

Corrigendum for: A General Account of Argumentation with Preferences

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1 Background

In Modgil and Prakken [3], it is claimed that the rationality postulates of Caminada and Amgoud [2], and in particular the *consistency* postulates, are satisfied under the assumption that the preference preordering \preceq over arguments is *reasonable* (Definition 18, [3]). Modgil and Prakken then study a number of ways of defining preference orderings over arguments, based on set comparative orderings over the arguments' ordinary (fallible) premises and/or defeasible rules. The latter set comparative orderings are in turn defined on the basis of a given partial ordering over ordinary premises and a distinct given partial ordering over the defeasible rules.

The set comparison ordering, denoted \trianglelefteq , is defined according to the so called *Eli-tist* and *Democratic* principles (Definition 19, [3]), and these set comparisons are in turn used to define a non-strict preference relation \preceq over arguments, using the last (Definition 20, [3]) or weakest (Definition 21, [3]) link principles.

It is then claimed that if the set ordering \trianglelefteq is *reasonable inducing* (Definition 22, [3]), then \preceq is reasonable. However, in a personal communication, Sjur. K. Dyrkolbotn recently pointed out a counterexample to this claim. We present the example below, and then observe that the example is also cited as a counter-example to the consistency postulates by P.M. Dung [1]. In this erratum, we then show that:

- the counter-example can be avoided by reverting to the way preferences are defined in the first version of the *ASPIC*⁺ framework in [5];

However, reverting to this previous way of defining preferences does not address the falsity of the claim that if \trianglelefteq is *reasonable inducing* then this guarantees that \preceq is reasonable. Hence, in this erratum we also:

- summarise modifications made to the definitions in [3] in order to ensure that if \trianglelefteq is *reasonable inducing* then this does guarantee that \preceq is *reasonable*, and the counter-example due to Dyrkolbotn (and a variation of this example that appears in Dung [1]) is avoided.

2 Excluding Counter-example to Consistency Postulates by Reverting to Preference Relations as defined in [5]

Suppose an argumentation theory consisting of a knowledge base that has the ordinary (i.e., fallible) premises p, q, r, s , the empty set of axiom premises, the strict rules (closed under contraposition):

$$\text{r1: } p, q, r \rightarrow \neg s$$

$$\text{r2: } p, q, s \rightarrow \neg r$$

$$\text{r3: } s, q, r \rightarrow \neg p$$

$$\text{r4: } p, s, r \rightarrow \neg q$$

and the empty set of defeasible rules. We then have the arguments:

$$S = [s]$$

$$R = [r]$$

$$P = [p]$$

$$Q = [q]$$

$$A1 = [p; q; r; p, q, r \rightarrow_{r1} \neg s]$$

$$A2 = [p; q; s; p, q, s \rightarrow_{r2} \neg r]$$

$$A3 = [s; q; r; s, q, r \rightarrow_{r3} \neg p]$$

$$A4 = [p; s; r; p, s, r \rightarrow_{r4} \neg q]$$

Moreover, assume $p \leq q, q \leq p$ and $r \leq s, s \leq r$ (i.e., $p \equiv q$ and $r \equiv s$). The Elitist set comparison (Definition 19, [3]) states that $\Gamma \preceq \Delta$ if there is at least one element in Γ that is \leq all elements in Δ , i.e.,

$$\Gamma \preceq \Delta \text{ if } \exists X \in \Gamma, \forall Y \in \Delta : X \leq Y \quad (1)$$

and then the strict counterpart $\Gamma \triangleleft \Delta$ is defined in the usual way:

$$\Gamma \triangleleft \Delta \text{ iff } \Gamma \preceq \Delta \text{ and } \Delta \not\preceq \Gamma. \quad (2)$$

In Modgil and Prakken, we say in general that $Y \prec X$ iff $Y \preceq X$ and $X \not\preceq Y$, where according to Definitions 21 and 22 in [3], $Y \preceq X$ is defined (under the last, respectively weakest, link principle) based on the set comparison \preceq . In our running example, under both last and weakest link principles:

1. $\{p, q, \mathbf{r}\} \preceq \{\mathbf{s}\}$ and $\{\mathbf{s}\} \not\preceq \{p, q, r\}$ (note $\{p, q, r\} \triangleleft \{\mathbf{s}\}$ by Eq. 2)
2. $\{p, q, \mathbf{s}\} \preceq \{\mathbf{r}\}$ and $\{\mathbf{r}\} \not\preceq \{p, q, s\}$ (note $\{p, q, s\} \triangleleft \{\mathbf{r}\}$ by Eq. 2)
3. $\{s, \mathbf{q}, r\} \preceq \{\mathbf{p}\}$ and $\{\mathbf{p}\} \not\preceq \{s, q, r\}$ (note $\{s, q, r\} \triangleleft \{\mathbf{p}\}$ by Eq. 2)
4. $\{\mathbf{p}, s, r\} \preceq \{\mathbf{q}\}$ and $\{\mathbf{q}\} \not\preceq \{p, s, r\}$ (note $\{p, s, r\} \triangleleft \{\mathbf{q}\}$ by Eq. 2)

and so:

1. $A1 \preceq S$ and $S \not\preceq A1$, hence $A1 \prec S$ and $A1 \not\rightsquigarrow S$
2. $A2 \preceq R$ and $R \not\preceq A2$, hence $A2 \prec R$ and $A2 \not\rightsquigarrow R$
3. $A3 \preceq P$ and $P \not\preceq A3$, hence $A3 \prec P$ and $A3 \not\rightsquigarrow P$
4. $A4 \preceq Q$ and $Q \not\preceq A4$, hence $A4 \prec Q$ and $A4 \not\rightsquigarrow Q$

As Dung points out in a variation on this example, the consistency postulate is then not satisfied since the obviously inconsistent $E = \{S, R, P, Q, A1, A2, A3, A4\}$ is a stable extension; it is conflict free since no two arguments defeat each other, and no arguments defeat the arguments in E .

This counter-example can be avoided by reverting to the way preferences are defined over arguments in the first published version of *ASPIC*⁺ in [5] (in which the closure and consistency postulates are shown to be satisfied). That is to say, the strict relation \prec over arguments is not defined in terms of the non-strict \preceq , which in turn is defined on the basis of the non-strict \trianglelefteq . Rather, we directly define \prec on the basis of \triangleleft , where \triangleleft is now *not* defined on the basis of \trianglelefteq as in Eq. 2, but rather: (for the Elitist comparison¹):

$$\Gamma \triangleleft \Delta \text{ if } \exists X \in \Gamma, \forall Y \in \Delta : X < Y \quad (3)$$

By Eq. 3, we now obtain that $\{p, q, r\} \not\triangleleft \{s\}$, $\{p, q, s\} \not\triangleleft \{r\}$, $\{s, q, r\} \not\triangleleft \{p\}$ and $\{p, s, r\} \not\triangleleft \{q\}$. Hence, directly defining \prec in terms of \triangleleft , which in our running example means that $A1(2, 3, 4) \prec S(R, P, Q)$ iff $\Gamma \triangleleft \Delta$, where Γ are the premises in $A1(2, 3, 4)$ and Δ the premises in $S(R, P, Q)$, we now obtain that $A1(2, 3, 4) \not\prec S(R, P, Q)$, and so all attacks succeed as defeats and $E = \{S, R, P, Q, A1, A2, A3, A4\}$ is not conflict free.

3 Correcting the Definition of ‘Reasonable Inducing’

By reference to the same running example, Sjur. K. Dyrkolbotn points out that the strict set relations

$$\{p, q, r\} \triangleleft \{s\} \text{ by Eq. 2}$$

$$\{p, q, s\} \triangleleft \{r\} \text{ by Eq. 2}$$

$$\{s, q, r\} \triangleleft \{p\} \text{ by Eq. 2}$$

$$\{p, s, r\} \triangleleft \{q\} \text{ by Eq. 2}$$

in the previous section verify that \trianglelefteq is *reasonable inducing* (Definition 22, [3]); that is to say that not only is \trianglelefteq transitive, but that, in general:

Let Γ be the set of premises (defeasible rules) in argument B_1, \dots, B_n and Δ the set of premises (defeasible rules) in argument A . If $\Gamma \triangleleft \Delta$ then:

¹For the Democratic comparison: $\Gamma \triangleleft \Delta$ if $\forall X \in \Gamma, \exists Y \in \Delta : X < Y$

1. for some $i = 1 \dots n$ such that $\Gamma' \subseteq \Gamma$ are the premises (defeasible rules) in B_i , then $\Gamma' \sqsubseteq \Delta$.
2. for some $i = 1 \dots n$ such that $\Gamma' \subseteq \Gamma$ are the premises (defeasible rules) in B_i , then $\Delta \not\sqsubseteq \Gamma'$.

Note that it is not assumed that the B_i in each of the above two cases is the same argument. Hence, under the existing definitions in Modgil and Prakken [3], our running example validates that the Elitist set comparison is reasonable inducing, since: 1) $\{r\} \sqsubseteq \{s\}$ and $\{s\} \not\sqsubseteq \{p\}$; 2) $\{s\} \sqsubseteq \{r\}$ and $\{r\} \not\sqsubseteq \{p\}$; 3) $\{q\} \sqsubseteq \{p\}$ and $\{p\} \not\sqsubseteq \{r\}$; 4) $\{p\} \sqsubseteq \{q\}$ and $\{q\} \not\sqsubseteq \{r\}$.

However, the defined argument preference ordering is not reasonable, since as observed in the previous section: $A1 \prec S$, $A2 \prec R$, $A3 \prec P$ and $A4 \prec Q$, and so we obtain the counter-example to consistency as illustrated in the previous section.

Hence, in addition to the proposed changed definitions of \triangleleft (Eq. 3), and defining \prec directly in terms of \triangleleft , rather than as the strict counterpart of \sqsubseteq , we also modify the definition of reasonable inducing so as to also account for the direct definition of \triangleleft . That is to say, we define \triangleleft to be reasonable inducing iff it is a strict partial ordering, and \triangleleft satisfies the following:

Let Γ be the set of premises (defeasible rules) in argument B_1, \dots, B_n and Δ the set of premises (defeasible rules) in argument A . If $\Gamma \triangleleft \Delta$ then:

1. for some $i = 1 \dots n$ such that $\Gamma' \subseteq \Gamma$ are the premises (defeasible rules) in B_i , then $\Gamma' \triangleleft \Delta$.
2. for some $i = 1 \dots n$ such that $\Gamma' \subseteq \Gamma$ are the premises (defeasible rules) in B_i , then $\Delta \not\triangleleft \Gamma'$.

where of course, given the asymmetry of \triangleleft , we can do away with condition 2, so that we now define \triangleleft to be reasonable inducing iff it is a strict partial ordering, and \triangleleft satisfies the following:

for some $i = 1 \dots n$ such that $\Gamma' \subseteq \Gamma$ are the premises (defeasible rules) in B_i , then $\Gamma' \triangleleft \Delta$. (4)

Now notice that according to the definition of \triangleleft in Eq. 3, $\{p, q, r\} \not\triangleleft \{s\}$, $\{p, q, s\} \not\triangleleft \{r\}$, $\{s, q, r\} \not\triangleleft \{p\}$, and $\{p, s, r\} \not\triangleleft \{q\}$, and so the example trivially verifies that \triangleleft is reasonable inducing according to the above definition.

4 Summary of Changes

To summarise, we have made the following changes in the corrected paper, which is available on ArXiv [4]. All definition and proposition numbers are the same in Modgil and Prakken [3] and in the corrected paper on ArXiv, and all changes made to Modgil and Prakken are highlighted in the corrected paper. Note also that all results in Modgil and Prakken are unaffected by the changes in the corrected paper. We now summarise the key changes made in the corrected paper:

1. Definition 19: In the corrected paper, we now directly define the Elitist and Democratic strict set comparisons \triangleleft in terms of the strict ordering $<$ over the elements in compared sets, and then define $\Gamma \leq \Delta$ iff $\Gamma \triangleleft \Delta$ or $\Gamma = \Delta^2$.

²Where '=' denotes identity.

2. Definitions 20 and 21: In the corrected paper, we now directly define $B \prec A$ by reference to strict comparisons \triangleleft over premises and/or defeasible inference rules in B and A . We then define $B \preceq A$ iff i) $B \prec A$ or ii) the premises/and or defeasible rules in A and B that are used to determine the preference relation over A and B , are the same.
3. Definition 22: We now define the notion of a reasonable inducing strict set comparison \triangleleft as one that is a strict partial order (irreflexive and transitive), and that satisfies (4) in Section 3.
4. Propositions 19 – 24 are then modified and proven, substituting reference to any reasonable inducing \preceq with reference to any reasonable inducing \triangleleft in Propositions 19 – 22, and substituting reference to any transitive relation \preceq with reference to any strict partial order \triangleleft in Propositions 23 and 24.

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References

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