Expressing obligations by strategic ability update

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Expressing obligations by strategic ability update

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3 Reasoning on the Opponents
   - The Subgame Operator
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4 Properties
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$\neg p$ $\implies$ $p$ $\implies$ $\neg p$
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Dynamic Deontic Logic (Meyer 1987) applies normative statements to the theory of computer programs;
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It makes use of violation constants to label undesirable properties of a transition systems.
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It makes use of violation constants to label undesirable properties of a transition systems.

John-Jules Ch. Meyer,
A different approach to deontic logic: deontic logic viewed as a variant of dynamic logic.
In Dynamic Deontic Logic the formula $Oa$ (*It is obligatory to do a*) is interpreted in a transition system, labeled by an action relation, and indicates that from a world $w$ all worlds reachable not executing action $a$, written as $[\sim a]$, lead to violation ($V$).

- $Oa = [\sim a]V$
- $Pa := \neg O \sim a$
- $Fp := \neg Pa$
Expresing obligations by strategic ability update

Motivation

Deontic Logic and Strategic Ability

The *dynamic turn* in Deontic Logic

- In Dynamic Deontic Logic the formula $Oa$ (*It is obligatory to do* $a$) is interpreted in a transition system, labeled by an action relation, and indicates that from a world $w$ all worlds reachable not executing action $a$, written as $[\sim a]$, lead to violation ($V$).

\[
Oa = [\sim a]V
\]

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Pa : = \neg O \sim a
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**Deontics in Games**

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### Deontics in Games

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- Games offer interesting applications of deontic logic:
  - The notion of economic optimality becomes suitable to interpret what it should be;
  - The notion of rational action become suitable to interpret what it should be done;
Deontics in Games

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- Games offer interesting applications of deontic logic:
  - The notion of economic optimality becomes suitable to interpret what it should be;
  - The notion of rational action become suitable to interpret what it should be done;

- Under this interpretation, deontic formulas can express complex interactions between preferences and strategic ability of players.
Pareto Optimality

\[
\begin{array}{c|cc}
  & C & D \\
  i & (4,4) & (0,4) \\
  j & (4,0) & (1,1) \\
\end{array}
\]

**Definition**

Given a set of outcomes \( W \), a set of agents \( Agt \) and a weak linear order (transitive and complete) \( \geq_i \) (\( >_i \) its strict counterpart) over \( W \), \( x \in W \) is *Weakly Pareto Optimal* if there is no \( y \in W \) for which \( y >_i x \) for all \( i \in Agt \).
**Definition**

Given a set of outcomes $W$, a set of agents $Agt$ and a weak linear order (transitive and complete) $\geq_i$ ($>i$ its strict counterpart) over $W$, $x \in W$ is *Strongly Pareto Optimal* if there is no $y \in W$ for which $y \geq_i x$ for all $i \in Agt$ and $y >i x$ for some.

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Constructing efficient policies

- Once we identify the optimal states we can mandate their achievement.
Constructing efficient policies

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- However...
  - Players are confronted with choices, i.e. sets of worlds to select;
Constructing efficient policies

- Once we identify the optimal states we can mandate their achievement.
- However...
  - Players are confronted with choices, i.e. sets of worlds to select;
  - Players can form coalitions.
Effectivity in games

- The plan is to model coalitional choices and preferences to provide a semantics for deontic logic that accounts for what coalitions should rationally and socially do.
The plan is to model coalitional choices and preferences to provide a semantics for deontic logic that accounts for what coalitions should rationally and socially do.


Barteld Kooi and Allard Tamminga, at DEON ’06; Normas ’08. 2006.
Definition (Dynamic Effectivity Function)

Given a finite set of agents $Agt$ and a set of states $W$, a dynamic effectivity function is a function $E : W \rightarrow (2^{Agt} \rightarrow 2^{2^W})$.

Marc Pauly
A logic for social software.
E is outcome monotonic

\[ X \subseteq Y \quad \text{and} \quad X \in E(C) \implies Y \in E(C) \]
E is outcome monotonic

\[ X \subseteq Y \text{ and } X \in E(C) \text{ implies } Y \in E(C) \]
E is outcome monotonic

\[ X \subseteq Y \text{ and } X \in E(C) \text{ implies } Y \in E(C) \]
Lifting Preferences

\[ X \geq_i Y \iff x \geq_i y \text{ for } x \in X, y \in Y \]
Lifting Preferences

\[ X \succeq_i Y \iff x \succeq_i y \text{ for } x \in X, y \in Y \]

\[ X \succeq_C Y \iff X \succeq_i Y \text{ for } i \in C \]
Lifting Preferences

\[ X \geq_i Y \iff x \geq_i y \text{ for } x \in X, y \in Y \]

\[ X \geq_C Y \iff X \geq_i Y \text{ for } i \in C \]

\[ X >_C Y \iff x >_i y \text{ for } x \in X, y \in Y, i \in C \]
Lifting Preferences

\[ X \geq_{i}^{\forall,\forall} Y \iff \forall x \in X, \forall y \in Y, x \geq_{i} y \]
Lifting Preferences

- $X \geq_i \forall Y \iff \forall x \in X, \forall y \in Y, x \geq_i y$
- $X \geq_i \exists Y \iff \forall x \in X, \exists y \in Y, x \geq_i y$
Lifting Preferences

- \( X \geq_i ^\forall \forall Y \equiv \forall x \in X, \forall y \in Y, x \geq_i y \)
- \( X \geq_i ^\forall ^\exists Y \equiv \forall x \in X, \exists y \in Y, x \geq_i y \)
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- $X \geq Y \iff \exists x \in X, \forall y \in Y, x \geq_i y$
- $X \geq Y \iff \exists x \in X, \exists y \in Y, x \geq_i y$

Fenrong Liu
Definition (Weak Pareto Optimal Choice)

Given an Effectivity Function $E(w)(C)$, a set $X$ is a $(\forall, \forall)$-Weak Pareto Optimal Choice for coalition $C$ in world $w$ (abbr. $POC_{C,w}^{\forall,\forall}$) if, and only if,

(i) $X \in E(w)(C)$
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(ii) for no $Y \in E(w)(C)$, $Y >^{\forall,\forall}_i X$ for all $i \in C$. 

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Pareto Optimal Choices

Definition (Weak Pareto Optimal Choice)

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(i) $X \in E(w)(C)$

(ii) for no $Y \in E(w)(C)$, $Y >_{i,\forall}^{\forall,\forall} X$ for all $i \in C$.

- Pareto Optimal Choice with quantified preference liftings and its Strong version are definable in similar manner.
Let $Agt$ be a finite set of agents and $Prop$ a countable set of atomic formulas. The language $\mathcal{L}^>$ is made by formulas that are defined as follows:

$$\phi ::= p | \neg \phi | \phi \land \phi | [C] \phi | \Diamond ^{\leq}_i \phi | \Diamond ^{>}_i \phi | A \phi$$

where $p$ ranges over $Prop$ and $C$ ranges over the subsets of $Agt$. I will refer to $\mathcal{L}$ as the fragment of $\mathcal{L}^>$ without the modality $\Diamond ^{>}_i$. 
Structures

Definition (Models)

A model is a quadruple

\((W, E, \{\geq i\}_{i \in \text{Agt}}, V)\)

where:

1. \(W\) is a nonempty set of states;
2. \(E : W \rightarrow (2^{Agt} \rightarrow 2^W)\) is an outcome monotonic effectivity function.
3. \(\geq_i \subseteq W \times W\) for each \(i \in \text{Agt}\), is the weakly linear preference relation.
4. \(V : W \rightarrow 2^{\text{Prop}}\) is the valuation function.
Semantics

\[ M, w \models p \iff p \in V(w) \]
\[ M, w \models \neg \phi \iff M, w \not\models \phi \]
\[ M, w \models \phi \land \psi \iff M, w \models \phi \text{ and } M, w \models \psi \]
\[ M, w \models [C] \phi \iff [[\phi]]^M \in E(w)(C) \]
\[ M, w \models A \phi \iff M, v \models \phi, \text{ for all } v \in W \]
\[ M, w \models \Diamond_{\leq} i \phi \iff M, w' \models \phi, \text{ for some } w' \text{ with } w \leq_i w' \]
\[ M, w \models \Diamond_{>} i \phi \iff M, w' \models \phi, \text{ for some } w' \text{ with } w' <_i w \]

\[ [[\phi]]^M \triangleq \{ w \in W \mid M, w \models \phi \} \]
Proposition

\( \phi^M \) is POC\( \{(\forall, \forall)\} \) iff \( M, w \models [C] \phi \land \langle C \rangle \bigvee_{i \in C} \diamond_i \leq \phi \)
Proposition

\( (Q_1, Q_2) \)-POC and \( (Q_1, Q_2) \)-SPOC are characterizable in \( \mathcal{L}^> \), for \( Q_1, Q_2 \in \{\forall, \exists\} \).
Obligations and Pareto Optimal Choices

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- $C_i$ is $POC_{i,w}^{\forall,\forall}$
Obligations and Pareto Optimal Choices

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- $C_i$ is $POC_{i,w}^{\forall,\forall}$
- $D_i$ is $POC_{i,w}^{\forall,\forall}$

$[[C_i]]^{PD} = \{ w | PD, w \models \text{coalition } \{i\} \text{ chooses } C \}$
Obligations and Pareto Optimal Choices

- When considering the interest of all the agents together, Pareto Optimal Choices provide an intuitive reference, this is no more true when acting in the interest of smaller coalitions.
Obligations and Pareto Optimal Choices

- When considering the interest of all the agents together Pareto Optimal Choices provide an intuitive reference, this is no more true when acting in the interest of smaller coalitions.

- Pareto Optimal Choices are independent of the possible reactions of one’s opponents.
Let us focus on the choice by $i$ of a dominant strategy:
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- If $j$ plays $D$, I had better play $D$. 
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Let us focus on the choice by $i$ of a dominant strategy:

- If $j$ plays $D$, I had better play $D$.
- If $j$ plays $C$, I had better play $D$.
- In conclusion, I had better play $D$. 
Let us focus on the choice by $i$ of a dominant strategy:

- If $j$ plays $D$, I had better play $D$.
- If $j$ plays $C$, I had better play $D$.
- In conclusion, I had better play $D$.

In Dominant Strategy Equilibria, strategic reasoning means reasoning about all game restrictions induced by the opponents’ moves.
Domination

Definition (Subchoice set)

If $X \in E(w)(\overline{C})$, then the $X$-subchoice set for $C$ in $w$ is given by

$$E^X(w)(C) = \{X \cap Y \mid Y \in E(w)(C)\}.$$
Back to the game

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\[
E^{(D_i)}(w)(j) = \{D_i \cap C_j, D_i \cap D_j\}
\]
Definition (Domination)

Given an effectivity function $E$, $X$ is *undominated* for $C$ in $w$ (abbr. $X\triangleright_{C,w}$) if, and only if,
Definition (Domination)

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(i) $X \in E(w)(C)$
Definition (Domination)

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(i) $X \in E(w)(C)$

(ii) for all $Y \in E(w)(\overline{C})$, $(X \cap Y)$ is Pareto Optimal in $E^Y(w)(C)$ for $C$. 

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\begin{array}{c|cc}
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\hline
i & (4, 4) & (0, 4) \\
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\]

\[
D_i \triangleright_{i,w}
\]
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- $D_i \succ i, w$
- not $C_i \succ j, w$
Semantics

\[ M, w \models [\text{rational}_C] \phi \iff [[\phi]]^{M \triangleright C, w} \]
What it should be done

\[ F(C, \phi) := [C] \phi \rightarrow \neg[rational_C] \phi \]
\[ P(C, \phi) := \neg F(C, \phi) \]
\[ O(C, \phi) := F(C, \neg \phi) \]
What it should be done

\[
F(C, \phi) := [C]\phi \rightarrow \neg[rational_C]\phi \quad F(\alpha) := [\alpha]V \\
P(C, \phi) := \neg F(C, \phi) \quad P(\alpha) := \neg F(\alpha) \\
O(C, \phi) := F(C, \neg \phi) \quad O(\alpha) := F(\sim \alpha)
\]

- Game Theory assigns a natural meaning to violations.
Can we express the operator \([\text{rational}_C]\) in terms of the operators \([C], \leq_i\)?
Can we express the operator $[\text{rational}_C]$ in terms of the operators $[C], \Diamond_i \leq$?

We introduce an operator able to define $[\text{rational}_C]$ together with $\Diamond_i \leq$ and expressible in terms of $[C]$. 
Semantics

\[ M, w \models [C \downarrow \psi] \phi \text{ iff } \psi^M \in E(w)(C) \text{ implies } M \downarrow (C, \psi^M, w), w \models \phi \]
Semantics

\[ M, w \models [C \downarrow \psi] \phi \iff \psi^M \in E(w)(C) \implies M \downarrow_{(C, \psi^M, w)}, w \models \phi \]

\[ M \downarrow_{(C, \psi^M, w)} \triangleq \langle W, E \downarrow_{(C, \psi^M, w)}, V \rangle \]
Exp ressing obligations b y strategic ability update
Reasoning on the Opp onents
The Subgame Operator

Semantics

Definition (Superset Closure)

Given a set of sets $\mathcal{X}$,

$$(\mathcal{X})^{\text{sup}} = \{ X \subseteq W | \text{there is } Y \in \mathcal{X} \text{ and } Y \subseteq X \subseteq W \}.$$
### Semantics

**Definition (Superset Closure)**

Given a set of sets $\mathcal{X}$, 

$$(\mathcal{X})^{\text{sup}} = \{ X \subseteq W \mid \text{there is } Y \in \mathcal{X} \text{ and } Y \subseteq X \subseteq W \}.$$  

**Definition (Choice Intersection)**

Given two sets of sets $\mathcal{X}, \mathcal{Y}$, 

$$\mathcal{X} \cap \mathcal{Y} = \{ X \cap Y \mid X \in \mathcal{X} \text{ and } Y \in \mathcal{Y} \}.$$
Semantics

\[ E \downarrow (C, \psi^M, w) \] is defined in the following way:
The Subgame Operator

Semantics

\[ E \downarrow_{(C, \psi^M, w)} \] is defined in the following way:

\[
E \downarrow_{(C, \psi^M, w)} (w)(D) \doteq (\{\psi^M\})^{\text{sup}} \quad \text{for} \quad D \cap C \neq \emptyset
\]
Semantics

\[ E \downarrow_{(C, \psi^M, w)} \text{ is defined in the following way:} \]

\[
E \downarrow_{(C, \psi^M, w)} (w)(D) \equiv (\{\psi^M\})^{\text{sup}} \quad \text{for } D \cap C \neq \emptyset
\]

\[
E \downarrow_{(C, \psi^M, w)} (w)(D) \equiv (E(w)(D) \cap \psi^M)^{\text{sup}} \quad \text{for } D \cap C = \emptyset
\]
Semantics

\(E \downarrow_{(C, \psi^M, w)}\) is defined in the following way:

\[
E \downarrow_{(C, \psi^M, w)} (w)(D) \equiv \begin{cases} 
\{\psi^M\}^{\text{sup}} & \text{for } D \cap C \neq \emptyset \\
(E(w)(D) \cap \psi^M)^{\text{sup}} & \text{for } D \cap C = \emptyset \\
E(w')(D) & \text{for } w' \neq w
\end{cases}
\]
Semantics

\[ E \downarrow (C, \psi^M, w) \] is defined in the following way:

\[
\begin{align*}
E \downarrow (C, \psi^M, w) (w)(D) & \doteq (\{\psi^M\})^{\text{sup}} & \text{for } D \cap C \neq \emptyset \\
E \downarrow (C, \psi^M, w) (w)(D) & \doteq (E(w)(D) \cap \psi^M)^{\text{sup}} & \text{for } D \cap C = \emptyset \\
E \downarrow (C, \psi^M, w) (w')(D) & \doteq E(w')(D) & \text{for } w' \neq w
\end{align*}
\]

I will refer to \( L \downarrow \) as the subgame operator free language \( L \) plus the subgame operator.
Semantics

- Dynamic perspective on strategic ability as in (van Benthem 2007);
Semantics

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Semantics

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Johan van Benthem,
In praise of strategies.

Wiebe van der Hoek, Wojciech Jamroga and Michael Wooldridge.
A logic for strategic reasoning.
AAMAS ’05: Proceedings of the fourth international joint conference on Autonomous agents and multiagent systems.
Proposition

Given \( \{\psi_1, \ldots, \psi_n\} = E(w)(\overline{C}) \),
\( \phi^M \triangleright_{C,w} \iff M, w \models \bigwedge_{\psi_i \in \{\psi_1, \ldots, \psi_n\}} [\overline{C} \downarrow \psi_i] POC_C(\phi \land \psi_i) \)
Proposition

For the class $\mathcal{C}$ of all frames based on the models described, the axiom $[C]\phi \rightarrow [\overline{C} \downarrow \xi]POC_C(\phi \land \xi)$ determines the following condition: $X \in E(w)(C)$ implies that $X \cap Y$ is Pareto Optimal in $E^Y(w)(C)$.
Axiomatization and Reduction

**Proposition**

*The language $\mathcal{L} \downarrow$ is finitely axiomatizable and reducible to $\mathcal{L}$.***
Breaking Down Pareto Optimal Choices

- $\phi^M$ is POC$_{C,w}^{(\forall,\forall)}$ iff $M, w \models [C] \phi \land \langle C \rangle \bigvee_{i \in C} \lozenge_{i} \phi$
Breaking Down Pareto Optimal Choices

- $\phi^M$ is POC$_{C,w}^{(\forall,\forall)}$ iff $M, w \models [C]\phi \land \langle C \rangle \bigvee_{i \in C} \diamond_i \leq \phi$
- $[C]\phi \land \langle C \rangle \bigvee_{i \in D} \diamond_i \leq \phi$ iff $\phi$ is choice by $C$ made in the interest of $D$. 
Deontic logic in strategic interaction can account for coalitionally rational action;
Conclusion

- Deontic logic in strategic interaction can account for coalitionally rational action;
- The framework is flexible enough to express choices and consequently obligations to protect other coalitions’ interests.
Thanks!