Modeling Legal Dynamics in Defeasible Logic

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Explicit vs Implicit Legal Modifications

- **Explicit:** The law introduces norms whose peculiar objective is to change the system by specifying what and how other existing norms should be modified.

- **Implicit:** the legal system is revised by introducing new norms which are not specifically meant to modify previous norms, but which change in fact the system because they are incompatible with such existing norms and prevail over them.
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- **Implicit:** The legal system is revised by introducing new norms which are not specifically meant to modify previous norms, but which change in fact the system because they are incompatible with such existing norms and prevail over them.
Types of Explicit Modifications (1)

- *Textual modifications*: e.g., substitution, which typically replaces some textual components of a provision with other textual components. Example (fictional from the Italian constitution):

  Article 3 (1) All citizens have equal social status and are equal before the law, without regard to their sex, race, language, religion, political opinions, and personal or social conditions.

  Modifying norm (enacted in 2011):
  The Article 3, paragraph 1, is modified as follows: “All human beings have . . .”

- *Modifications on norm validity and existence*: repeal, abrogation, annulment.

  Example (judicial annulment; real!):
  Legislative Act n. 124, 23 July 2008
  Art. 1. With the exception of the cases mentioned under the Articles 90 and 96 of the Constitution, criminal proceedings against the President of the Republic, the President of the Senate, the President of the House of Representatives, and the Prime Minister, are suspended for the entire duration of tenure. [. . . ]
  On account of Art. 3 of the Constitution [. . . ] the Constitutional Court hereby declares the constitutional illegitimacy of Art. 1 of the Legislative Act n. 124, 23 July 2008 [. . . ]
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Modifying norm (enacted in 2011):
In derogation to the provisions set out in Article 3, paragraph 1, of the Constitution, the citizens who are resident in Bologna may have different social status.

- *Temporal modifications*: they change norms in regard to their time of force, efficacy, or applicability. Example (fictional from a real case):

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Defeasible Logic

Natural representation of legal systems.

- Defeasible Theory
  - Facts
  - Strict rules \((A \rightarrow B)\)
  - Defeasible rules \((A \Rightarrow B)\)
  - Defeaters \((A \sim B)\)
  - Superiority relation over rules

- Derive (plausible) conclusions with the minimum amount of information.
  1. \(+\Delta q\), which means that \(q\) is strictly provable in \(D\);
  2. \(-\Delta q\), which means that \(q\) is not strictly provable in \(D\);
  3. \(+\partial q\), which means that \(q\) is defeasibly provable in \(D\);
  4. \(-\partial q\), which means that \(q\) is not defeasibly provable in \(D\).

- Theory extension

\[
E(T) = (\Delta^+(T), \Delta^-(T), \partial^+(T), \partial^-(T))
\]
Revision in DL: Substitution

Art.3 : Citizen ⇒ Equal Status
Revision in DL: Substitution

\[ \text{Art.3} : \text{Citizen} \Rightarrow \text{Equal Status} \]
Revision in DL: Substitution

\[
\begin{align*}
\text{Art.3 : Citizen} & \Rightarrow \text{Equal Status} \\
\downarrow \\
\text{Art.3 : Human} & \Rightarrow \text{Equal Status}
\end{align*}
\]
Revision in DL: Substitution

\[ T_{r:A \Rightarrow B, C/A}^{\text{sub}} = \begin{cases} T & \text{if } B \in \partial^+(T) \text{ whenever } C \in \partial^+(T) \\ (F, R', \succ') & \text{otherwise} \end{cases} \]

where

\[ R' = R - \{ r : A \Rightarrow B \} \cup \{ r : C \Rightarrow B \} \]

\[ \succ' = \succ \]
Revision in DL: Derogation

Art.3 : Citizen $\Rightarrow$ Equal_Status
Revision in DL: Derogation

Art. 3: Citizen ⇒ Equal_Status

BUT
Revision in DL: Derogation

Art. 3: Citizen $\Rightarrow$ Equal_Status

BUT

$mn: Citizen, Resident\_Bologna \sim \lnot Equal\_Status$
Revision in DL: Derogation

Art. 3: Citizen $\Rightarrow$ Equal \_Status

BUT

mn: Citizen, Resident \_Bologna $\sim\sim \neg$Equal \_Status

\[ T_{r:A\Rightarrow B,C}^{\text{derog}} = \begin{cases} 
T & \text{if } B \notin E(T) \\
(F, R', \succ') & \text{otherwise}
\end{cases} \]

\[ R' = R \cup \{ r' : A, C \sim\sim \sim B \} \quad \text{and} \]

\[ \succ' = \succ \cup \{ r' \succ r \} \cup \{ s \succ r' | s \in R[B] - \{ r \} \}. \]
Revision of Norms: Annulment as Rule Removal

\[ T_{r}^{\text{annul}} = (F, R - \{r\}, \succ) \]

Contraction of the rule: simple and close to legal practice!
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Contraction of the rule: simple and close to legal practice!

- Annulments remove a norm from the legal system
- The effect of annulments apply *ex tunc*, as the annulled norms are prevented to produce all their legal effects, independently of when they are obtained (retroactivity).
Revision of Norms: Annulment as Rule Removal

\[ T^{\text{annul}}_r = (F, R \setminus \{r\}, \succ) \]

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Abrogation?
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Abrogation?

- Abrogations operate \textit{ex nunc} and so do not cancel the effects that were obtained before the modification (non-retroactivity);
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Abrogation?

- Abrogations operate \textit{ex nunc} and so do not cancel the effects that were obtained before the modification (non-retroactivity);
- Abrogations are not properly norm removals: a norm \( n \), abrogated in 2009, will anyway produce its effects if the facts happened in 2008.
The Limits of Revision

- Retroactivity?
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- Retroactivity?
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- Temporal modifications?
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- Modifications can be conditioned to some preconditions (conditional modifying norms)
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- Retroactivity?
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- Temporal modifications?
- Modifications are parametrized with time
- Modifications can be conditioned to some preconditions (conditional modifying norms)
- Conflicts between modifying norms: modifications are defeasible
Retroactive Modifications

$T_0$ initial normative system
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$T_1$ add $r$ to $T_0$: $T_1 = (T_0)_r^+$
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$T_0$ initial normative system

$T_1$ add $r$ to $T_0$: $T_1 = (T_0)^+_r$

$T_2$ remove $s$ from $T_0$: undo–change–redo. $T_2 = (((((T_0)^+_r)_r)_r)_r)^-$.

Assuming $(T^+_r)_r^- = T$. 
Retroactive Modifications

$T_0$ initial normative system
$T_1$ add $r$ to $T_0$: $T_1 = (T_0)^+$
$T_2$ remove $s$ from $T_0$: undo–change–redo. $T_2 = (((((T_0)^+)^-)_s^-)_r^+$. Assuming $(T_r^+)^- = T$. 
Retroactive Modifications

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$T_1$ add $r$ to $T_0$: $T_1 = (T_0)_r^+$

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$T_0$ is just the fact $A$. $T_1$ is obtained from $T_0$ by retroactively adding two rules $A \Rightarrow B$ and $B \Rightarrow C$ and these rules are effective in $T_0$. Then the next transformation, leading to $T_2$ is the removal of $A \Rightarrow B$ from $T_0$. 
Moving to TDL: Normative Systems

\[ LS(t_1), LS(t_2), \ldots, LS(t_j), \ldots \]

\[ t_0 \rightarrow t' \rightarrow t'' \]
Rules in TDL

A rule is identified by a unique label and gives conditions to derive a (legal) provision at a particular time.
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\[ r_1 : (IncomeThreshold_{31Jan} \Rightarrow HighMarginalRate_{(28\text{Feb}, \tau)}^{(1Jan, \pi)})_{(31\text{Dec}, \pi)} \]

\[ r_2 : (HighMarginalRate_{28\text{Feb}} \Rightarrow Pay50\%_{(1\text{March}, \pi)})_{(1Jan, \pi)}_{(31\text{Dec}, \pi)} \]
Meta-Rules in TDL

A meta-rule gives conditions to establish that a rule is effective (and when it is), with respect to a particular time.
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$$mr : (JoinEU^{21\text{March}} \Rightarrow$$

$$r1 : (IncomeThreshold^{31\text{Jan}} \Rightarrow HighMarginalRate^{(28\text{Feb},\tau)}^{(1\text{Jan},\pi)})^{(1\text{Jan},\pi)}$$
Temporal Model

\[ t_0 \rightarrow t' \rightarrow t'' \]

\[ LS(t') \]

\[ t_0 \rightarrow t' \rightarrow t'' \]

\[ LS(t'') \]
Rule Persistence

\[ t_0 \rightarrow t' \rightarrow \cdots \rightarrow r \rightarrow t'' \rightarrow t_0 \]

\[ t_0 \rightarrow t' \rightarrow \cdots \rightarrow r \rightarrow t'' \rightarrow t_0 \]

\[ LS(t') \]

\[ LS(t'') \]

\[ t_0 \rightarrow r^{t''} @ t' \rightarrow \cdots \rightarrow \cdots \rightarrow r^{t''} @ t'' \]

\[ t_0 \rightarrow t' \rightarrow \cdots \rightarrow t'' \rightarrow t_0 \]
Conclusion Persistence

\[ LS(t') + \partial a \]

\[ LS(t'') \]
Persistence in Normative Systems

Given

\[ a^{10} \]

\[ r_1 : (a^{10} \Rightarrow b^{(20, \pi)})^{(5, ?)} @ v_1 \]

When can we prove \( b \)?
Persistence in Normative Systems

Given

\[ a^{10} \]

\[ r_1 : (a^{10} \Rightarrow b^{(20, \pi)})^{(5, ?)} \circ v_1 \]

When can we prove \( b \)?

1. Can we prove \( b^{20} \) from viewpoint 4?

2. Can we prove \( b^{20} \) from viewpoint 5?

3. Can we prove \( b^{25} \) from viewpoint 5?

4. Can we prove \( b^{20} \) from viewpoint 10?

5. What about if \( r_1 \) ceases to be effective at 9? Can we still prove \( b^{20} \) from viewpoint 10, and prove it from viewpoint 5?

???

6. Can we prove \( b^{20} \) from viewpoint 5 in a successive version of the normative system (\( v_2 \)) and what about if \( v_2 \) no longer contains \( r_1 \)?

???
Persistence in Normative Systems

Given

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\[ r_1 : (a^{10} \Rightarrow b^{(20, \pi)})^{(5, \?) \circ v_1} \]

When can we prove \( b \)?

1. Can we prove \( b^{20} \) from viewpoint 4? No
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\[ r1 : (a^{10} \implies b^{(20,\pi)})^{(5,?)} @ v1 \]

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2. Can we prove \( b^{20} \) from viewpoint 5?
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\[ r1 : (a^{10} \Rightarrow b^{(20,\pi)}(5,?)) @ v1 \]

When can we prove \( b \)?

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6. Can we prove \( b^{20} \) from viewpoint 5 in a successive version of the normative system (\( v_2 \))? and what about if \( v_2 \) no longer contains \( r_1 \)?
Persistency in Normative Systems

Given

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When can we prove \( b \)?

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6. Can we prove \( b^{20} \) from viewpoint 5 in a successive version of the normative system (\( v_2 \))? and what about if \( v_2 \) no longer contains \( r_1 \)? ??????
Temporal Modifications: Time of Force

Norm: \( r^{t'''} @ t' \)
Modifying norm: \( (⇒ r^t) @ t'' \)

\[
\begin{align*}
& r^{t'''} @ t' \\
& \Rightarrow r^t @ t'' \\
& \text{LS}(t') \quad \text{LS}(t'')
\end{align*}
\]
Abrogation

\[ t_0 \rightarrow t' \rightarrow t'' \rightarrow r \rightarrow +\partial B \rightarrow +\partial B \]

\[ t_0 \rightarrow t'_{t_v} \rightarrow t'' \rightarrow r \rightarrow +\partial B \rightarrow +\partial B \]

\[ \text{LS}(t') \]

\[ \text{LS}(t'') \]

\[ t_0 \rightarrow r^{t_v}@t' \rightarrow t' \rightarrow \text{abrog}(r)^{t_a}@t'' \rightarrow t'' \]
Annulment

\[ \text{LS}(t') \]

\[ \text{annul}(r)^{t_a @ t''} \]

\[ + \partial B \]
Conflicts between Modifications

\[ r : (A^{t_1} \Rightarrow B^{(t_2, \tau)})^{(t_3, \pi)} @ (t_4, \pi) \]
Conflicts between Modifications

\[ r : (A^{t_1} \Rightarrow B^{(t_2, \tau)})^{(t_3, \pi)} \odot (t_4, \pi) \]

- Indisputable cases of conflict

\[ mr_1 : ( \Rightarrow r : (C^{t_1} \Rightarrow B^{(t_2, \tau)})^{(t_3, \pi)}) \odot (t_5, \pi) \]
\[ mr_2 : ( \Rightarrow \text{annul}(r)^{(t_3, \pi)}) \odot (t_5, \pi)) \]
Conflicts between Modifications

\[ r : (A^{t_1} \Rightarrow B^{(t_2, \tau)}(t_3, \pi)) @ (t_4, \pi) \]

- Indisputable cases of conflict

\[ m_{r_1} : ( \Rightarrow r : (C^{t_1} \Rightarrow B^{(t_2, \tau)}(t_3, \pi)) @ (t_5, \pi) \]
\[ m_{r_2} : ( \Rightarrow \text{annul}(r)^{(t_3, \pi)}) @ (t_5, \pi) ) \]

- Disputable cases of conflict

\[ m_{r_1} : ( \Rightarrow r : (C^{t_1} \Rightarrow B^{(t_2, \tau)}(t_3, \pi)) @ (t_5, \pi) \]
\[ m_{r_2} : ( \Rightarrow r : (A^{t_1} \Rightarrow Q^{(t_2, \tau)}(t_3, \pi)) @ (t_5, \pi) \]
\[ \downarrow \]
\[ r : (C^{t_1} \Rightarrow Q^{(t_2, \tau)}(t_3, \pi)) @ (t_5, \pi) \]
Conclusions

- Logical model to capture modifications in legal systems
- It handles retroactivity, time-forking
- It handles modifications of modifications
- Experiment with other temporal models (intervals, duration, periodicity), and causality
- Complexity