Formalising arguments about norms

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Abstract. In most attempts to model legal systems as formal argumentation systems, legal norms are viewed as an argumentation's system inference rules. Since in formal argumentation systems inference rules are generally assumed to be fixed and independent from the inferences they enable, this approach fails to capture the dialectical connection between norms and arguments, where on the one hand legal arguments are based on norms, and on the other hand the validity of norms depends on arguments. The validity of a new norm can be supported by referring to authoritative sources, such as legislation or precedent, but also through interpretations of such sources, or through analogies or *a contrario* arguments based on existing authoritative norms. In this contribution arguments about norms are modelled as the application of argument schemes to knowledge bases of facts and norms.

Keywords. Argumentation, Legal norms, Interpretation, analogy

1. Introduction: The derivation of new norms though legislative, interpretive and analogical arguments

Through legal arguments we support claims that certain norms are valid, and, a result of establishing the validity of such norms, we obtain the entitlement to use them in our legal reasoning, for deriving their legal consequences. Thus, by establishing that certain norms are legally valid, through appropriate arguments, we make such norms conclusions of our legal reasoning, which can be asserted in legal discourses and used to support further legal claims. The derivation of legal norms also has a recursive aspect: derived norms may enable the derivation of further norms, which may enable the derivation of further norms, and so on. Thus, a comprehensive model of legal reasoning must integrate arguments supporting the validity of norms and arguments using such norms for establishing specific claims. The model should include not only the derivation of norms, but also the derivation of the properties of them that are relevant to their use in reasoning, such as their applicability and their priorities with regard to other norms.

While various authors have discussed the derivation of legal norms through legal arguments, and have modelled this derivation in specific contexts, no comprehensive logical model yet covers this field. Providing such a comprehensive model, and showing how it can be applied to different kinds of norm-producing arguments, is the aim of this paper. For this purpose, first of all, we shall briefly describe some typical arguments dealing with the derivation of valid norms in the context of legislation, interpretation, analogy and a-contrario reasoning. Then we shall provide a formal model of multilevel legal reasoning, as an instance of the ASPIC$^+$ framework for structured argumentation of [8], and apply the model to different typical norm-productive arguments.

2. Legal validity as an argumentative claim

All norm-productive arguments that we shall consider lead to the same conclusion, namely, that a norm is legally valid. There exists a huge jurisprudential debate concerning the idea of legal validity. For our purposes, however, we do not need to enter this debate, since we shall consider just the inferential role of the concept of validity, a role that consists exactly in the fact that establishing that a norm is legally valid licenses us (and indeed obliges us, if the norm is relevant) to use the norm in legal reasoning (for such an approach, see [12]). In other words, the following inference pattern should hold, for any norm ϕ named z: if z is legally valid, than it can be inferred, for the purposes of legal reasoning, that ϕ. Consider, for instance the following norm (normative content) "It is forbidden to smoke in closed spaces", and assume that it is named C_1. Assume also that it can be established that C_1 is legally valid, for instance, since it is a fragment of a legislative act by the Italian Parliament, namely, art. 52 of Law n. 3/2003. This should entitle us to conclude that indeed, according to Italian law, it is forbidden to smoke in closed spaces. Given that the validity of a norm males it derivable in legal reasoning, the crucial issue becomes establishing when a norm is valid. For this purposes different kinds of arguments can be used, as we shall see in the following.

Legislative arguments. First of all, the claim that a norm is valid can be supported by a legislative argument, namely, an argument appealing to the fact that that the norm at issue was produced by a competent legal authority. Assume that PhD student Giulia enters the office of her supervisor Mario, is annoyed by his smoke, and wants to remind him of his legal obligations. A full argument to that effect would be that Mario is under the prohibition of smoking in his office, since it is prohibited to smoke in closed spaces, such as Mario's office, and this is the case since there is a valid legal norm to this effect, namely Art. 51 of the law n. 3/2003, which is valid since it was issued by the Italian Parliament, which has a general law-making power. If Giulia wanted to annoy her supervisor, she could also say that the Parliament has this power since this is conferred to it by another valid law, namely, art. 70 of the Italian constitution, which confers the Parliament a general legislative power. This argument makes use of a power-conferring, or competence norm, namely, Art. 70 of the Italian constitution. There is a vast debate concerning the nature of competence norms and the ways to formalise them, but for our purposes we need to focus only on the inferential role of such norms, which consists in licensing the conclusion that a norm is valid, given that it has been stated in a certain way, by a certain body or person. In other words a norm of competence may be viewed as conditional, whose antecedent if the fact that a norm has been stated in a certain way and whose conclusion is the validity of the norm so stated. Thus for instance the rule of the Italian constitution that establishes the legislative competence of the Parliament can be modelled as the following conditional: for every norm z, if z is stated by the Parliament, according to the parliamentary procedure, then z is valid. This universal conditional, given that norm C_1, the prohibition to smoke in Art. 51 of Law n.3 of 2003, was stated by the Parliament, according to the parliamentary procedure, allows us to conclude that C_1 is valid.

Interpretive arguments The claim that a norm is valid can also be supported by interpretive arguments. In the legislative argument presented above, it assumed that the legislator, by using a certain linguistic vehicle, e.g., the sentence "It is forbidden to smoke in closed spaces", directly conveys an unquestionable normative content, e.g., the prohibition to smoke in such spaces. On the contrary interpretive arguments are based on the distinction between the linguistic vehicle used by the legislator, a legislative provision in an authoritative source, and the normative content so conveyed, the legislative norm; they support the validity of a certain norm as being the most preferable interpretation of a certain legislative provision.

For instance, the issue has been raised whether the prohibition to smoke also covers the use of electronic cigarettes, namely, whether the text of Art. 51, “it is forbidden to smoke in closed spaces”, only covers inhaling and exhaling burning substances, such as in tobacco, or also includes other heated suspensions. Thus if Julia wanted to argue that John is forbidden to use his electronic cigarette in his office she would have to mount an interpretive argument claiming that the valid norm stated through art. 51 should be interpreted the general prohibition to inhale and exhale any heated suspensions in closed spaces. This prohibition is valid being the best interpretation of the text "it is forbidden to smoke in closed spaces", a text that is authoritative, having been stated by the legislator according to the legislative procedure.

Thus interpretive arguments seem to correspond to a general inference scheme: given that t is a text fragment of a binding legal source, and that norm z is the best interpretation of t, we can conclude that z is legally valid. In this pattern, two names appear, the name of the text fragment being interpreted and the name of the norm expressing the interpretation. For modelling such arguments we need a way of referring to fragments of sources and to their possible contents, i.e., the norms conveyed by such sources. In ordinary legal language fragments in sources are identified using citations (e.g. the text for the prohibition to smoke may be denoted by “the first sentence of Art. 5.1 of law n.3/2003”), and norms are identified by various linguistic device (e.g., “the broadest interpretation of law n.3/2003”). Here we use for simplicity short labels $T_1, \ldots T_n$ for referring to texts, and $C_1, \ldots C_m$ for referring to norms. Assume that T_1 is Art. 5 of law n. 3 of 2003, that C_1 is the norm according to which it is forbidden to inhale/exhale burning tobacco in closed spaces, C_2 is the norms according to which it is forbidden to inhale/exhale burning substances in closed spaces, while C_3 is the norm according to which it is forbidden to inhale and exhale any hot vapours in closed spaces. Assume also that we have established both that T_1 is a binding legal source and that C_2 is the best interpretation of T_1. Then we can conclude that C_2 is valid, and that consequently, that according to the law, it is forbidden to inhale and exhale any burning substances in closed spaces.

Analogy arguments and a contrario *arguments.* While in interpretive arguments the validity of a norm is claimed on the basis of the authoritativeness of a source and the interpretive preferability of the norm with regard to that source, in the arguments we shall consider here the validity of a norm is argued for on the basis of (a) the validity of other norms and (b) a content connection between the norm claimed to be valid and the already valid ones.

The linkage between analogy and legal validity may be expressed through the following inference scheme: given that a norm z_1 is valid, being the best interpretation of an authoritative source, and that another norm z_2 is relevantly similar to z_1, we can conclude z_2 also valid. For instance, assume that it is agreed that the best interpretation of the provision of Art. 51.1 of Law 3 of 2003, is the norm C_1 above, namely, the prohibition to inhale and exhale burning substances in closed spaces. Assume also that it is established that a prohibition to inhale or exhale hot vapours containing psychotropic particles, let us call it C_4 is relevantly similar to C_2. Then the scheme for analogical reasoning would enable us to conclude that C_4 is also valid.

Similarly, in *a contrario* arguments, given the validity of a set of norms $z_1, \ldots z_m$ establishing the same conclusion ψ respectively under conditions $\phi_1, \ldots, \phi_m$, we conclude for the validity of a new norm w establishing conclusion $\neg\psi$ when none of this conditions hold. For instance, assume that we affirm the validity of just two smoking prohibitions, established by the legislator, C_1, saying that if x is a burning substance then it is forbidden to inhale/exhale x in closed spaces, and C_2, saying that if x is psychotropic substance then it is forbidden to inhale/exhale x in closed spaces. On the basis of (the validity of) C_1 and C_1 of we may conclude for the validity of an additional norm C_3, stating that if x is not a burning substance and x is not a psychotropic substance, then it not forbidden, i.e., it is permitted, to inhale/exhale x in closed spaces.

3. An ASPIC$^+$-based framework for norm-generating arguments

In the following a formal model shall be provided for capturing all inferences schemes we have just presented. For this purpose we shall use the ASPIC$^+$ framework as defined in [8], with the extension for reasoning about preferences provided in [7]. As explained above we assume that legal norms are not part of the argumentation system, but rather components of the system's knowledge base. The inference rules of the system provide inference schemes for deriving further norms from a knowledge base of norms and facts. After introducing ASPIC$^+$, we shall provide a basic framework, using the smallest set of inference rules and then we shall expand that set for dealing also with interpretations, analogical and *a contrario* arguments.

The ASPIC$^+$ framework Let us first introduce the ASPIC$^+$ basic framework, by using the definition in [8], but replacing their general notion of a contrariness relation with the special case of ordinary negation, for simplicity's sake.

Definition 1 (ASPIC$^+$ argumentation system) *An argumentation system is a tuple* $L_S = (\mathscr{L}, ^-, \mathscr{R}, n)$ *where*

1. $\mathscr{L}$ *is a logical language closed under negation* $\neg$. *We call any pair of formulas* φ *and* $\neg\varphi$ *each other's contradictories.*
2. $\mathscr{R} = \mathscr{R}_s \cup \mathscr{R}_d$ *is a set of strict* ($\mathscr{R}_s$) *and defeasible* ($\mathscr{R}_d$) *inference rules of the form* $\phi_1, \ldots, \phi_n \rightarrow \phi$ *and* $\phi_1, \ldots, \phi_n \Rightarrow \phi$, *respectively, such that* $\mathscr{R}_s \cap \mathscr{R}_d = 0$.
3. n *is a naming convention* $\mathscr{R}_d \mapsto \mathscr{L}$ *for all defeasible rules in* $\mathscr{R}_d$

The knowledge base of an ASPIC$^+$ argumentation is defined as follows.

Definition 2 (ASPIC$^+$ knowledge base) *A knowledge base in an argumentation system* $(\mathscr{L}, ^-, \mathscr{R}, n)$ *is a set* $\mathscr{K} \subseteq \mathscr{L}$ *consisting of two disjoint subsets,* $\mathscr{K}_n$*, the necessary premises and* $\mathscr{K}_p$*, the ordinary premises*

Arguments can be constructed by using the information in the knowledge base in combination with the inference rules whose antecedents are satisfied. These arguments can attack and defeat each other, so that an extended Dung-style semantics can be constructed (see [8]). Three types of attacks are defined: (1) argument A undercuts argument B if A concludes $\neg n(r)$, i.e., if it declares r is inapplicable, where r is the defeasible top rule of a subargument of B, (2) A rebuts B if A concludes for the contradictory of the conclusion of a defeasible rule in B, (3) A undermines B if A concludes for the contradictory of an ordinary premise of B. For instance, given defeasible rules $R_d = \{r \Rightarrow p, s \Rightarrow \neg p, t \Rightarrow \neg n(r_1)\}$, named respectively r_1, r_2, and r_3 and $K = \{r, s, t\}$, we have that argument $A_1 = \{r, r \Rightarrow p\}$ rebuts and is rebutted by $A_2 = \{s, s \Rightarrow \neg p\}$, while argument $A_3 = \{t, t \rightarrow \neg r_1\}$ undercuts argument A_1. Thus Argument A_2 is justified, since its only attacker has been defeated by an argument (A_3) having no counterarguments.

A dynamic legal argumentation system Let us now proceed to define a legal argumentation system, as a specification of an ASPIC$^+$ argumentation system. For our purpose of addressing norm-producing arguments, a simple representation for legal rules as universally quantified conditionals of literals will suffice.

Definition 3 (Dynamic Legal Argumentation System (DLAS)) *Let a dynamic legal argumentation system be an argumentation systems $L_S = (\mathscr{L}, ^-, \mathscr{R}, n)$ where*

1. *$\mathscr{L}$ is a language including symbols for predicates, functions, constants and variables, $=$ for equality, connectives $\neg$ for negation and $\rightsquigarrow$ for normative conditionals, and the universal quantifier $\forall$. An atom is a predicate followed by corresponding terms. A literal is an atom or the negation of an atom. A norm has the form $\forall(L_1 \wedge \ldots \wedge L_n \rightsquigarrow L)$, where $L_1 \ldots L_n, L$ are literals and $\forall$ applies to all variables in $L_1 \ldots L_n, L$. α is wff of $\mathscr{L}$ if α is a literal or α is a norm. In particular, $\mathscr{L}$ includes the following special symbols:*
   (a) *unary predicate symbols $Valid(x)$ and $Applicable(x)$*
   (b) *function symbols $N(x)$ and $DMP(x)$.*
   (c) *binary predicate symbols $\succ'$ and $\succ$, expressing a strict order relationship over norms and over inference rules respectively.*
2. *The set $\mathscr{R}$ of inference rules includes the following schemes, where a rule scheme is the set of all rules instantiating the scheme.*
   (a) *Validity: $Valid(N(\phi)) \rightarrow \phi$, where $N(\phi)$ is the name of norm ϕ*
   (b) *Instantiation : $\forall(\phi \rightsquigarrow \psi) \rightarrow (\phi \rightsquigarrow \psi)[x_1/t_1, \ldots, t_n]$, where $[x_1/t_1, \ldots, x_n/t_n]$ is a substitution of variables $x_1, \ldots x_n$ with ground terms $t_1, \ldots t_n$.*
   (c) *Defeasible modus ponens (DMP): $\phi_1, \ldots, \phi_n, (\phi_1 \wedge \ldots \wedge \phi_n \rightsquigarrow \psi) \Rightarrow \psi$.*
   (d) *Undercutting: $\neg Applicable(w) \rightarrow \neg DMP(w)$ where w is is the name of norm $\phi_1 \wedge \ldots \wedge \phi_n \rightsquigarrow \psi$ and $DMP(w)$ is the name of the DMP inference rule $\phi_1, \ldots, \phi_n, \phi_1 \wedge \ldots \wedge \phi_n \rightsquigarrow \psi \Rightarrow \psi$.*
   (e) *Preference: All rules having the form: $z_1 \succ' z_2 \rightarrow DMP(z_1) \succ DMP(z_2)$.*

Definition 4 (DLAS knowledge base) *A knowledge base for our dynamic legal argumentation system $L_S = (\mathscr{L}, ^-, \mathscr{R}, n)$ is a set $\mathscr{K} \supseteq \mathscr{L}$ consisting of ordinary premises such that:*

1. *every norm $\forall\phi$ occurring in $\mathscr{K}$ is given a unique name by a naming premise having the fom $N(\forall\phi) = c$, where c is constant symbol. By naming a norm, also names for all ground instances of the norm are provided: the name for instance $\phi[x_1/t_1, \ldots, x_n/t_n]$ of norm c, where $x_1, \ldots x_n$ are all free variables in ϕ*

and $t_1, \dots t_n$ are ground terms, is the functional expression $c(t_1, \dots, t_n)$. We denote all norm names for norms in $\mathscr{K}$ as $Norms_{\mathscr{K}}$.

2. *The function $DMP : N(Norms_{\mathscr{K}}) \mapsto n$ maps each norm-name $N(\phi_1 \wedge \dots \wedge \phi_n \rightsquigarrow \psi)$ into the DMP-rule name $n(\phi_1, \dots, \phi_n, \phi_1 \wedge \dots \wedge \phi_n \rightsquigarrow \psi) \Rightarrow \psi)$.*
3. *at least one premise is a validity statement $Valid(x)$, where $x \in Norms_{\mathscr{K}}$.*

Running example To illustrate the definitions just introduced, we shall use an example concerning a smoking regulation. Let use the following abbreviated predicates:

$ICS(x)$: "x is in a closed space"
$FFS(x)$: "x is forbidden from smoking"
$IMPS(x)$: "x is in a merely private space"
$NSMG(x)$: "x needs to smoke on medical grounds"
$EP(z)$: "Parliament enacts norm z"
$BS(x)$: "x is a burning substance"
$FIE(x,y)$: "x inhales or exhales y"

Here is a knowledge base constructed according to Definition 4 above, to which we will refer in the following.

$N(EP(w) \rightsquigarrow Valid(w)) = C_0;\ N(ICS(x) \rightsquigarrow FFS(x)) = C_1;$
$N(IMPS(x) \rightsquigarrow \neg(Applicable(C_1(x))) = C_2;\ N(NSMG(x) \rightsquigarrow \neg FFS(x)) = C_3;$
$Valid(C_0);\ C_3 \succ' C_1;$
$EP(C_1);\ EP(C_2);\ EP(C_3);\ ICS(John);\ ICS(Mary);\ ICS(Tom);\ NSMG(Mary);\ IMPS(Tom)$

The first four premises provide names for norms $C_0, \dots C_3$. C_0 is a competence norm: if the Parliament issue a norm w, then w is valid. C_1 is an obligational norm: if a person x is a closed space, then x is prohibited to smoke. C_2 is an exception norm: if a person x is in a merely private space, then prohibition C_1 is inapplicable to x; C_3 is a permissive norm: if an x need to smoke on medical grounds, then x is not prohibited (he or she is permitted) to smoke. The following two premises state properties of such norms: C_0 is valid, and C_3 is stronger than C_1. Finally we have factual premises: C_1, C_2 and C_3 have been enacted by the Parliament, John, Mary and Tom are in closed spaces, Mary needs to smoke on medical ground, and Tom is in a merely private space.

Let us now analyse the inference schemes specified in Definition 3, by applying them to this example. The validity scheme in Definition 3(2a) enables the derivation of norms, on the basis of their validity. For instance, if we established $Valid(C_1)$, we could conclude that $\forall(x)(ICS(x) \rightsquigarrow FFS(x))$, according to rule $Valid(C_1) \rightarrow \forall(ICS(x) \rightsquigarrow FFS(x))$.

The instantiations scheme allows us to derive instances of general norms. For instance, from the general norm $\forall(ICS(x) \rightsquigarrow FFS(x))$, named C_1, we could derive the specific norm, $ICS(John) \Rightarrow FFS(John)$, named $C_1(John)$, according to inference rule $\forall(ICS(x) \rightsquigarrow FFS(x)) \rightarrow ICS(John) \rightsquigarrow FFS(John)$.

The DMP scheme allows us to apply the rule-instances we have obtained. For instance, from $ICS(John)$ and $ICS(John) \Rightarrow FFS(John)$, we could conclude for $FFS(John)$, according to the rule $ICS(John), ICS(John) \rightsquigarrow FFS(John) \Rightarrow FFS(John)$, named $DMP(C_1(John)$.

The undercutting scheme links the inapplicability of a norm and the inapplicability of the corresponding DMP rule. For instance, given that the norm C_1 is not applicable to Tom, i.e., that $\neg Applicable(C_1(Tom))$, we could conclude that the corresponding DMP rule, i.e., $DMP(C_1(Tom)$ is inapplicable to Tom, i.e., that $\neg DMP(C_1(Tom))$, according to the rule $\neg Applicable(C_1(Tom) \rightarrow \neg DMP(C_1(Tom))$ The conclusion of this rule, according to ASPIC$^+$, undercuts the argument culminating in the inference rule

$DMP(C_1(Tom))$ (i.e., the argument that Tom should not smoke since he is in closed space).

The preference scheme links priorities between norms to priorities between the corresponding modus-ponens rules. For instance given that norm C_3, allowing smoke on medical grounds, is stronger than the prohibition to smoke C_1, i.e., given that $C_3 \succ' C_1$, we can conclude, that also the rule $DMP(C_3)$ is superior to the corresponding rule $DMP(C_1)$, according to the rule $C_3 \succ' C_1 \rightarrow DMP(C_3) \succ DMP(C_1)$. This means that, according to the ASPIC$^+$ framework, an argument culminating in an instance of $DMP(C_3)$ would be able to strictly defeat a contradictory argument culminating in an instance of $DMP(C_2)$.

4. Modelling arguments about norms

ASPIC+ arguments can be constructed, on the basis of the framework just introduced, to deal with legislative, analogical and *a contrario* arguments.

Legislative arguments. As an example of a legislative argument, let us consider the following A_1, supporting the claim that John is forbidden from smoking ($FFS(John)$). Remember that an ASPIC+ argument is a chain of premises and inference rules, where each antecedent element of an inference rule matches a preceding premise or the conclusion of a preceding inference rule (for a formal definition see [8]).

1. $Valid(C_0)$	[Premise]
2. $Valid(C_0) \rightarrow \forall(EP(w) \rightsquigarrow Valid(w))$	[Validity]
3. $\forall(EP(w) \rightsquigarrow Valid(w)) \rightarrow (EP(C_1) \rightsquigarrow Valid(C_1))$	[Instantiation]
4. $EP(C_1)$	[Premise]
5. $EP(C_1), EP(C_1) \rightsquigarrow Valid(C_1) \Rightarrow Valid(C_1)$	[DMP]
6. $Valid(C_1) \rightarrow \forall(ICS(x) \rightsquigarrow FFS(x))$	[IR Validity]
7. $\forall(ICS(x) \rightsquigarrow FFS(x)) \rightarrow (ICS(John) \rightsquigarrow FFS(John))$	[Instantiation]
8. $ICS(John)$	[Premise]
9. $ICS(John), (ICS(John) \rightsquigarrow FFS(John)) \Rightarrow FFS(John)$	[DMP]

Let us clarify some steps of this inference. The validity rule of step 2, allows us to derive the norm granting a general legislative power to the Parliament. The instantiation rule of step 3 says that if Parliament has the power of issuing any norm, then it also has the power of issuing C_1. The DMP rule of step 5 says that if Parliament enacted C_1 (the prohibition to smoke in closed spaces) and it has the power of enacting C_1, then C_1 is valid. The following steps concern the derivation of C_1's content, i.e., the prohibition to smoke in closed spaces, and its application to John's case, providing the conclusion that John is forbidden to smoke. The reader can check that given the knowledge base above, we can construct similar arguments A_2 for $FFS(Mary)$ and A_3 for $FFS(Tom)$. In the case of May, however, an argument A_4 can be constructed also for $\neg FFS(Mary)$, which rebuts the argument A_3. Arguments A_3 and A_4 defeat one-another, unless a preference for one against the other can be established. In the case of *Tom* an argument A_5 can be built for $\neg DMP(C_1(Tom))$, which undercuts A_2, and strictly defeats it, given that we can infer, using a preference rule, that $DMP(C_3) \succ DMP(C_1)$. The reader can also check that the derivation of valid norms is recursive. If norm C_0 is substituted with norm $\forall(InConstitution(w) \rightsquigarrow Valid(w)$ and fact $InConstitution(C_0)$, then we can derive C_0's validity and then on the basis of C_0 derive C_1's validity (the recursiveness of legal validity was emphasised by Hans Kelsen[5]).

Interpretive arguments. To embed interpretive arguments in our framework, we just need to add two inference rules to the argumentation system of Definition 3.

Definition 5 (DLAS with Interpretative Arguments) *A dynamic legal argumentation system with interpretation is a tuple $L_S = (\mathscr{L}, ^-, \mathscr{R}, n)$ as in Definition 3, with the addition of the following schemes:*

- *Validity by interpretation: $AuthSource(x), BestInt(w,x) \rightarrow Valid(w)$*
- *Interpretive choice: $BestInt(z_1,x), z_2 \neq z_1 \rightarrow \neg BestInt(z_2,x)$*

The first scheme states that a norm is valid if it is the best interpretation of an authoritative source; the second says that alternative interpretations of the same legal source are incompatible. The latter constraint is needed in the so-called operative interpretation, finalised to the decision of a legal issue, where a choice is necessary between interpretations supporting different decisions (on operative interpretation, see [6, 11ff]), a choice that must be supported by arguing that the chosen interpretation is the best one. The rules of a legal knowledge base need to be changed, or integrated to provide input to interpretive arguments. Let as assume that the source fragments "it is forbidden to smoke in public spaces", "the prohibition to smoke does not apply to merely private spaces" and "it is permitted to smoke if this is needed for medical reasons" are named T_1, T_2, T_3. Consider the following knowledge base.

$$N(EP(w) \rightsquigarrow Valid(w)) = C_0;\ N(ICS(x) \rightsquigarrow FFS(x)) = C_1;$$
$$N(IMPS(x) \rightsquigarrow \neg(Applicable(C_1(x))) = C_2;\ N(NSMG(x) \rightsquigarrow \neg FFS(x)) = C_3;$$
$$\forall z(EP(z) \rightsquigarrow Authoritative(z));$$
$$Valid(C_0);\ EP(T_1);\ EP(T_2);\ EP(T_3);$$
$$BestInt(T_1,C_1);\ BestInt(T_2,C_2);\ BestInt(T_3,C_3);$$
$$ICS(John);\ ICS(Mary);\ ICS(Tom);\ NSMG(Mary);\ IMPS(Tom);$$

Using this knowledge base, the reader can easily construct arguments supporting the validity of norms C_1, C_2, and C_3. In this formalisation, competence norms have been rephrased as empowerments to enact authoritative texts, i.e., sources whose best interpretation provides valid norms. Arguments claiming the preferability of particular interpretations can also be articulated, if appropriate premises are added. Assume for instance that premise $BestInt(C_1,T_1)$ is substituted with the following premises:

$$N(\forall(OrdLangMeaning(w,x)) \rightsquigarrow BestInt(w,x)) = C_{ol}; Valid(C_{ol}); OrdLangMeaning(C_5,T_1)$$

where C_{ol} is the canon supporting interpretations corresponding to the ordinary language w meaning of a text x. Then it could be argued that $BestInt(C_1,T_1)$, on the basis of the DMP rule that supports this conclusion given that (a) C_1 is the ordinary language interpretation T_1, (b) if C_1 is T_1's ordinary language interpretation, then C_1 is T_1's best interpretation. Condition (b) would result from the instantiation of the valid interpretive canon C_{ol}. If multiple interpretive canons where applicable to the case, argumentation should address their convergence or their conflict, aspects that cannot be dealt with here, but which can be straightforwardly modelled in the framework of the present logic, expanded with a method for dealing with the accrual of arguments (see [9]).

Analogical and a contrario *arguments.* To address analogical and *a contrario* arguments, we can complement the framework of Definition 5 with the following rules.

Definition 6 (DLAS with Analogical and A Contrario Arguments.) *A dynamic legal argumentation system providing for analogical and* a contrario *arguments is a tuple* $L_S = (\mathscr{L}, ^-, \mathscr{R}, n)$ *as in Definition 5, with the addition of the following rule schemata:*

- *Authoritative validity:* $AuthSource(z), BestInt(w,z) \rightarrow AuthValid(w)$
- *Interpretive choice:* $BestInt(z_1,x), z_2 \neq z_1 \rightarrow \neg BestInt(z_2,x)$
- *Analogical validity:* $AuthValid(z_1), Similar(z_2,z_1) \Rightarrow Valid(z_2)$
- *A contrario validity:* $AuthValid(N(\phi_1 \rightsquigarrow \psi)), \ldots, AuthValid(N(\phi_n \rightsquigarrow \psi)) \Rightarrow Valid(N(\neg\phi_1 \wedge \ldots \wedge \neg\phi_n \rightsquigarrow \neg\psi))$

The first rule characterises authoritative validity as resulting from the best interpretation of an authoritative text (these are the norms that are valid according to Definition 5). The third says that we can infer that a norm is valid if is similar to an authoritatively valid norm. The fourth says that we can infer that a norm is valid if it denies the conclusion established by a set of authoritatively valid rules in a case not contemplated by any of them. For instance, assume that C_6 is a norm forbidding the use of electronic cigarettes in closed spaces, and that it is established that this norm is similar to norm C_1, forbidding smoke in closed spaces, and that Tom is smoking an electronic cigarette in a closed space. It is easy to see that on the basis of Definition 6 we build an argument to the effect that Tom is prohibited from smoking the cigarette, including a subargument to the effect that C_6 is valid by analogy. Similarly, given that we have an authoritative norm C_1 prohibiting smoke in closed spaces, we can conclude, *a contrario* that a norm is valid, let us call it C_7, which allows smoking in places which are not closed. The *a contrario* arguments here presented are premised on a set of pre-existing unidirectional conditionals, assumed to express the authoritative content of legal sources, and thus they can be distinguished from *a contrario* interpretive arguments, which argue that a single legal source expresses a biconditional norm, rather than a simple conditional, as its authoritative content (though in practice it may difficult to classify *a contrario* arguments one way or the other). Our model could be further refined, so that, for instance, authoritatively valid norms prevail over, or undercut norms whose validity is supported by analogy or *a contrario* arguments, but these and other refinements must be left to further research.

5. Conclusion and related works

This paper has the modest objective of showing how the ASPIC$^+$ framework enables us to express arguments establishing new norms, and to embed them, as proper sub arguments, into the arguments establishing the conclusions of such norms. No substantive analysis has been provided of the preconditions for the application of the schemes we have identified (when is an authority empowered to produce new norms? what is the best interpretation of a text? what does it means for two rules to be similar?), since these preconditions constitute the inputs to the top-level schemes that we have identified, and so they are outside the scope of this work. We have combined ideas that have been already been discussed in various contributions. The logical basis for our argumentation framework is provided by the ASPIC$^+$ framework of [8]. The combination of inferences establishing the validity of norms with inferences using valid norms was proposed in [15]. The view that valid norms are defeasible reasons for legal conclusions was at the core of the Reason Based Logic of [4]. The derivation of rule priorities through argumentation was provided in [11] and formalised in the ASPIC$^+$ framework in [7]. Arguments about applicability and inapplicability are discussed in [1], [11], and [4]. The idea that from the validity of a rule we can infer that rule was discussed in [12] and formalised by [2] in the framework of Carneades and by [10] in the framework of ASPIC$^+$. Modelling reasoning with norms though the application of argument schemes, as suggested by [14], was formalised in [13] and within ASPIC$^+$ by [10]. The dynamics of normative systems has been addressed in a number of contributions, such as [3].

The present attempt may be useful for integrating various aspects so far presented separately, providing a general framework for legal reasoning about norms and their validity. It can be extended in various directions, for instance, enriching the underlying logical language with a larger subset of predicate logic, with a weak negation, with formal accounts of modalities, such as obligation, time and action, or adding further patterns for norm creation and elimination and for the validity/acceptability of empirical information. But this must be left to future research.

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