Bargaining

With patience

Symmetric impatience

Asymmetric impatience

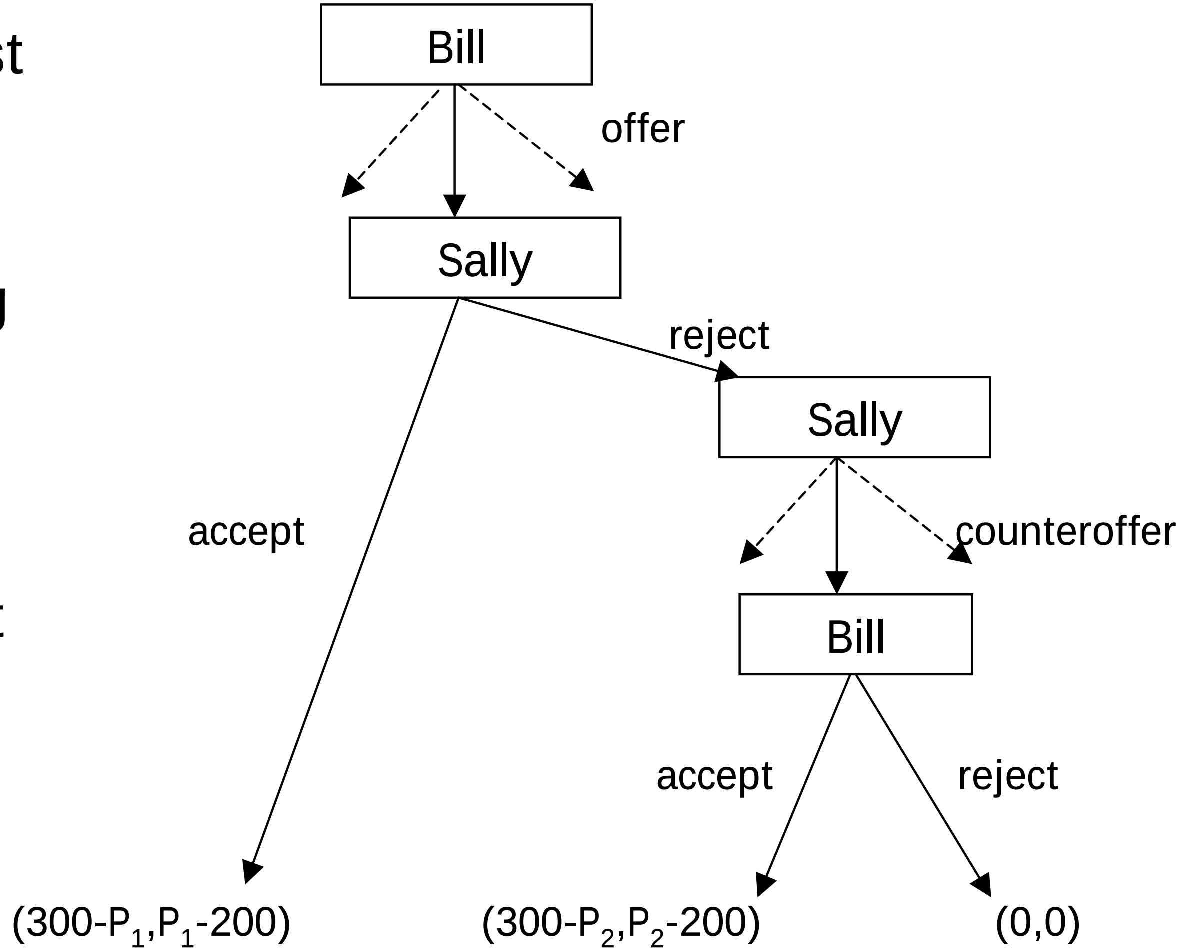
Example I

Bill pays at most $300

Sally will not accept anything below $200

If either is indifferent between accept and reject than they accept

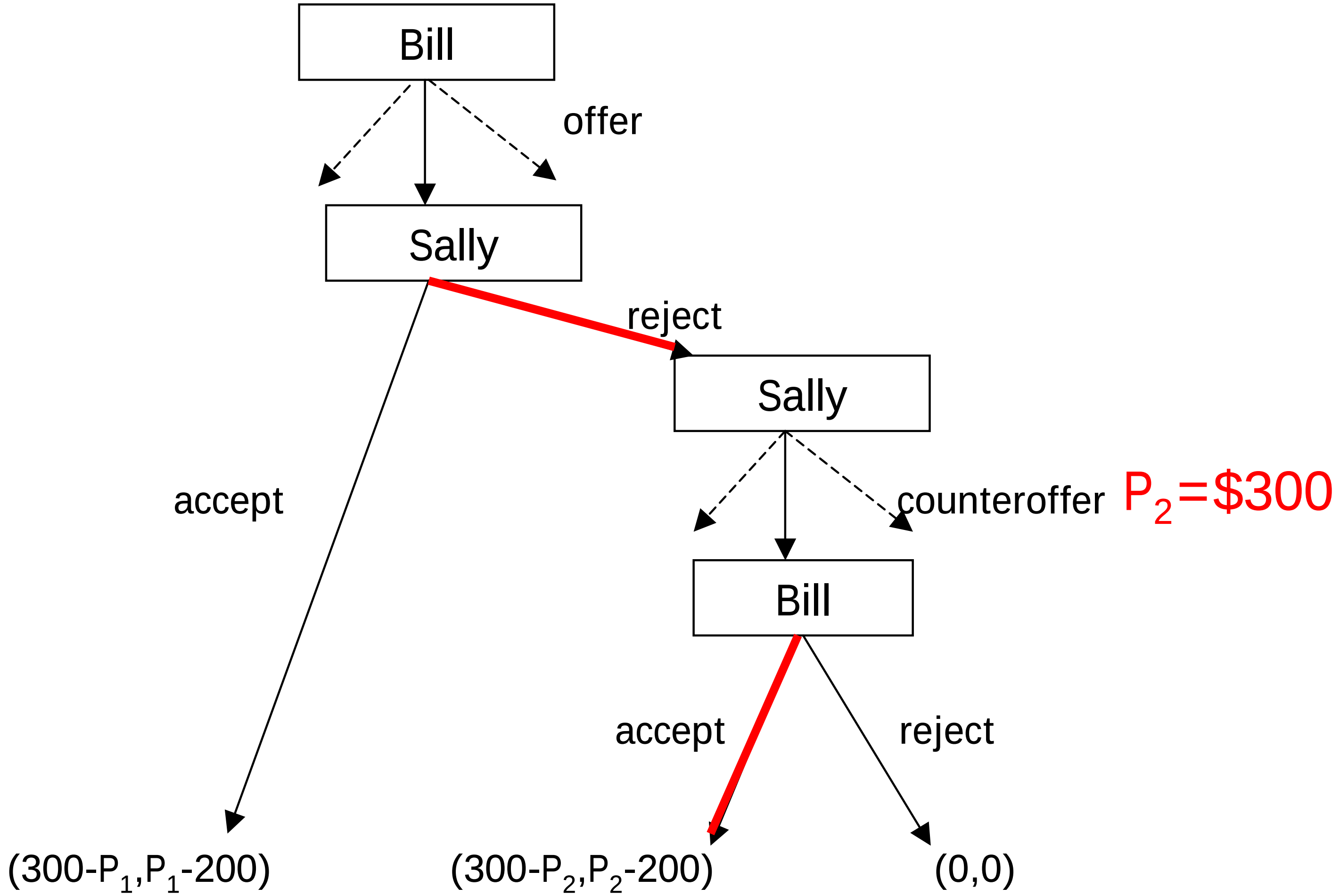
This is common knowledge



Example I

With backward induction:

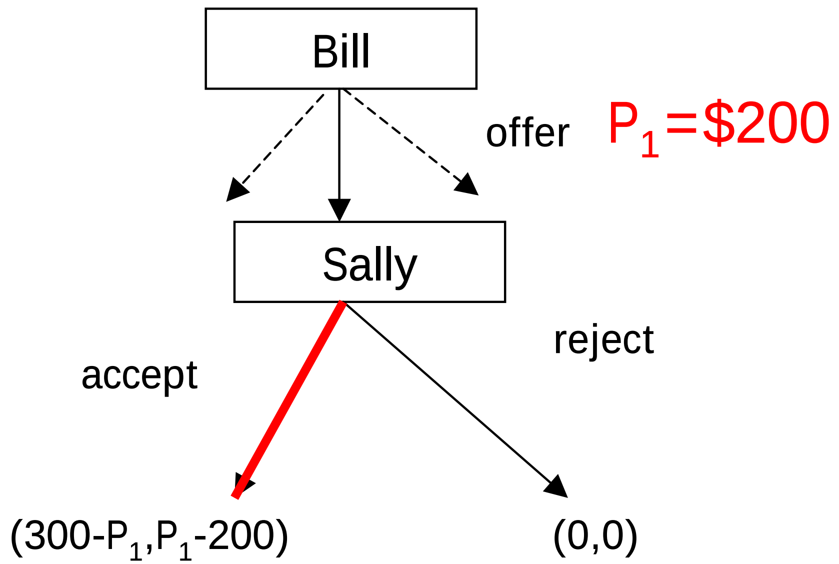
Sally gets $300



Example II

With backward induction:

Sally gets $200



Last/first mover's advantage

1. If Bill makes the first offer and the number of rounds is odd then Bill's pay-off is equal to the complete surplus
2. If Bill makes the first offer and the number of rounds is even then Sally's pay-off is equal to the complete surplus

Symmetric impatience

- There are costs connected to each round in the bargaining process
- Suppose both players endure the same costs for each round and that the costs are a percentage of the pay-off of the trade

Example III (100 rounds)

Round number	Offer maker	Sally's share	Bill's share
100	Sally	100 %	0 %
99	Bill	97 %	3 %
98	Sally	97.1 %	2.9 %
97	Bill	94.2 %	5.8 %
2	Sally	53.2 %	46.8 %
1	Bill	51.7%	48.3 %

Last mover's advantage

- Sally still has a small last mover's advantage
- Bill now uses his power to prolong the bargaining to get the price down

Asymmetric impatience

- There are costs connected to each round in the bargaining process
- Suppose both players endure different costs for each round and that the costs are a percentage of the pay-off of the trade
- The costs for Sally will be 6% for Bill 3%

Example IV (100 rounds)

Round number	Offer maker	Sally's share	Bill's share
100	Sally	100 %	0 %
99	Bill	94 %	6 %
98	Sally	94.2 %	5.8 %
97	Bill	88.5 %	11.5 %
2	Sally	34.7 %	65.3 %
1	Bill	32.6%	67.4 %

Last mover's advantage

- Sally still has a small last mover's advantage
- The ratio in the split is almost 2:1 in favour of Bill
- This is approximately the reverse of the ratio of the costs

General model

- Let x be the costs (as a percentage of the trade profit) then 1-x is the discount factor
- The buyer's discount factor is denoted by δ_B
- The seller's discount factor is denoted by δ_S
- The higher this factor, the more patient the agent!

Theorem

Suppose players S and B bargain over a surplus.

Player B makes the first offer.

There is no limit to the number of offers

The discount factors of the players are δ_S and δ (both between 0 and 1)

Players accept offers when they are indifferent between accepting or rejecting them.

Theorem (cont.)

This bargaining game has a unique subgame perfect equilibrium in which player B immediately offers S the fraction:

$$\delta_S.(1-\delta_B)/(1-\delta_S.\delta_B)$$

of the surplus, retaining

$$(1-\delta_S)/(1-\delta_S.\delta_B)$$

for himself and S accepts.

Observations

1. $\delta_S.(1-\delta_B)/(1-\delta_S.\delta_B) + (1-\delta_S)/(1-\delta_S.\delta_B) = 1$
2. If $\delta_S=\delta_B$ then $\delta_S.(1-\delta_B)< (1-\delta_S)$

   Proof: let $\delta_S=\delta_B$ and let $\delta_B<1$

   $(\delta_B-1)^2>0 \Rightarrow$

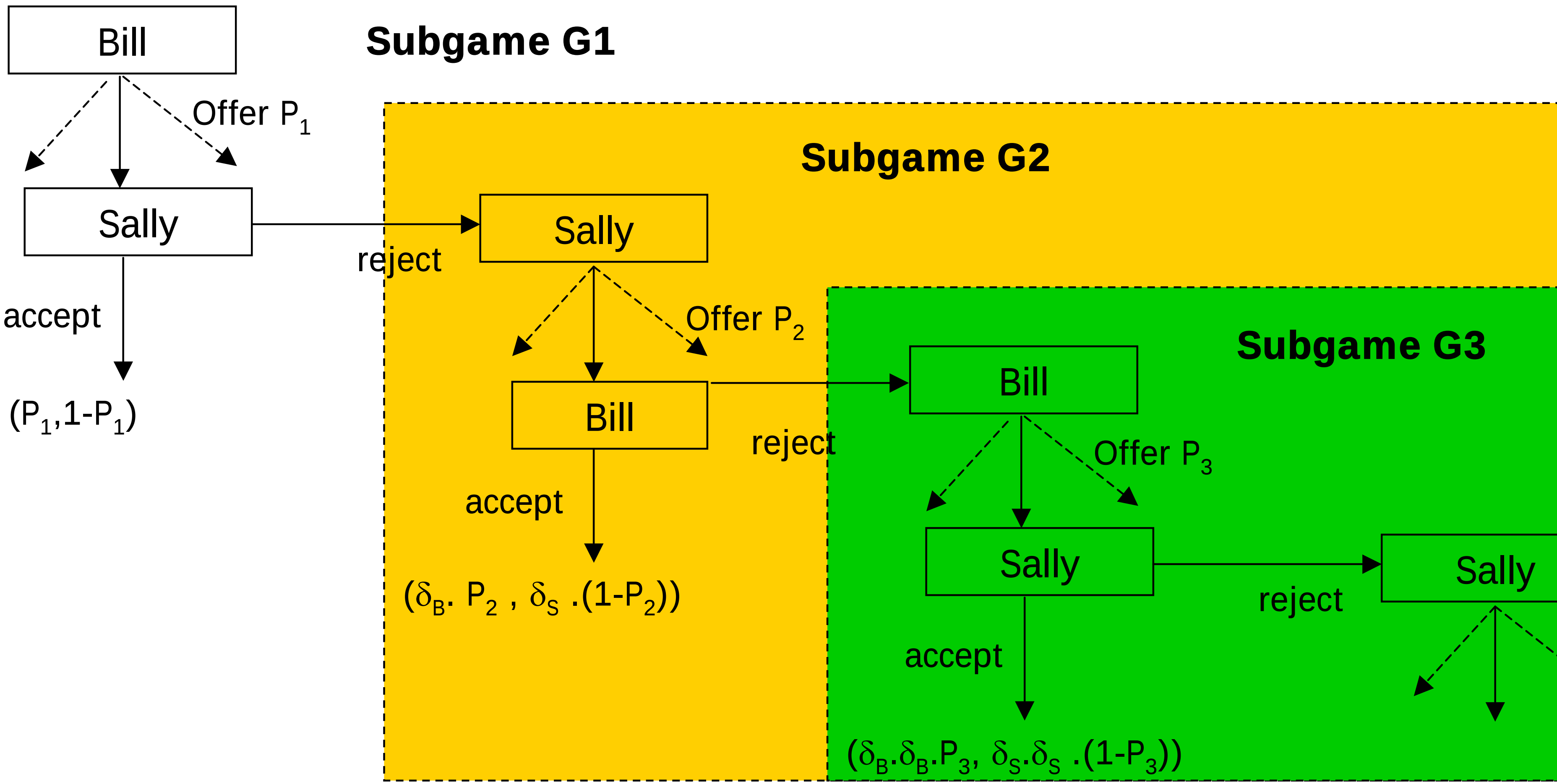
   $\delta_B^2 -2\delta_B + 1 >0 \Rightarrow$

   $-\delta_B^2 +2\delta_B - 1 <0 \Rightarrow$

   $\delta_B.(1-\delta_B) - (1-\delta_B) <0 \Rightarrow$

   $\delta_B.(1-\delta_B)< (1-\delta_B)$
3. First mover’s advantage!

Proof



Bill
Subgame G1
Offer P1
Sally
reject
accept
(P1,1-P1)
Subgame G2
Sally
Offer P2
Bill
reject
accept
(δB. P2 , δS .(1-P2))
Subgame G3
Bill
Offer P3
Sally
reject
Sally
accept
(δB.δB.P3, δS.δS .(1-P3))

Proof (cont.)

- Let the highest pay-off of the buyer in game G1 be Q_B and the lowest pay-off be q_B.
- Let the highest pay-off of the seller in game G2 be Q_S and the lowest pay-off be q_S.
- The game G3 is strategically equivalent to G1. Thus Q_B is the highest pay-off for the buyer and q_B is the lowest in G3.

Proof (cont.)

- In G1 the seller should get at least $\delta_S.q_S$ (q_S is the worst pay-off from G2, but is discounted because the seller does not have to wait).

$$Q_B \leq 1-(\delta_S.q_S) \quad [1]$$

- Similarly:

$$q_B \geq 1-(\delta_S.Q_S) \quad [2]$$

Proof (cont.)

- Reversing the logic about the buyer and seller we also have:

$$Q_S \leq 1-(\delta_B.q_B) \qquad [3]$$

- and

$$q_S \geq 1-(\delta_B.Q_B) \qquad [4]$$

$$\Leftrightarrow$$

$$1- \delta_S.q_S \leq 1- \delta_S.(1-(\delta_B.Q_B)\ )$$

- Together with [1]:

$$Q_B \leq 1- \delta_S.(1-(\delta_B.Q_B)\ )$$

$$\Leftrightarrow$$

$$Q_B \leq (1-\delta_S)/(1-\delta_S.\delta_B) \qquad [5]$$

Proof (cont.)

- From [3] (multiply with - δ_S and adding 1:

$$1- \delta_S.Q_S \geq 1- \delta_S.(1-(\delta_B.q_B)) \quad [6]$$

- Together with [2]

$$q_B \geq 1- \delta_S.(1-(\delta_B.q_B)) \quad [7]$$

$$\Leftrightarrow$$

$$q_B \geq (1-\delta_S)/(1-\delta_S.\delta_B) \quad [8]$$

- But [5] and [8] together imply:

$$Q_B \leq (1-\delta_S)/(1-\delta_S.\delta_B) \leq q_B \quad [9]$$

Proof (cont.)

- Per definition it holds that:

$$q_B \leq Q_B$$

- Together with [9] this means:

$$q_B = Q_B = (1-\delta_S)/(1-\delta_S.\delta_B) \qquad [10]$$

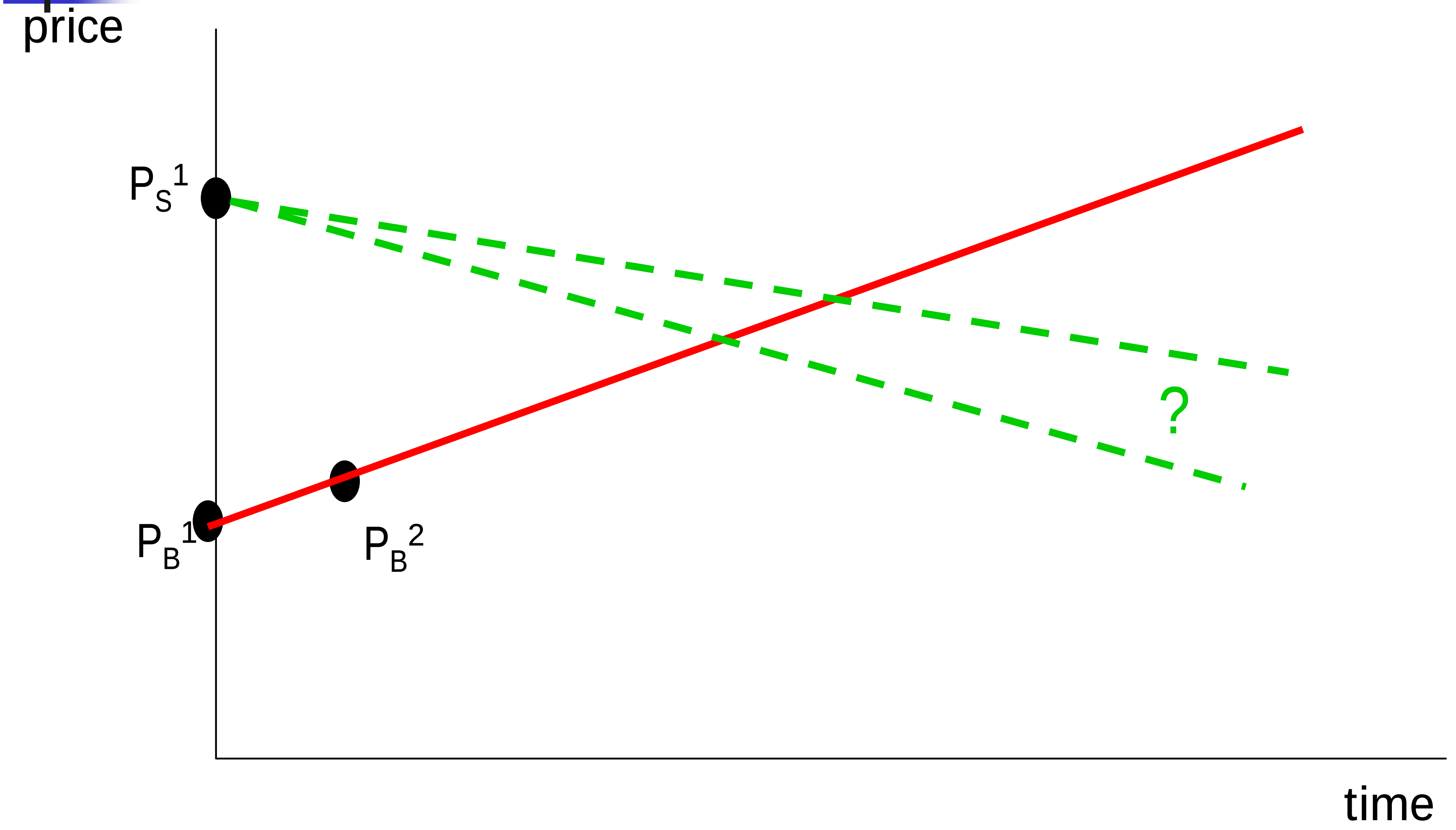
- Similarly one can derive that:

$$q_S = Q_S = 1- (1-\delta_S)/(1-\delta_S.\delta_B) \qquad [11]$$

Strategy

- Suppose buyer offers P_B^1 and then selle offers P_S^1
- If now the buyer offers P_B^2 then the seller might derive δ_B (= $(P_B^2/P_B^1).100$)
- We get the following picture:

Strategy



Strategy: But what if...