

Chapter 8

Conclusions

In this thesis we have studied aspects of simulating interactively deformable objects, in the context of training systems for surgical procedures. It is said that such training systems should reproduce mechanical behavior accurately, which motivated our choice for the Finite Element Method (FEM) as a discretization and modeling technique. The linear elasticity approximation for mechanical FEM problems is attractive, since part of the solution can be computed in advance, yielding a guaranteed performance. Unfortunately, this technique cannot be used with more advanced, nonlinear material models and it is impractical when the mesh is large or is changed online. Motivated by this observation, we have proposed solution schemes based on static iterative relaxation methods.

In general, all computational techniques have a tradeoff between accuracy and computational cost. For example, the discretization error in a FEM approximation depends on the granularity of the mesh. More refined meshes lead to more accurate solutions, but also have increased costs due to their increased size. This tradeoff holds even more strongly for iterative solution techniques: their rate of convergence is also decreased by adverse mesh characteristics.

The influence of mesh characteristics and convergence speed is a common theme in this thesis. In Chapter 3 we have seen that flat elements significantly slow down the convergence speed of the CG algorithm, and in Chapter 4, we have seen that this slowdown is not limited to the linear CG algorithm. In Chapter 6 we have observed that information travels faster over coarser mesh parts, and that this can lead to improved convergence. Since size and quality of the mesh influence convergence speed, it is important to keep quality high, and size low while changing it. Hence, Chapter 5 shows a method for making cuts in meshes that produces better and smaller meshes than element subdivision, the most common technique for incorporating cuts in meshes. In Chapter 6 we showed a simple yet effective technique for locally refining meshes while maintaining element quality. That chapter also explored the connection between computational cost and accuracy more deeply. In the scenario of needle insertion, the accuracy of the solution, and the mesh resolution—and hence the update rates of the system—are closely coupled. The difference in the update rates can be large.

In literature on surgery simulations, systems are often described with qualifications such as “real-time”, or “runs at 25 Hz.” The performance of such systems is hard to evaluate with these numbers by themselves. In the first place, these numbers measure computational cost, and this measurement depends on the quality of the implementation, and the hardware and supporting software used. For this reason, Chapter 4 analyzes the speed of our static relaxation by comparing it to another algorithm coded within the same framework, and measures the speed of the implementation by comparing it to the speed of the machine/compiler combination. In the second place, even with an accurate indication of computational cost, an indication of accuracy is needed to evaluate the total cost of a solution. For this reason, Chapter 6 takes into account the accuracy of the solution when listing computation speeds.

We recall that our original choice for the FEM as a method for surgery simulation was motivated by its promise of higher realism. In other words, it was driven by the need for accuracy in the simulation. When we look at possible future extensions, we see that many other mathematical techniques are available that might improve accuracy or decrease computational costs. This is not surprising; tissue mechanics are complex, while an interactive simulation allows only small amounts of computation. There are virtually infinitely many areas of improvement. For determining which technical improvements really improve the overall solution, a broader view should be taken. Technical improvements should not only be mathematically justified, but also be tested against the problem being solved, i.e., simulation of surgical procedures for virtual training environments.