Game Theory

Exercises strategic games

Auctions

- players {1,..,n} bid on an object Ob
- Private value of i is v_i and
- $v_1 > v_2 > \ldots > v_n > 0$
- sealed bid: simultaneous, one shot
- Highest bid with lowest index wins Ob

Second prize auction

- players {1,..,n} bid on an object Ob
- Private value of i is v_i and
- $v_1 > v_2 > \dots > v_n > 0$
- sealed bid: simultaneous, one shot
- Highest bid with lowest index wins Ob
- For the price of the second bid!

Second prize auction

- $v_1 > v_2 > \dots > v_n > 0$
- Action $b_i \in [0,\infty)$
- payoff for i:
  - $v_i - b_k$
    - if b_i is highest bid with lowest index, and b_k is second highest bid
  - 0
    - otherwise

Second prize auction

- $v_1 > v_2 > \ldots > v_n > 0$, Action $b_i \in [0,\infty)$
- payoff for i: $v_i - b_k$
    - if b_i is highest bid with lowest index, and b_k is second highest bid
  - 0, otherwise
- Then $b_i = v_i$ is a dominant strategy!

Second prize auction

- $v_1 > \dots > v_n > 0$, Action $b_i \in [0,\infty)$
- payoff for i: $v_i - b_k$
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- let $mo(i) = \max_{k \neq i} b_k$
- and let $b_i \neq v_i$

Second prize auction

- $v_1 > \ldots > v_n > 0$, Action $b_i \in [0,\infty)$
- payoff for i: $v_i - b_k$
  - if b_i is highest bid with lowest index, and b_k is second highest bid
  - 0, otherwise
- Then $b_i = v_i$ is a dominant strategy!

- let $mo(i) = \max_{k \neq i} b_k$
- and let $b_i \neq v_i$
- $mo(i) \geq v_i$
  - Either i does not get Ob
  - or $(b_i \geq v_i)$ $u_i \leq 0$
    - While at v_i, $u_i = 0$
- so: $\perp$

Second prize auction

- $v_1 > \ldots > v_n > 0$, Action $b_i \in [0,\infty)$
- payoff for i: $v_i - b_k$
  - if b_i is highest bid with lowest index, and b_k is second highest bid
  - 0, otherwise
- Then $b_i = v_i$ is a dominant strategy!

- let $mo(i) = \max_{k \neq i} b_k$
- And let $b_i \neq v_i$
- $mo(i) \geq v_i \Rightarrow \perp$
- $mo(i) < v_i$
  - with $b_i = v_i$,
    - $u_i = v_i - mo(i) > 0$
  - with $b_i \neq v_i$,
    - u_i is equal, or 0

War of attrition

- two players are interested in object Ob, private value of Ob for i is v_i
- Each player can give Ob to the other; if they do that simultaneous, Ob is divided, otherwise the one who did not give it wins Ob
- Until the moment t when some player gives Ob to other each player loses t

War of attrition

- two players are interested in object Ob, private value of Ob for i is v_i
- Each player can give Ob to the other; if they do that simulatneous, Ob is divided, otherwise the one who did not give it wins Ob
- Until the moment t when some player gives Ob to other each player loses t
- analyse the Nash equilibria

War of attrition

- two players are interested in object Ob, private value of Ob for i is v_i
- Each player can give Ob to the other; if they do that simultaneous, Ob is divided, otherwise the one who did not give it wins Ob
- Until the moment t when some player gives Ob to other each player loses t

- players: 1,2
- Actions: $t_i \in [0,\infty)$
- payoff:
- $u_i(t_1,t_2) =$
  - $-t_i$ if $t_i < t_k$
  - $v_i/2 - t_i$ if $t_i = t_k$
  - $v_i - t_k$ otherwise

War of attrition

- $t_1 = t_2 = \alpha$

	0	…	α	…
0	…	…	…	…
…	…	…	…	…
α	…	…	$v_1/2 - \alpha$ $v_2/2 - \alpha$	…
…	…	…	…	…

War of attrition

- t_1 = t_2 = α
- 1 should wait

	0	…	α	…
0	…	…	…	…
…	…	…	…	…
α	…	…	$v_1/2-\alpha$ $v_2/2-\alpha$	…
$\alpha+\varepsilon$	…	…	$v_1-\alpha$ $-\alpha$	…

War of attrition

$t_1 = t_2 = \alpha \Rightarrow \perp$

$0 < t_1 < t_2$

	0	…	α	…
0	…	…	…	…
…	…	…	…	…
α	…	…	…	…
	…	…	…	…

War of attrition

$t_1 = t_2 = \alpha \Rightarrow \bot$

$0 < t_1 = \alpha < t_2 = \beta$

	0	α	β	…
0	…	…	…	…
α	…	…	…	…
β	…	…	…	…
	…	…	…	…

War of attrition

$t_1 = t_2 = \alpha \Rightarrow \perp$

$0 < t_1 = \alpha < t_2 = \beta$

	0	…	α	β
0	…	…	…	…
…	…	…	…	…
α	…	…	…	$-\alpha$ $v_2 - \alpha$
β	…	…	…	…

War of attrition

$t_1 = t_2 = \alpha \Rightarrow \bot$

$0 < t_1 = \alpha < t_2 = \beta$

- 1 should play $\alpha - \varepsilon$

	0	…	α	β
0	…	…	…	…
$\alpha-\varepsilon$	…	…	…	$-(\alpha-\varepsilon)$ ….
α	…	…	…	$-\alpha$ $v_2 - \alpha$
β	…	…	…	…

War of attrition

$t_1 = t_2 = \alpha \Rightarrow \perp$

$0 < t_1 < t_2 \Rightarrow \perp$

$0 = t_1 < t_2 = \beta$

	0	…	β	…
0	…	…	…	…
…	…	…	…	…
β	…	…	…	…
…	…	…	…	…

War of attrition

$t_1 = t_2 = \alpha \Rightarrow \perp$

$0 < t_1 < t_2 \Rightarrow \perp$

$0 = t_1 < t_2 = \beta$

	0	…	β	…
0	…	…	0,v_2	…
…	…	…	…	…
β	…	…	…	…
…	…	…	…	…

War of attrition

$t_1 = t_2 = \alpha \Rightarrow \perp$

$0 < t_1 < t_2 \Rightarrow \perp$

$0 = t_1 < t_2 = \beta$

- better for 1 is
- $\beta+\varepsilon$
- As long as $v_1 > t_2$

(t_1, t_2) is equilibrium if $t_1=0$ and $t_2 \geq v_1$

	0	…	β	…
0	…	…	$0, v_2$	…
…	…	…	…	…
β	…	…	…	…
$\beta+\varepsilon$	…	…	$v_1-\beta$, $-\beta$	…

War of attrition

(t_1, t_2) is equilibrium if $t_1 = 0$ and $t_2 \geq v_1$

(t_1, t_2) is equilibrium if $t_2 = 0$ and $t_1 \geq v_2$

	0	…	α	…
0	…	…	…	…
…	…	…	…	…
α	…	…	…	…
	…	…	…	…

Auctions

- players {1,..,n} bid for an object O
- Private value of i is v_i and
- $v_1 > v_2 > \ldots > v_n > 0$
- sealed bid: simultaneous, one shot
- Highest bid with lowest index wins O

First prize auction

- players {1,..,n} bid for an object O
- Private value of i is v_i and
- $v_1 > v_2 > \ldots > v_n > 0$
- sealed bid: simultaneous, one shot
- Highest bid with lowest index wins O
- For the price of the bid!

First prize auction

- Formulate this as a strategic game
- Give the Nash equilibria, and show who gets the object in these equilibria
- First do it for two players with the following actions:
- $0 < a_i < b_i < x_i < d_i < e_i < y_i < g_i < h_i$
- with x value of 2 and y value of 1