Solving the gate assignment at Amsterdam Airport Schiphol

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Gate assignment at Schiphol

Gate assignment consists of three stages:

- Seasonal planning
- One-day-ahead planning
- Tactical planning
Gate assignment at Schiphol

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We look at the One-day-ahead planning problem
Problem description

For each flight we know:

- Its arrival and departure time;
- The type of plane that flies it;
- Origin and destination;
- The preferences of the airline;
- The handling company.
Problem description (2)

Possibility of splitting flights

- Stays longer than 3 hours may be split
- Stays with different origin and destination region may be split
Problem description (3)

Goal: find a robust solution subject to

- a number of single-gate constraints;
- a number of multiple-gate constraints;

The problem instance can be partitioned according to handling company: KLM groundservice versus Non-KLM groundservice.
Model - Cost function

Several possibilities

- We use square idle time
- Based on variance
- Need to know the immediate predecessor
Model - Cost function

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Cost function refinement:

- **Punctuality-factor** \((i, j)\)
Approach: ILP

- Standard formulation: assign a flight to a position on a gate; this requires many variables.
- Multiple-gate constraints yield many constraints.
- Problems with 30 flights are solvable.

We need a non-standard approach.
Two-phase ILP approach

Phase 1: find for each set of gates of the same type the right number of ‘gate plans’, while taking into account the single-gate constraints.

Phase 2: assign the gate plans to the actual gates, while taking into account the multiple-gate constraints.

We need to distinguish between several types of gates because of size, security, etc.
Phase 1 - Model

A gate plan of type $h$ is feasible if

- the minimum idle time is at least 20 minutes
- each flight can be assigned to a gate of type $h$
Phase 1 - Model (2)

Suppose all feasible gate plans of all types are known.

Gate plan \( j \) is characterized by

\[
g_{ij} = \begin{cases} 
1 & \text{if gate plan } j \text{ contains flight } i; \\
0 & \text{otherwise.} 
\end{cases}
\]

\[
t_{hj} = \begin{cases} 
1 & \text{if gate plan } j \text{ is of type } h; \\
0 & \text{otherwise.} 
\end{cases}
\]
Model - Constraints

Basically two constraints are needed:

- Each flight must be assigned exactly once
- The number of gateplans of type $h$ is equal to the number of gates of type $h$
Minimise \( \sum_{j=1}^{n} c_j x_j \) s.t.

\[ \sum_{j=1}^{n} g_{ij} x_j = 1 \quad \text{for } i = 1, \ldots, m \]

\[ \sum_{j=1}^{n} t_{hj} x_j = T_h \quad \text{for } h = 1, \ldots, H \]

\( x_j \in \{0, 1\} \quad (j = 1, \ldots, n) \)
Model - Constraints (3)

Possibility of preferences:

- El Al has designated gates
- Some airlines have *own* gates

\[ \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{h=1}^{H} p_{ihr} t_{h} g_{ij} x_{j} \geq P_{r} \text{ for } r = 1, \ldots, R \]
Possibility of not assigning flights:

\[ \sum_{j=1}^{n} g_{ij} x_j + U_i = 1 \quad \text{for } i = 1, \ldots, m \]

This is only used for finding a feasible solution quickly.
Solving - First phase

Overview solving first phase:

- Relax original GA problem to LP-relaxation
- Solve LP-relaxation with column generation
- Solve ILP problem for the set of gate plans discovered in solving the LP-relaxation
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\[ Z_{LP} \leq Z_{GA} \leq Z_{ILP} \]
Pricing problem

Use graph $G_h = (V, E)$ for representing gate plans for gate of type $h$

- Vertex for every possible flight on gate of type $h$
- Start and End vertex
- Arc for all possible successive flights
- Path through graph is feasible gate plan
Pricing problem (2)

Set the cost of the arcs such that the length of the path is equal to the reduced cost of the corresponding gate plan.

- Assumption: Flights are sorted on arrival times
- Result: flight indices form topological ordering
- Finding shortest path: $O(|V| + |E|)$
Enhancement

Creating additional columns

1. Remove one flight in the currently optimal gate plan from the graph (once for each flight)
2. Recalculate shortest path
3. Put newly found gate plan in additional pool

- May lead to a better solution
- More importantly: this speeds up the ILP tremendously.
Solving - Second phase

Assigning gate plans to physical gates

- Impossible constraints in first phase:
  - Overlapping wingspans
  - Combining 2 smaller gates
  - Flights with same departure time on neighboring gates
  - Flights with same departure time across 'Opposing groups of passengers' close to each other
  - Usage of additional gate details
Testing

Implementation of algorithm tested with:

- Random data
- Seasonal data provided by Schiphol
- Detailed data for one day, provided by Schiphol
Results

- Maximum needed time for solving LP: 1000 seconds.
- Maximum generated columns: 2600
- Size extra columnpool: 190,000
- Maximum needed time for solving ILP: 2300 seconds.
- Typical integrality-gap: $0.00\% - 0.04\%$

Acceptable running times (speed-up with a factor of 3 by using new version of CPLEX)
Conclusion

- New ILP formulation
- Realistic constraints
- Integrality gap relatively small
- Acceptable running time
- Promising results with real life input data
Questions?