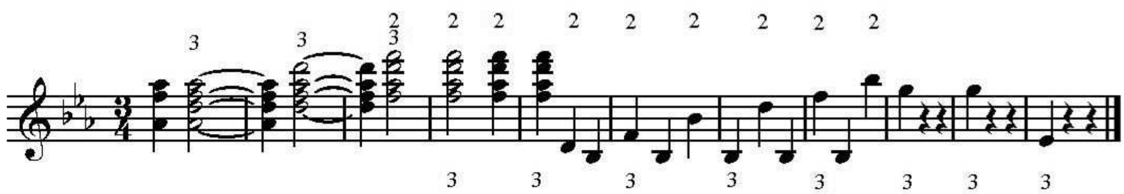

Figure 2: Three different pulses of Harald Krebs' analysis of Beethoven's *Eroica*, 1st movement, bars 681-690.

Inner Metric Analysis defines the pulses of a piece of music solely on the base of note onsets. The model considers all the pulses, called *local metres*, that can overlay with each other with very different periods and shifted at all phases. It does not take into account features often said to be

[5]Hence pulse in this context does not denote a single event, but a series of events.

important for the induction of metre such as dynamic accents or changes in the melodic and harmonic domains. Perhaps surprisingly, the time organization given by note onsets often provides highly differentiated metric information. Figure 3 shows an example illustrating the local metres using the first four bars of the middle section of Schubert's Moment Musical Op. 94 No. 4.

In order to determine the local metres of a given piece of music, the notes are projected onto the set On of all onsets of notes. The elements of the set On of the example in Figure 3 are given in the first row below the notes, denoting each onset as $\star$. By enumerating the onsets as multiples of sixteenth notes starting with 0 for the first onset, the set On consists of the elements $0, 1, 2, 6, 8, 9, 10, 14, 16, 17, 18, 22, 24, 25, 26,$ and 30. Within this set all subsets $m \subset On$ of equally spaced onsets are the candidates for the considered pulses which are called *local metres*. We consider a subset m as a local metre, if it contains at least three onsets and is maximal, meaning that it is not a subset of any other subset consisting of equally distanced onsets. Figure 3 shows all local metres enumerated as A, B, C, ... S. The dark circles $\bullet$ indicate the local metre, the triangles $\triangle$ illustrate the extension of the local metre throughout the entire piece that is discussed in the next section.

The condition of maximality can be easily illustrated by means of the local metre H. This local metre consists of the onsets $2, 6, 10, 14, 18, 22, 26,$ and 30. The local metre contains, for instance, the subsets $H' = \{2, 6, 10\}$ and $H'' = \{2, 10, 18, 26\}$ of equally distanced onsets which are therefore not maximal and hence not considered as local metres.

Each local metre can be identified with three parameters: the starting point or first onset s; the distance d between the consecutive onsets of the local metre (the period); and the number of repetitions k of the period which equals the number of onsets the local metre consists of minus 1, called the length. For instance, the local metre H starts at point $s = 2$, has a period of $d = 4$, consists of eight onsets and has hence a length of $k = 7$. Formally any local metre can be denoted as $m_{s,d,k} = \{s + id, i = 0, \ldots, k\}$, hence $H = m_{2,4,7}$. In Table 1 all local metres of the example are listed with their corresponding parameters s, d and k. The listed phase ph of a local metre is calculated as $ph = s \; modulo \; d$.

2.2 Metric weight

Based on the detection of all local metres in a given piece a metric weight for each onset is defined that reflects the amount of local metres that coincide at this onset. Hence, the basic idea is similar to the explanation of metric accent patterns arising from the superposition of pulses according to Figure 1. Onsets where many pulses coincide get a greater weight than onsets where fewer pulses coincide. Moreover, the intuition modelled in the metric weight is that longer repetitions should contribute more weight than shorter ones. The more stably established in a chain of successive and regular events, the more significant the onset. Hence, the weight does not simply count the

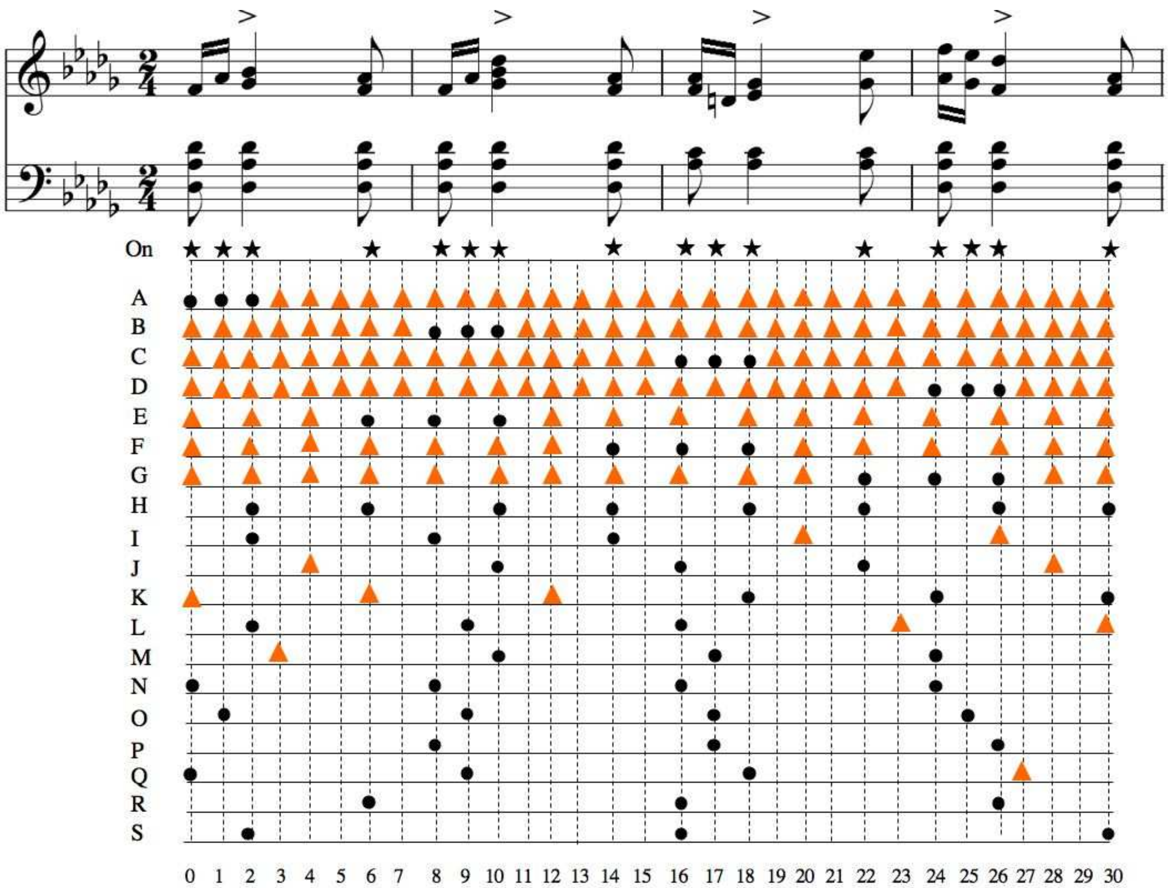

Figure 3: The Set On $\star$ and all local metres $\bullet$ and their extension $\triangle$ of bars 62-65 of Schubert's Moment Musical Op. 94 No. 4 (Beginning of the middle part).

number of local metres coinciding at a given onset but considers their length k.

local meter m	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
start onset s	0	8	16	24	6	14	22	2	2	10	18	2	10	0	1	8	0	6	2
period d	1	1	1	1	2	2	2	4	6	6	6	7	7	8	8	9	9	10	14
length k	2	2	2	2	2	2	2	7	2	2	2	2	2	3	3	2	2	2	2
phase ph	0	0	0	0	0	0	0	2	2	4	0	2	3	0	1	8	0	6	2

Table 1: List of all local meters of the example in Figure 3 with their corresponding parameters.

We first define the weight w_p of a local metre $m_{s,d,k}$ as the power function $w_p(m_{s,d,k}) = k^p$ with p as a variable parameter. We use the power function k^p in order to be able to test different amounts of influence of local metres depending on their length. The higher the value of p the greater the weight of longer local metres in comparison to shorter local metres.

The metric weight for each onset o is now calculated as the sum of weights $w_p(m_{s,d,k})$ of those local metres that inhere the onset o. The very first onset 0 in the example of Figure 3 participates in the local metres A, N and Q, which all coincide at the first onset. The sum of weights of the local metres w_p in this case equals $2^p + 3^p + 2^p$ since the length k of both A and Q is 2 and N has the length 3. We introduce a further variable parameter to the model which regulates the minimum length of the local metres denoted by ℓ. (Since a local metre must contain at least three onsets, ℓ cannot be smaller than 2). Local metres shorter than the minimum length ℓ are not considered in the calculation of the metric weight. Hence, the metric weight of a given onset o is the weighted sum of the length of all those local metres that coincide at o and have a length of a least ℓ. All examples in this paper are based on the parameters $p = \ell = 2$. Hence, the minimum length ℓ does not exclude any of the local metres; the influence of the local metre's length on the weight is quadratic.[6]

In mathematical terms, let $M(\ell)$ be the set of all local metres of the piece of size at least ℓ, that is to say, $M(\ell) = \{m_{s,d,k} : k \geq \ell\}$. The general metric weight of an onset, $o \in On$, is as follows:

$$W_{\ell,p}(o) = \sum_{\{m \in M(\ell): o \in m\}} k^p. \tag{1}$$

2.3 Spectral weight

The spectral weight (see Nestke & Noll, 2001) is based on the extension of each local metre throughout the entire piece. This extension corresponds to a certain extent to an idea discussed by Krebs (1999). He argues that pulses do not only affect the metric structure in regions where

[6]For a systematic variation of these parameters see Fleischer, 2002 and Fleischer, 2003.

they are active. Even after a pulse has stopped we may continue to count along this pulse in the following section. In the case of the spectral weights the local metres are extended also to the beginning of the piece. This extension models the effect of reinterpreting the past of a given event based on new incoming events.

The extension of a local metre $m_{s,d,k}$ is denoted as $ext(m_{s,d,k}) = \{s + id, \forall i\}$ with i as integer numbers. The additional elements of each local metre in the extension in the example of Figure 3 are indicated as triangles $\triangle$ such that the extension $ext(m_{s,d,k})$ consists of all dark circles $\bullet$ and triangles $\triangle$ in one row. In the spectral weight approach each local metre contributes a weight to all events in its extension. For example, the contribution $w_p(A)$ of the local metre A is added to the weight of all time points on a grid of sixteenth notes, because the extension of a local metre with period $d = 1$ meets all time points of this grid. The spectral weight allows the assignment of weights to silences as well, in contrast to the metric weight that is defined only on note onsets.

Hence, the difference between the metric and spectral weights for the very first onset is as follows. The metric weight of this onset depends on the contributions w_p of the local metres A, N and Q. The spectral weight depends in addition to these local metres on the contributions of the local metres B, C, D, E, F, G, and K since their extensions coincide at this first onset as well.

The spectral weight of a given onset or silence event t is the weighted sum of the length of all those local metres whose extensions coincide at t and have a length of a least ℓ. In mathematical terms, the spectral weight is defined as:

$$SW_{\ell,p}(t) = \sum_{\{m \in M(\ell): t \in ext(m)\}} k^p. \tag{2}$$

2.4 Metric and Spectral weights applied to Schubert's Moment Musical Op. 94 No. 4

To illustrate the use of the model, this section discusses the metric and spectral weight profiles of the excerpt from Schubert's Moment Musical Op. 94 No. 4 as shown in Figure 3. Both melody and rhythm of the middle section in D flat Major of this piece seem to suggest that the actual downbeat is located on the quarter note with the accent mark. This note forms a syncopation, since it is placed on a metrically weak position, while the following strong second beat position of the bar has no note onset. [7]

Figures 4 (a) and (b) show the metric and spectral weight profiles using the parameters $p = \ell = 2$. The higher the line, the greater the corresponding weight; the light-dark boundaries of the grayscale background indicate the bar lines as notated in the score.

[7] This notion of syncopation follows the characterization of syncopation as given in Grove Music Online: "Syncopation usually occurs in lines in which the strong beats receive no articulation. This means either that they are silent ... or that each note is articulated on a weak beat (or between two beats) and tied over to the next beat ..."

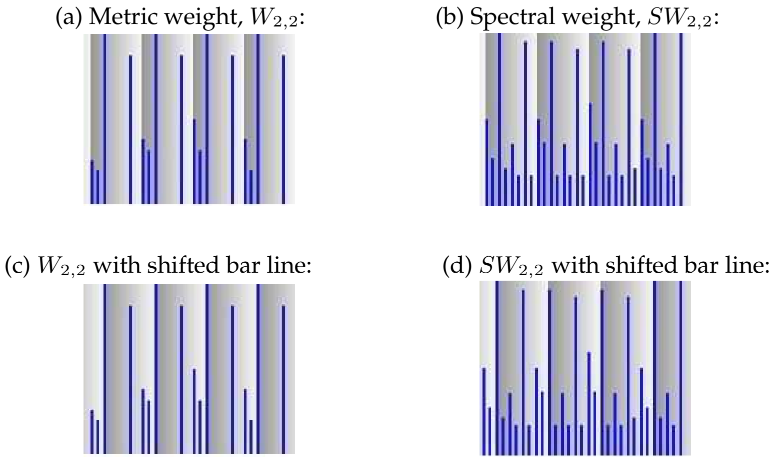

Figure 4: Upper Row: Metric (left) and spectral (right) weight profiles of bars 62-65 of Schubert's Moment Musical Op. 94 No. 4. Lower Row: The same weights as in the upper row with shifted bar lines in the background.

The metric weight profile of Figure 4 (a) reveals three different layers according to the height of the lines. A highest layer is built upon the weights of the second eighth note positions of all bars. A next layer is built upon the weights of the last notes in all bars (located at the fourth eighth note position). The lowest layer corresponds to the weights of the first two sixteenth notes in all bars. The spectral weight profile in Figure 4 (b) assigns weights to all silence events along the finest grid of sixteenth notes. All of these weights of the silence events contribute to the lowest layer as observed in the metric weight profile. However, within the spectral weight profile the first onsets in all bars form a separate layer.

Shifting the bar lines in the background by one eighth note shown in Figures 4 (c) and (d) reveals a correspondence between these layers and the hierarchy of the typical accent schema of a $\frac{2}{4}$ metre. The highest layers correspond to the highest layer of the first and second quarter notes of these shifted bars. In a typical $\frac{2}{4}$ metre, the second and fourth eighth note positions would form the next layer. In this case, the fourth eighth notes of all shifted bars (the original first onsets) gain prominence in the spectral weight profile over the second eighth note positions. Similarly, the last sixteenth note positions gain greater weights than the second, fourth and sixth eighth note positions of all shifted bars in the spectral weight profile of Figure 4 (d).

Hence, the inner metric structure is in line with the observation that not the first onset of the notated bars, but the quarter note with the accent mark might be understood as the actual downbeat by a listener[8]. This example illustrates that, remarkably, a graph that originates in

[8]The analysis of the entire middle section of the Moment Musical in Fleischer et al. (2000) reveals the same charac-

calculating the strength of the pulses to which each onset belongs, generates a metric hierarchy. Furthermore, the example shows that layers according to the inner metric structure may differ from the weight layers as induced by the outer metric structure. Due to the syncopation in all bars on the second eighth note positions the greatest weights are shifted to the second and fourth eighth note positions of the original score.

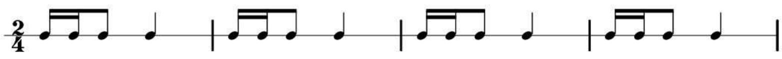

Figure 5: Adapted rhythm avoiding the syncopation in Schubert's Moment Musical Op. 94 No. 4.

Avoiding the syncopation by swapping the note values of the quarter and eighth notes in all bars (see note example in Figure 5) leads to different layers in the metric and spectral weight profiles. Figure 6 shows that the greatest metric weights coincide with the bar lines, followed by the layer of the second beat in all bars. Hence, in this case the inner metric structure is synchronous with the accents according to the outer metric structure.

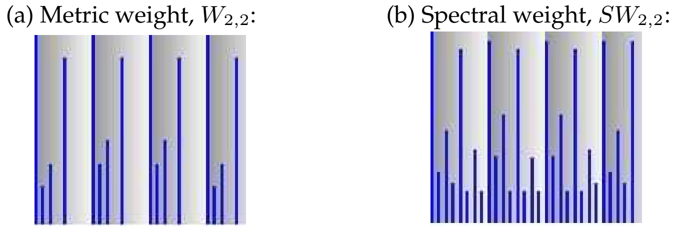

Figure 6: Metric (left) and spectral (right) weight profiles of the adapted rhythm avoiding the syncopation in Schubert's Moment Musical shown in Figure 5.

3 Local and global perspectives on the study of syncopation

The syncopation in the Schubert example of the previous section has a significant effect on the inner metric structure. Within music theory, the phenomenon of syncopation has hardly been formalized. Often an accent displacement is said to be responsible for the experience of syncopation, such that a metrically weak position gets a strong accent.[9] Longuet-Higgins and Lee (1984) introduce the calculation of a syncopation measure of rhythmic patterns, which is based on the

teristic.

[9]See, for instance, the characterization of syncopation in Scholes and Nagley (2008): "The displacement of the normal musical accent from a strong beat to a weak one".

assumption that a note is perceived as syncopated if the ensuing stronger metrical position has no note onset. This measure has been tested within a recognition and reproduction task of short rhythmic patterns in $\frac{4}{4}$ metre by Fitch and Rosenfeld (2007). Rhythm complexity measures (including syncopation measures) have been compared by Thul and Toussaint (2008) using different data sets, all containing short monophonic rhythmic patterns. However, within real musical compositions, syncopation often takes place within a context that consists of more than one monophonic line. The study of syncopation in this paper extends these studies based on short monophonic rhythmic patterns by analyzing complex ragtime pieces.

In Fleischer (2003) I have used the metric weight approach to introduce a concept of *metric coherence*. This concept denotes cases where the inner and outer metric structures concur—that is, where the metric structure evoked by the notes is synchronous with the normative metric state given by the bar lines, such as in the weight profiles of Figure 6. The application of Inner Metric Analysis to compositions by Dufay, Ockeghem, Wilbye, Morley, Händel, Bach, Mozart, Beethoven, Schubert, Schumann, and others in Fleischer (2003) demonstrates that metric coherence is usual in a number of works noted for their strict metricity. However, the example from Schubert's *Moment Musical* in the previous section illustrates that a syncopation leads to a discrepancy between the layers according to the inner and outer metric structures respectively. Avoiding the syncopation of the example leads to layers in the weight profiles that are synchronous with the metric levels according to the outer metric structure. Hence, syncopation prevents metric coherence.

The syncopation in the Schubert example leads to a shift of weight layers. The example from Beethoven's *Eroica* shown in Figure 2 illustrate a different effect on how the syncopation influences the inner metric structure. The metric and spectral weight profiles in the upper row of Figure 7 do not reveal weight layers corresponding to a $\frac{3}{4}$ metre that are shifted towards the syncopation as in the Schubert example. In some of the bars the greatest weight is located on the first beat, in others on the second or on the third beat. Hence, the syncopation in this context has different effects.

As Krebs (1999) points out, the phrase contains pulses that have competing periods of 2 and 3 respectively shown in Figure 2. The syncopation in the first two bars contribute to a pulse of period 3, the syncopation in the third bar contributes to this pulse of period 3 as well as to a pulse of period 2. The exclusion of local metres of period 2 and 3 respectively demonstrates that the overlapping of these competing periods prevents metric coherence in this example.

Excluding all local metres of period 2 leads to metric coherence in the weight profiles as shown in the middle row of Figure 7. The highest weights are located on the first beats of bars 6-9 in the metric weight profile (corresponding to bars 686-689 of the score). This structure is extended throughout the entire segment in the spectral weight profile (with the exception of the third bar).

The exclusion of all local metres of period 3 leads to weight layers in bars 6-9 of the metric weight that corresponds to a $\frac{2}{4}$ metre (lowest row in Figure 7). The spectral weight profile contin-

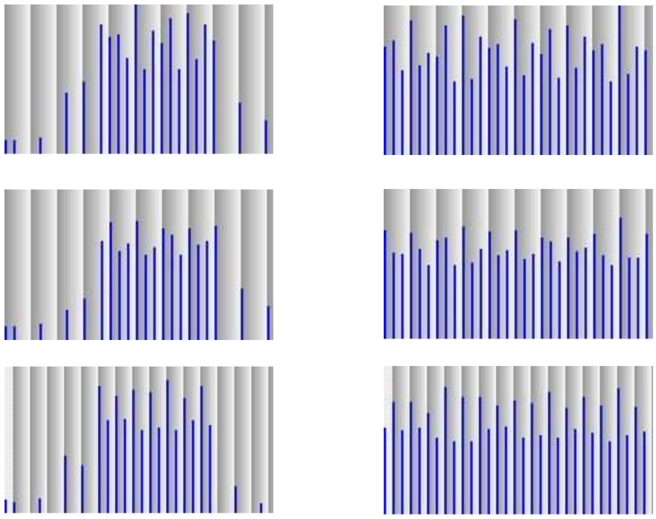

Figure 7: Upper Row: Metric (left) and spectral (right) weight profiles of bars 681-690 of Beethoven's *Eroica*, 1st mvmt. Middle Row: The same weights as in the upper row after excluding local meters of period 2. Bottom Row: The same weights after excluding local meters of period 3, interpreted with bar lines as a $\frac{2}{4}$.

ues this structure throughout the entire segment showing the predominant metric characteristic of this segment. Hence, competing local metres of periods 3 and 2 prevent the generation of weight layers according to a $\frac{3}{4}$ metre.

The comparison of the metric and spectral weight profiles of the last example points to different analytic perspectives on the inner metric structure. While the metric weight exhibits the respective layers only in the middle part of this segment, the spectral weight ignores local changes and shows these layers throughout the entire segment. As shown in Volk (2008a), the metric and spectral weights provide different perspectives on the metric structure with respect to local and global information. The metric weight is sensitive to local changes and can distinguish sections or units of differing metric characteristics. The spectral weight on the contrast reacts more robust on local changes and hence describes the most dominant metric characteristic of a given piece or section. In the following these different perspectives are used in order to characterise different amounts of syncopation in ragtimes.

The first 36 bars of the *Nonpareil Ragtime* by Scott Joplin have been used in an experiment by Snyder and Krumhansl (2001) to be discussed in section 4. The metric weight profile of the right-hand part of this segment in the upper row of Figure 8 assigns the highest metric weights to the first beats of all bars. Interestingly there is no layer associated with the second beat of the bars, as might be expected in a $\frac{2}{4}$ metre.

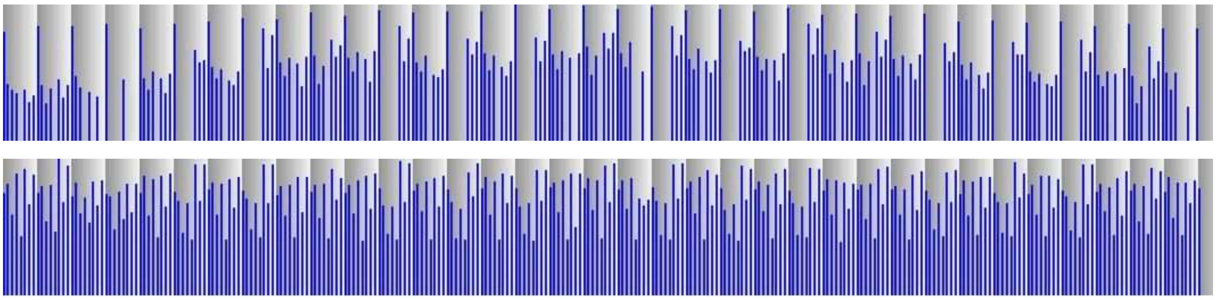

Figure 8: Metric (above) and Spectral (below) weights of the *Nonpareil Rag* by Scott Joplin for the right-hand part (analysis of an excerpt of the piece used by Snyder and Krumhansl, 2001).

In contrast to this, the spectral weight profile in the lower row of Figure 8 assigns in most of the bars the highest weights to the second, fourth, sixth and eighth sixteenth notes instead of the first beat. Hence, an offbeat structure is created. The striking difference between the metric and spectral weights is mainly due to 13 short local metres of the phase $ph = 1/8$ and the period $d = 1/8$. In the local perspective of the metric weight these local metres are too short in order to affect the role of the strong first beats of all bars because the latter form a very long local metre. However since all of the 13 short local metres have the same phase, the 13 corresponding extensions contribute to the weights of the same onsets – the second, fourth, sixth and eighth sixteenth notes in each bar. Hence, the weights of these sixteenth notes in the spectral weight are significantly higher than the other weights within the bars. The formation of these local metres is due to the syncopation of the right-hand part. Tying either the fourth sixteenth note to the following note or the first main beat to the fifth sixteenth note creates the syncopation as shown in the left example of Figure 10. Nevertheless, the syncopation does not prevent great metric weights on the first beat of all bars in the local perspective of the metric weight profile.

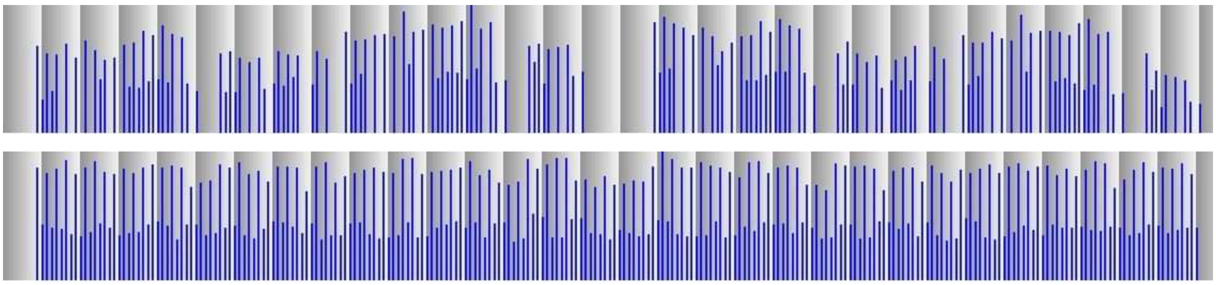

Figure 9: Metric (above) and Spectral (below) weights of Joseph Lamb's *American Beauty Rag* for the right-hand part (analysis of an excerpt of the piece used by Snyder and Krumhansl, 2001).

By contrast, the analysis of the right-hand part of the first 31 bars of Joseph Lamb's *American*

Beauty Rag used by Snyder and Krumhansl (2001) reveals an offbeat structure in both the metric and spectral weight profiles due to high weights on the second, fourth, sixth and eighth sixteenth notes (see Figure 9). Hence, even from a local perspective the bar lines are not detected. An analysis of the score shows that the amount of syncopation in this piece is much more exaggerated than in the *Nonpareil Rag*, as the short excerpt for the beginning in the right example of Figure 10 illustrates. The syncopations are not restricted to ties in the middle of the bar as in the *Nonpareil Rag*, as syncopations on the second and sixth sixteenth notes of the bars in the left example of Figure 10 show. They create in many bars an offbeat pattern in the right-hand part.

Hence, the different relations between the global and local perspectives concerning these two ragtimes expressed by the metric and spectral weights respectively reflect the different amount of syncopation present in the compositions.

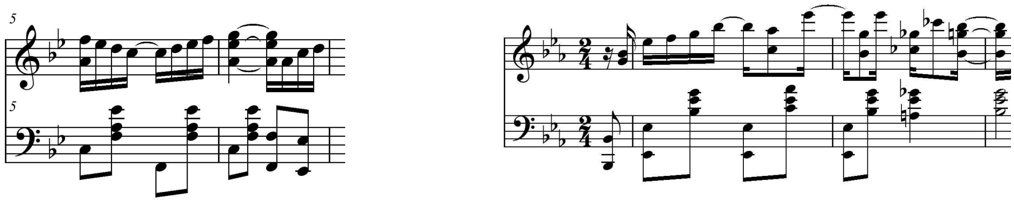

Figure 10: Left: Bars 5 and 6 of the *Nonpareil Rag*. Right: Bars 1 and following of the *American Beauty Rag*.

This section has demonstrated how syncopation effects the inner metric structure. While the syncopation in Schubert's *Moment Musical* leads to a shift of the layers of the inner metric structure, syncopation prevents the emergence of layers according to the $\frac{3}{4}$ metre in the Beethoven example. The comparison of the metric and spectral weight profiles of the ragtimes has revealed different amounts of syncopation in different pieces. Furthermore, the occurrence of great weights on weak metrical positions is a formalization of the common characterization of syncopation as a shifting of accent.

4 Revisiting Tapping to Ragtimes

Ragtimes are characterized by syncopation in the right-hand part. They have been used in a tapping experiment by Snyder and Krumhansl (2001) in order to study the perception of metric structures. Concerning the model's potential to address the perception of metric structures, Inner Metric Analysis has been tested by Fleischer (2003) on short rhythmical patterns used within a rhythm reproduction task by Povel and Essens (1985). This section applies Inner Metric Analysis to the rhythmically more complex ragtimes and relates the influence of syncopation on the inner metric structure to the observed tapping performance.

The listening experiments by Snyder and Krumhansl (2001) investigated the role of *pitch* and the contribution of the right-hand versus left-hand parts on the perception of metric structures using ragtimes. Hence, listeners tapped to different versions of these ragtimes. First, tapping performances along ragtimes with pitch information were compared to tapping performances along ragtimes with no pitch information. Second, tapping along versions using right- and left-hand parts *together* were compared to tapping along versions with right-hand parts *only*. This section examines the results obtained with seven different ragtime pieces, namely Scott Joplin's *Lily Queen*, *Nonpareil* and *Chrysanthemum Rags*, Will Nash's *Glad Cat Rag*, Raymond Birch's *Blue Goose Rag* and Joseph Lamb's *American Beauty* and *Sensation Rags*.[10]

As a result of the experiment no significant difference was detected concerning the ability of the listeners to tap to the pitch version versus the non-pitch version. Hence, pitch in these examples did not serve as a major clue for metre detection. This is contrary to earlier experimental findings about the role of pitch information reported by Snyder and Krumhansl (2001) that served as a motivation for this experiment. However, a significant decrease in the ability of participants to tap along the music was stated when the right-hand part only was presented in comparison to the full texture of the left- and right-hand parts.

Most listeners tapped to a pulse on the first and fifth sixteenth note positions in the bars. In order to explain the prominence of this pulse, Snyder and Krumhansl investigated the isolation of different musical dimensions that might have been cues for the listeners. The authors compared mean distributions at each sixteenth-note metrical position of musical events for different dimensions. For the right-hand part the number of notes per onset reveals moderate peaks at the strong first and fifth sixteenth notes in correspondence to the pulse that listeners tapped to. However, another peak occurs at the seventh sixteenth note, which is in contrast to this pulse. Moreover, the strongest peaks for the number of onsets for the left hand part appear on the third and seventh sixteenth notes. Hence the number of note onsets in this case did not prove to be an appropriate means to detect the downbeats.

Furthermore, Snyder and Krumhansl calculated autocorrelation coefficients for time lags in order to isolate musical dimensions that are responsible for giving cues to the listeners as to where the downbeats are located. The analysis for the left hand indeed explains the period of the pulse that most listeners tapped to, the phase of the pulse remains an open question.

In the following it is argued that Inner Metric Analysis can provide a deeper understanding of the experimental results by means of more detailed analytical perspectives on the ragtimes. The model helps explaining the most salient beat listeners tapped to, the importance of the time

[10] In Snyder and Krumhansl (2001) these results are obtained within the second experiment.

organization as well as the different roles of the right-and left-hand parts for the perception of the metric structure.

4.1 Resume of the Results using Inner Metric Analysis for the Study of Ragtimes

This section discusses the analyses of the left-hand part and the combined right- and left-hand parts of the ragtimes in order to explain the most prominent tapping pulse of the listeners.

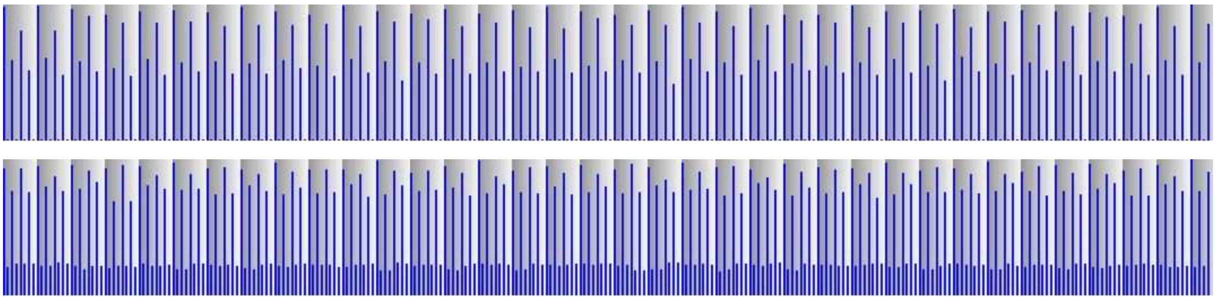

Figure 11: Upper Row: Spectral weight of Scott Joplin's *Nonpareil Rag* for the left-hand part. Lower Row: Spectral weight for both hands.

The analysis of the left-hand part of Scott Joplin's *Nonpareil Rag*[11] in the upper row of Figure 11 shows that the left-hand part reveals the down beat pulse on which most listeners tapped to. The greatest weights are located on the first and second quarter notes of the $\frac{2}{4}$ metre. This is in contrast to the analyses of the right-hand part (see Figure 8). The metric weight profile does not show great metric weights on the second beat of the bars for the right-hand part. The spectral weight profile shows neither on the first nor on the second beat great metric weights. Since the left-hand part is characterised by metric coherence this part mediates the strong downbeats of the $\frac{2}{4}$ bars of the ragtime. The same applies to the other ragtimes used in the experiment. In five out of seven cases the left-hand part is characterised by metric coherence, hence the layers of the weight profiles are synchronous with the metric levels implied by the $\frac{2}{4}$ metre.[12] Examples of excerpts are given in Figure 12 concerning the *Blue Goose Rag*, the *Chrysanthemum Rag* and the *American Beauty Rag*. Given that the pitch structure of the left hand often seems to provide important clues as to where the downbeats are located (with low pitches on the downbeats, see Figure 10), it is rather surprising that the time organization alone provides sufficient information on the metric structure.

Furthermore, the analysis of *both* hands of the *Nonpareil Rag* in the lower row of Figure 11

[11] All ragtime analyses in this and the previous sections concern the segments used in the second experiment in Snyder and Krumhansl (2001).

[12] The analyses of all ragtime pieces are listed in the appendix in Volk (2008a).

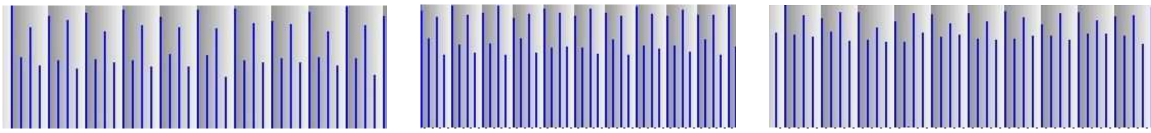

Figure 12: Excerpts from spectral weights of the left-hand parts of the ragtimes *Blue Goose* (left), *Chrysanthemum* (middle) and *American Beauty* (right).

results in the same highest layers as for the left-hand part while adding a layer on the weak eighteenth note positions. The highest weights are located on the first and third beats, the second and fourth eighth notes gain smaller weights. Further examples showing excerpts from the corresponding weights of the combined left- and right-hand parts in Figure 13 demonstrate that this is the regular case. In five out of seven ragtimes the inner metric structure of both hands reveals metric coherence. Since Inner Metric Analysis does not depend on pitch structure, these findings explain that the time information is sufficient for the understanding of the metric structure of these pieces. Therefore, no significant difference could be stated concerning the ability of the listeners to tap along the downbeats after removing the pitch information.

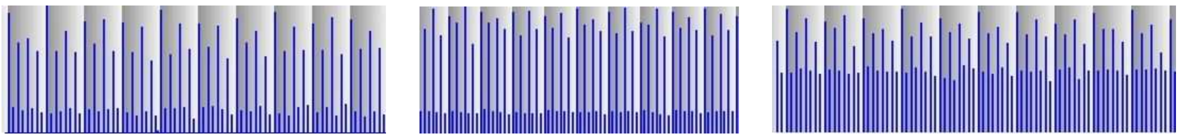

Figure 13: Excerpts from spectral weights of both hands (*Blue Goose Rag*, *Chrysanthemum Rag*, *American Beauty Rag*).

4.2 Syncopation in the right-hand part

Presenting the right-hand part only to the listeners resulted in a significant decrease of the tapping accuracy. The discussion of the analyses of the right-hand part of the *Nonpareil Rag* and the *American Beauty Rag* in section 3 has illustrated how syncopation influences the inner metric structure by preventing the emergence of metric coherence. This section compares these different effects of syncopation in the right-hand parts of the other ragtimes used in the experiment.

The lowest amount of syncopation in the right-hand part can be found in the *Chrysanthemum Rag*, *Blue Goose Rag* and *Lily Queen Rag*. Figure 14 shows examples concerning the *Chrysanthemum Rag*. The local perspective of the metric weight profile in the upper row of Figure 14 results in high weights on the first beats. The weights of the third, fifth and seventh sixteenth notes tend to create a second layer in some of the bars that is not very distinct. In contrast to this, the spectral weight profile in the lower row of Figure 14 does not show a strong first beat of the bar with a weight significantly above the other weights. Hence, from the global perspective the strength of the first beat is degraded in comparison to the local perspective. However, an offbeat, such as in the weight profiles of the *Nonpareil Rag* in Figure 8 and the *American Beauty Rag* in Figure 9, cannot be observed. The high weights located on the first, third, fifth and seventh sixteenth notes indicate a considerably smaller amount of syncopation in the right-hand part of this piece.

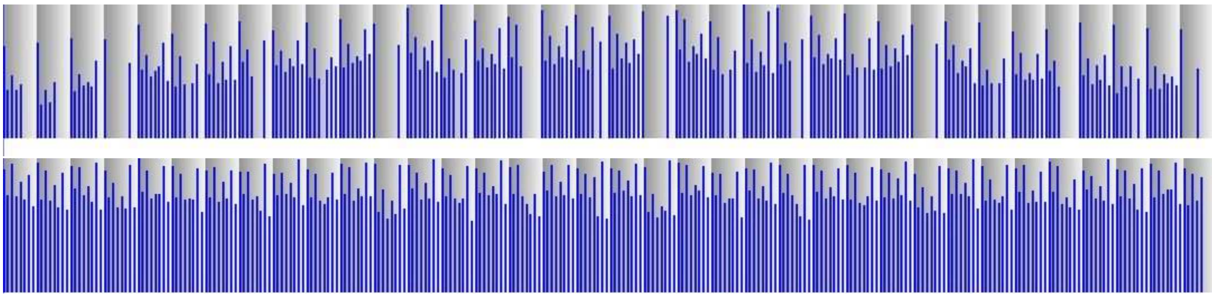

Figure 14: Metric (above) and Spectral (below) weights of the *Chrysanthemum Rag* for the right-hand part.

The metric structure of the right-hand part of the *Sensation Rag* by Joseph Lamb is similar to that of the *American Beauty Rag*. Even the metric weight profile shown in the upper row of Figure 15 does not reveal high weights on the first beats of the bar, but on the offbeat (the second, fourth, sixth and eighth sixteenth notes). The spectral weight profile confirms this metric characteristic in the lower row of Figure 15. One can then conclude that the amount of syncopation in the right-hand part of the *Sensation Rag* as well as of the *American Beauty Rag* exceeds those of the *Nonpareil* or *Chrysanthemum* Rags. Even the local perspective of the metric weight results in great metric weights on weak onsets with respect to the bar for both the *Sensation* and *American Beauty Rag*.

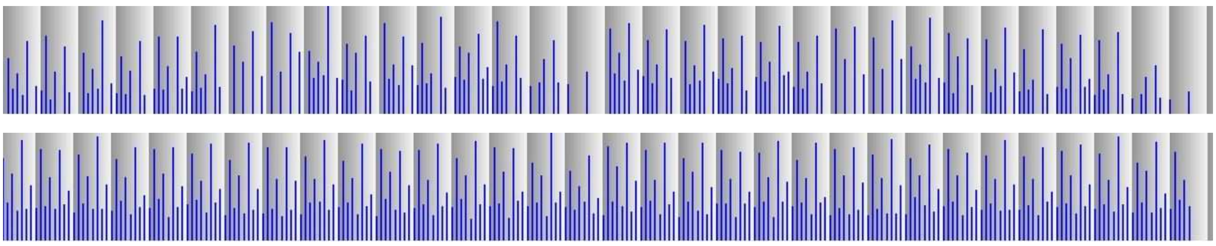

Figure 15: Metric (above) and Spectral (below) weights of the *Sensation Rag* by Joseph Lamb for the right-hand part.

In general the isolated analyses of the right-hand parts do not result in high weights on both the first and second main beats of the bar in contrast to the analyses of the left-hand parts. This explains the significant difference observed in the experiments concerning the ability to tap to the downbeats while listening to the right-hand part only. The *American Beauty Rag* and the *Sensation Rag* resulted both in very poor listeners' performances after the left-hand parts were removed. In each of these two pieces the strong syncopation observed in the analysis of the right-hand part caused significantly higher weights on the weak metric positions of the bar.

4.3 Syncopation in the left-hand part

Inner Metric Analysis revealed in five out of the seven ragtimes the downbeats on the first and second beats of the bars in the analysis of the left-hand part. This section discusses the *Sensation Rag* and *Glad Cat Rag*, which are not characterised by metric coherence in the left-hand part.

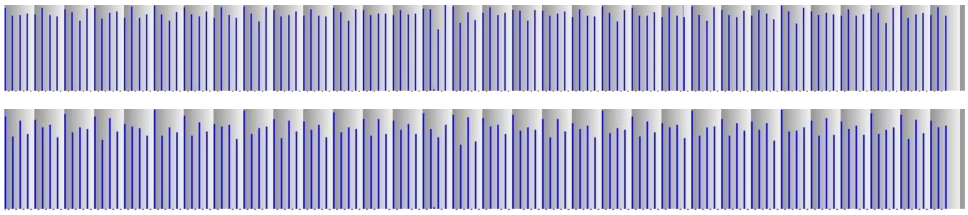

Figure 16: Above: Spectral weight of the *Sensation Rag* by Joseph Lamb for the left-hand part. Below: Spectral weight of the left-hand part after excluding two local meters.

The lack of metric coherence in the analysis of the left-hand part of the *Sensation Rag* (see upper row of Figure 16) is due to syncopation contributing to two local metres that are not in sync with the downbeats of the bars. The effect of these two long local metres on the inner metric structure is demonstrated in the lower picture of Figure 16 that shows the weight profile after the subtraction of their contributions. Here great weights are located on the strong first and second beats in many but not all bars. A syncopation prevents the emergence of significantly higher weights on the downbeats in the left-hand part of the *Glad Cat Rag* as well. Hence, the occurrence of syncopation is responsible that in these two ragtimes the left-hand part does not reveal the highest weights on the downbeats. However, only for the *Sensation Rag* consistently poor tapping performances have been observed by Snyder and Krumhansl (2001).

The application of Inner Metric Analysis to ragtimes in this section helped to understand important observations obtained in listening experiments by Snyder and Krumhansl. The metric and spectral weight profiles do not consider pitch information but show in most of the examples metric coherence in the analysis of both hands. This explains that the time information is sufficient in order to convey the metric structure of the pieces to the listeners. Hence, the comparison of the listeners' ability to tap along the full-pitch version of the ragtimes to the version with no pitch information revealed no significant difference in the experiment. The isolated analysis of the left-and right-hand parts demonstrated that the left hand is characterised by metric coherence, while the syncopated right hand does in most of the cases not show coherence. Interestingly, the strong syncopation in the right-hand part as a typical characteristic of ragtimes does not lead to a shift of accents within the inner metric structure within the polyphonic context of the combined left- and right-hand parts. This difference between the isolated right-hand part and the combined left- and right-hand parts as observed within the structural description of the ragtime pieces using Inner Metric Analysis corresponds to the significant difference observed in the performance of the tapping task by listeners using both hands parts versus using the right-hand part only.

4.4 Future direction: the unfolding of metric hierarchy over time

Both the metric and spectral weight profiles require a degree of abstraction, since the weight assigned to a given onset is not primarily a description of that note's perception within a moment to moment listening process. The weight of a note depends on past and future events in the score and uses the maximum of information available for a structural description of the piece.

However, within the listening processes this information might not always be available for the listener. In future research a combination of the non-procedural approach applied in this paper with a procedural method as introduced in Volk (2005) might allow a better approximation to listening (and tapping) processes. The procedural method in Volk (2005) uses both a sliding and a cumulative window approach. For instance, the cumulative window approach considers for a given note only its past, hence expectation or information about future events are excluded. For a piece consisting of n note onsets the cumulative window approach produces analysis windows $w_{t,\ t=2,\ldots,n}$, each containing the weight profile for all onsets $o_1, o_2, \ldots, o_t$.

Figure 17 shows six example windows using the cumulative window approach for the metric weight or the right-hand part of the *Nonpareil Rag*. The characteristic of the metric weight profile with great metric weights on the first beats of all bars (as in Figure 8) is achieved relatively late within the procedural approach. Even after 16 bars from the beginning (top picture of Figure 17) only the first four bars show a modest peak on the first beats of the bars. The following windows (from top to down) in Figure 17 reflect a cumulative strengthening of the role of the first onsets by constantly growing weights. However, other pieces studied with this approach in Volk (2005) gain stable metric layers much earlier when using this process of cumulating information. Those pieces might be experienced differently in their unfolding of metric hierarchy over time while listening to the piece. Hence, in future research the investigation of the emergence and re-organization of metric layers within such a procedural approach might capture a more detailed understanding of the perception of metric structures within listening processes.

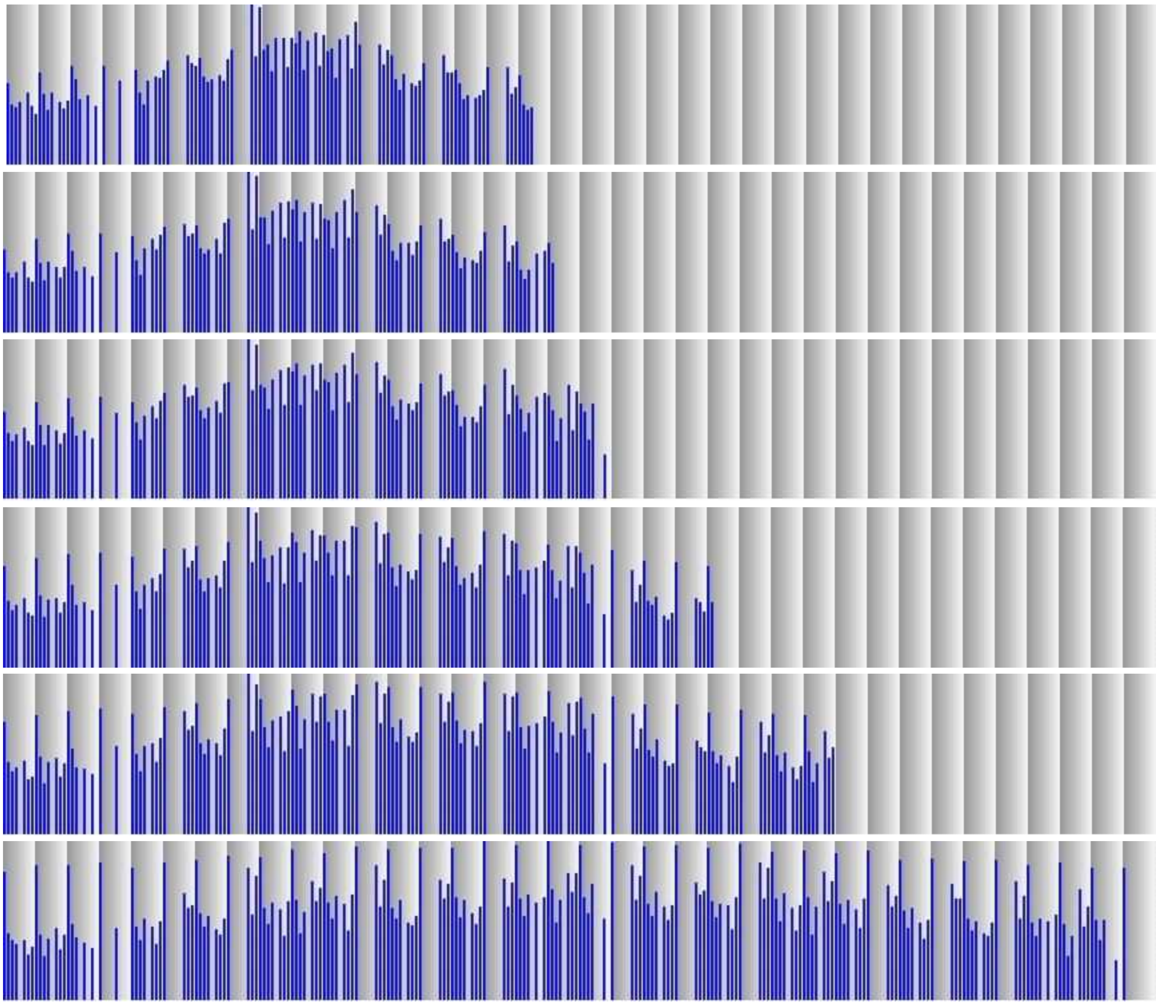

Figure 17: Six windows from the cumulative window approach for the right-hand part of the *Nonpareil Rag*.

5 Conclusion

This paper applied the model of Inner Metric Analysis to a structural description of musical pieces and to experimental findings obtained within a tapping task. Inner Metric Analysis realises a quantitative method of creating weight profiles based on pulse descriptions. The weight profiles exhibit layers such that a metric hierarchy is induced. The investigation in this paper has shown that syncopation prevents the concurrence of the weight layers and the metric levels associated with the bar lines. Hence, syncopation prevents metric coherence. The comparison of the metric and spectral weights allows to differentiate between varying amounts of syncopation present in a piece. Furthermore, the comparison of the left- and right-hand parts to the overall metric structure of the pieces allows the study of syncopation within polyphonic contexts.

The re-examination of the ragtimes applied in a tapping experiment using Inner Metric Analysis helps explaining listeners' ability to tap to these pieces. These findings show that Inner Metric Analysis introduces a new reference point for perceptual metric models. The bar lines do not always reveal the most plausible metric interpretation a listener might assign to a given note sequence. In the case of the experiment conducted by Snyder and Krumhansl (2001), the inner metric structure is a more appropriate means to describe the perceived structure. However, a performer might choose to deliberately stress the location of the bar lines in order to mediate the intended conflict. Hence, Inner Metric Analysis suggests that the relation between the inner and outer metric structures has to be considered in order to model the structure that the listener perceives. The application of Inner Metric Analysis to ragtimes demonstrates that the model successfully relates music theoretic structures to perceptual studies.

Acknowledgments. This research has been funded by the University of Southern California Women in Science and Engineering Postdoctoral Fellowship Award. This work made use of Integrated Media Systems Center Shared Facilities supported by the National Science Foundation under Cooperative Agreement No. EEC-9529152. Furthermore, part of this research has been funded by the WITCHCRAFT-project NWO 640-003-501 at Utrecht University. I especially thank Joel Snyder for the MIDI files of the ragtimes used in the listening experiment.

Glossary

inner metric structure: metric structure evoked by the notes

outer metric structure: metric structure associated with the time signature

local metre: maximal set of equally spaced onsets

p: power value for the weight of each local metre

ℓ: minimal length for local metres considered

$m_{s,d,k}$: a local metre m with starting point s, period d and length k

$w_p(m_{s,d,k}) = k^p$: the weight of the local metre $m_{s,d,k}$

$W_{\ell,p}(o)$: the metric weight of the onset o depending on the parameters ℓ and p

$ext(m_{s,d,k})$: extension of the local metre $m_{s,d,k}$ throughout the piece
$SW_{\ell,p}(t)$: the spectral weight of the time point t depending on the parameters ℓ and p

References

Chew, E. (2006). Regards on Two Regards by Messiaen: post-tonal music segmentation using pitch context distances in the Spiral Array. *Journal of New Music Research*, *34*, 341-354.

Cohn, R. (1992). The Dramatization of Hypermetric Conflicts in the Scherzo of Beethoven's Ninth Symphony. *19th-Century Music*, *15*, 188-206.

Davies, M., & Plumbley, M. (2007). Context-Dependent Beat Tracking of Musical Audio. *IEEE Transactions on Audio, Speech, and Language Processing*, *15*, 1009-1020.

Desain, P., & Honing, H. (1999). Computational Models of Beat Induction: The Rule Based Approach. *Journal of New Music Research*, *28*, 29-42.

Dixon, S. (2001). Automatic Extraction of Tempo and Beat From Expressive Performances. *Journal of New Music Research*, *30*, 39-58.

Eck, D. (2001). A Positive-Evidence Model for Rhythmical Beat Induction. *Journal of New Music Research*, *30*, 187-200.

Ellis, D. P. W. (2007). Beat Tracking by Dynamic Programming. *Journal of New Music Research*, *36*, 51-60.

Fitch, W., & Rosenfeld, A. (2007). Perception and Production of Syncopated Rhythms. *Music Perception*, *25*, 43-58.

Fleischer (Volk), A. (2002). A Model of Metric Coherence. In *Proceedings of the 2nd Conference Understanding and Creating Music.* Caserta.

Fleischer (Volk), A. (2003). *Die analytische Interpretation. Schritte zur Erschließung eines Forschungsfeldes am Beispiel der Metrik.* Berlin: dissertation.de - Verlag im Internet Gmbh.

Fleischer (Volk), A., Mazzola, G., & Noll, T. (2000). Computergestützte Musiktheorie. *Musiktheorie*, *4*, 314-325.

Fleischer (Volk), A., & Noll, T. (2002). Analytic Coherence and Performance Control. *Journal of New Music Research*, *31*, 239-247.

Goto, M., & Muraoka, Y. (1999). Real-time beat tracking for drumless audio signals. *Speech Communication*, *27*, 331-335.

Horlacher, G. (1992). The Rhythms of Reiteration: Formal Development in Stravinsky's Ostinati. *Music Theory Spectrum*, *14*, 171-187.

Huron, D. (2006). *Sweet Anticipation*. Cambridge, Massachusetts: MIT Press.

Klapuri, A. P., Eronen, A., & Astola, J. (2006). Analysis of the meter of acoustic musical signals. *IEEE Transaction on Speech and Audio Processing*, *14*, 342 - 355.

Krebs, H. (1999). *Fantasy Pieces: Metrical Dissonance in the Music of Robert Schumann*. Oxford University Press.

Large, E. W. (2000). On synchronizing movements to music. *Human Movement Science*, *19*, 527-566.

Large, E. W., & Kolen, J. (1994). Resonance and the Perception of Musical Meter. *Connection Science*, *6*, 177-208.

Large, E. W., & Palmer, C. (2002). Perceiving temporal regularity in music. *Cognitive Science*, *26*, 1-37.

Lerdahl, F., & Jackendoff, R. (1983). *A Generative Theory of Tonal Music*. Cambridge: MIT Press.

Lewin, D. (1981). On Harmony and Meter in Brahms's Op. 76, No. 8. *19th-Century Music*, *4*, 261-265.

Longuet-Higgins, H., & Lee, C. (1984). The Rhythmic Interpretation of Monophonic Music. *Music Perception*, *1*, 424-441.

Mazzola, G. (1985). *Gruppen und Kategorien in der Musik*. Berlin: Heldermann.

Mazzola, G. (2002). *The Topos of Music*. Basel: Birkhäuser.

McAuly, J., & Semple, P. (1999). The effect of tempo and musical experience on perceived beat. *Australian Journal of Psychology*, *51*, 176-187.

McKinney, M. F., & Moelants, D. (2004). Deviations from the resonance theory of tempo induction. In *Proceedings of the conference on interdisciplinary musicology.* Graz.

McKinney, M. F., Moelants, D., Davies, M. E. P., & Klapuri, A. (2007). Evaluation of Audio Beat Tracking and Music Tempo Extraction Algorithms. *Journal of New Music Research*, *36*, 1-16.

Morrison, C. (1992). Syncopation as Motive in Schoenberg's Op. 19, Nos 2, 3 and 4. *Music Analysis*, *11*, 75-93.

Nestke, A. (2004). Paradigmatic Motivic Analysis. In G. Mazzola, T. Noll, & E. Lluis-Puebla (Eds.), (p. 343-365). Osnabrück: epOs Music.

Nestke, A., & Noll, T. (2001). Inner Metric Analysis. *Tatra Mountains Publications*, 91-111.

Parncutt, R. (1987). The Perception of Pulse in Musical Rhythm. *Action and Perception in Rhythm and Music*, *55*, 127-138.

Parncutt, R. (1994). A Perceptual Model of Pulse Salience and Metrical Accent in Musical Rhythms. *Music Perception*, *11*, 409-464.

Povel, D. J., & Essens, P. (1985). Perception of Temporal Patterns. *Music Perception*, *2*, 411-440.

Roeder, J. (2004). Rhythmic Process and Form in Bartok's 'Syncopation'. *College Music Symposium*, *44*.

Scheirer, E. (1998). Tempo and beat analysis of acoustic musical signals. *JASA*, *103*, 588-601.

Scholes, P., & Nagley, J. (2008). *Syncopation. The Oxford Companion to Music. Oxford Music Online.*

Snyder, J., & Krumhansl, C. (2001). Tapping to Ragtime: Cues to Pulse Finding. *Music Perception*, *18*, 455-489.

Spanier, E. (1966). *Algebraic Topology*. New York: McGraw-Hill.

Steedman, M. (1977). The Perception of Musical Rhythm and Metre. *Quarterly Journal of Experimental Psychology*, *6*, 555-569.

Syncopation. Grove Music Online. Oxford Music Online. (2008).

Temperley, D. (1999). Syncopation in Rock: A Perceptual Perspective. *Popular Music*, *18*, 19-40.

Temperley, D. (2001). *The Cognition of Basic Musical Structures*. Cambridge, Massachusetts: MIT Press.

Thul, E., & Toussaint, G. (2008). Rhythm complexity measures: a comparison of mathematical models of human perception and performance. In *Proceedings of the ISMIR conference* (p. 663-668). Philadelphia.

Toiviainen, P., & Snyder, J. S. (2003). Tapping to Bach: Resonance-Based Modeling of Pulse. *Music Perception*, *21*, 43-80.

Tzanetakis, G., Essl, G., & Cook, P. (2002). Human Perception and Computer Extraction of Musical Beat Strength. In *Proceedings of the 5th International Conference on Digital Audio Effects.* Hamburg.

Volk, A. (2004a). Exploring the Interaction of Pulse Layers Regarding their Influence on Metrical Accents. In *Proceedings of the 8th International Conference on Music Perception and Cognition* (p. 435-438). Evanston, Illinois.

Volk, A. (2004b). Metric Investigations in Brahms' Symphonies. In G. Mazzola, T. Noll, & E. Lluis-Puebla (Eds.), *Perspectives of Mathematical and Computational Music Theory* (p. 300-329). Osnabrück: epOs Music.

Volk, A. (2005). Modeling a Processive Perspective on Meter in Music. In *Proceedings of the International Computer Music Conference* (p. 251-254). Barcelona.

Volk, A. (2007). Applying Inner Metric Analysis to 20th century compositions. In *Proceedings of the 1st Conference of the Society for Mathematics and Computation in Music.* Berlin.

Volk, A. (2008a). *The Generating of Metric Hierarchies using Inner Metric Analysis* (Tech. Rep. No. UU-CS-2008-006). Utrecht University.

Volk, A. (2008b). Persistence and Change: Local and Global Components of Meter Induction using Inner Metric Analysis. *Journal of Mathematics and Music*, *2:2*, 99-115.

Volk, A., & Chew, E. (2008). Reconsidering the Affinity between Metric and Tonal Structures in Brahms' Capriccio Op. 76, No. 8. *Computing in Musicology*, *15*, 138-171.

Yeston, M. (1976). *The Stratification of Musical Rhythm*. New Haven and London: Yale University Press.