Auctions Versus Negotiations

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Which is the more profitable way to sell a company: an auction with no reserve price or an optimally-structured negotiation with one less bidder? We show under reasonable assumptions that the auction is always preferable when bidders' signals are independent. For affiliated signals, the result holds under certain restrictions on the seller's choice of negotiating mechanism. The result suggests that the value of negotiating skill is small relative to the value of additional competition. The paper also shows how the analogies between monopoly theory and auction theory can help derive new results in auction theory. (JEL D44, G34)

There are close analogies between standard price theory and the theory of auctions. In an absolute English auction, in which the price rises continuously until only one bidder remains and the seller is required to accept the final bid, the sale price equals the lowest competitive price at which supply equals demand. In the theory of optimal auctions the seller is treated as a monopolist who can choose any mechanism, such as establishing a minimum sale (or reserve) price, to maximize expected profit. As in monopoly theory, optimal auction theory assigns all bargaining power to the seller, subject to the constraint that she does not have access to buyers' private information about an asset's value.

This paper shows how the analogies between monopoly theory and auction theory can help derive new results in auction theory. Specifically, we are able to put a fairly tight bound on the value of any seller's bargaining power: a seller with no bargaining power who can only run an English auction with no reserve price among $N + 1$ symmetric bidders will earn more in expectation than a seller with all the bargaining power, including the ability

to make binding commitments, who can hold an optimal auction with N buyers. This is true under standard assumptions if buyers have private values, common values, or something intermediate. No amount of bargaining power is as valuable to the seller as attracting one extra bona fide bidder.

Since the informational demands for computing optimal mechanisms are substantial, and the computations involved are complex, this result suggests that it will often be more worthwhile for a seller to devote resources to expanding the market than to collecting the information and making the calculations required to figure out the best mechanism.[1]

Our analysis also has policy implications for when the directors of a public company should be allowed to privately negotiate its sale. Our result shows that a single extra bidder more than makes up for any diminution in negotiating power. This means that there is no merit in arguments that negotiation should be restricted to one or a few bidders to allow the seller to maintain more control of the negotiating process, or to credibly withdraw the company from the market.[2]

[1] Similarly, in a procurement context, competitive bidding by suppliers will yield lower average prices than negotiating with a smaller number of suppliers. See R. Preston McAfee and John McMillan (1987b) for examples. More broadly, our results are supportive of the view that optimal regulation of an industry may be less important than attracting additional entry.

[2] Opening negotiations with additional bidders makes

Similarly, a seller should not accept any "lock-up" agreement that a buyer is willing to offer in return for the seller not beginning negotiations with additional potential acquirers. For example, in late 1993 Paramount agreed to sell itself to Viacom, knowing that QVC was interested in bidding for Paramount. Paramount and Viacom agreed to terms that gave Viacom options to buy 24 million shares of Paramount and a $100 million break-up fee in the event that any other company were to purchase Paramount. The boards argued that in return for effectively excluding other bidders, Paramount had been able to negotiate a higher price than it could have expected in an open auction. QVC contested the terms of the deal, contending that holding an auction would have been the appropriate way to maximize shareholder value. The Delaware courts subsequently agreed with QVC. Our analysis supports that decision.[3]

We begin in Section I by developing the intuition for our results, and informally deriving them in the simple and familiar case of buyers with independent private values.

We develop our general model in Section II. We extend Bulow and John D. Roberts's (1989) interpretation of auctions, based on marginal revenues, from their independent private values model to this general model. We use this to show (Theorem 1) that an English auction with $N + 1$ bidders but no reserve yields higher revenue in expectation than an English auction with N bidders, culminating with a final optimal take-it-or-leave-it offer to the last remaining bidder. Under mild assumptions, this result holds regardless of whether bidders' signals are independent or affiliated.[4]

We then show (Theorem 2) that with N risk-neutral bidders with independent signals, it is optimal for the seller to use the N-bidder mechanism described above, with a final offer that generally depends on the prices at which the low bidders dropped out. With independent signals and risk-neutral bidders, therefore, an auction with $N + 1$ bidders dominates any negotiation with N bidders.

With affiliated (but nonindependent) signals an English auction plus final take-it-or-leave-it offer does not maximize expected revenue among all conceivable selling mechanisms, but it does maximize expected revenue subject to some restrictions on the seller's choice of mechanism.[5] It therefore remains true that an auction with $N + 1$ bidders beats any standard mechanism for selling to N bidders.[6,7]

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it harder to use negotiating tactics such as credibly threatening not to sell if the bidders' offers are too low. If a board approaches many bidders, it may be legally forced to hold an open auction and cede its power to control the form of negotiations, see e.g., Jesse H. Choper et al. (1989). In any event, the business of a company whose future ownership is thought to be uncertain may be damaged until the question of ownership is resolved so it may be hard to credibly withdraw the company from the market. See, e.g., Andrei Shleifer and Robert W. Vishny (1988).

[3] Under dominant U.S. takeover law, a company can negotiate its sale to a purchaser and decline to hold an auction if (i) the board is acting in good faith to maximize shareholder value and has conducted a reasonable investigation of whether the price is adequate, and (ii) the price attained through negotiations was high enough to be worth the cost of any lock-up provisions and other prohibitions necessary to secure the offer.

[4] Signals are affiliated if, as a bidder's signal rises, he expects others' signals to rise as well, in the sense that higher values for other bidders' signals become relatively more likely. See Paul Milgrom and Robert J. Weber (1982).

[5] Giuseppe Lopomo (1995) shows that the English auction plus reserve price maximizes the seller's expected profit in Milgrom and Weber's (1982) "general symmetric model" among all mechanisms where losers do not pay anything and in equilibrium the winner (if anyone) is the bidder with the highest signal and his payments are weakly increasing in his own signal for any realization of other bidders' signals.

[6] The results of the one-shot seller-optimal mechanism-design literature extend straightforwardly to dynamic games in which the seller's discount rate is at least as high as the buyers', so a seller cannot do better in any multiperiod game than in the one-shot game. (Using delay is simply equivalent to a commitment to not sell with some probability—see, for example, Peter C. Cramton [1985] and Milgrom [1987].) If the seller's discount rate is lower than the buyers', then screening over time can allow the seller to extract a larger surplus than one can obtain from a one-shot mechanism. In the extreme case in which the seller does not discount the future at all and the buyers do, then the seller should run an extremely slow "Dutch" auction, in which the price begins high and is gradually reduced, and this will extract arbitrarily close to all surplus. We do, however, ignore any time costs of accumu-

We also note that if a seller could negotiate with N bidders while maintaining the right to subsequently hold an English auction without a reserve price and with an additional bidder, the seller would always do better to proceed directly to the auction. Thus a seller should generally focus on maximizing the number of bidders, and should refuse to bargain with bidders who wish to preempt the auction process.[8]

Finally we extend our result to multiple units and show that the price-theoretic analogy of this extension gives an interesting result about the value to firms of restricting competition relative to the value of expanding demand.

I. An Example with Independent Private Values

We begin with a simple problem and then generalize. Seller A has one "serious" potential buyer, with a value that is at least as high as the seller's. For example, A's value is zero

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lating buyers, and any differences in the costs of running different sales mechanisms. See Ruqu Wang (1993).

[7] We do not analyze how the number of bidders may be affected by a firm's choice of mechanism. However, a public auction may not only attract extra bidders through the extra publicity, but may also attract more bidders if bidding is costly. For example, with symmetric bidders who simultaneously decide whether or not to pay the costs of participating in an auction before learning their signals, an auction with a reserve price may attract fewer bidders than an auction without reserve. Our analysis will speak directly to this case. On the other hand, with sequential entry of potential bidders who decide in turn whether or not to pay the costs of acquiring a signal and then making a bid, bidders may make preemptive bids to make it unprofitable for any future bidder to enter. In this case, there is no trade-off between using a reserve price and attracting bidders: a higher reserve price earns more money because it both allows the seller to extract rents and increases the expected number of buyers who will participate by making a preemptive bid more difficult. See McAfee and McMillan (1987a). For further analysis of optimal seller strategies with costly bidder participation, see Richard Engelbrecht-Wiggans (1993), McAfee and Daniel Vincent (1992), and Shleifer and Vishny (1986).

[8] We assume a single seller, interested only in expected revenue, so in the context of selling a company we are abstracting from issues such as shareholders' individual incentives to sell (see, e.g., Sanford J. Grossman and Oliver D. Hart, 1980; David Hirshleifer and Sheridan Titman, 1990; and Bengt Holmstrom and Barry Nalebuff, 1992), or management's interest in retaining control (see, e.g., Milton Harris and Artur Raviv, 1988).

and the buyer's value, which is private information to the buyer, is drawn from a uniform distribution on [0, 1]. Both parties are risk neutral. It is easy to show that the optimal strategy for A in negotiating with her buyer is to offer a take-it-or-leave-it price of .5; the offer will be accepted half of the time, yielding an expected profit of .25. Seller B also has a value of 0, but differs from A in two respects: first, she has two "serious" bidders, each with private values drawn independently on [0, 1]; second, she may hold only an English auction with no reserve.[9] In this auction, the expected profit to the seller will be the expectation of the lower of the two bidders' values, which is the point in the auction where the lower bidder will drop out. That expected profit is 1/3, so the extra bidder is worth more than the reserve price.

How can we generalize this result? The difficulty can be illustrated in our numerical example. It is clear that in some cases (namely those when the first bidder's value is greater than or equal to .5, and the second bidder's value is less than .5) the reserve price is worth more ex post than the second bidder, but in all other cases the seller is better off with the extra bidder. The question is whether there is some way to group the potential cases so that the seller with two bidders does better in expectation within every subgrouping, and therefore better on the whole.

The most natural thing to try is to divide up the cases into those where the first bidder has a value above the reserve price of .5 and those where the first bidder's value is below .5. At first glance, this methodology does not work, even in our simple numerical example. Contingent on the first bidder having a value below .5, of course the seller with two bidders and no reserve price will earn more than the seller

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[9] Throughout the paper, an English auction is an auction in which the price rises continuously until only one bidder remains. At every price all bidders know how many other bidders remain active. A *reserve price* is a minimum price below which the object will not be sold. (In an auction with a reserve price the seller makes a final take-it-or-leave-it offer equal to the reserve price to the final bidder, if the final bid is below the reserve price.) An *absolute auction* is an auction with no reserve price; that is, the seller is required to accept the final bid.

with one bidder (who will earn zero). But contingent on the first bidder having a value between .5 and 1, expected revenue will be .5 with the reserve price and 11/24 with the extra bidder.[10]

Clearly, we need to use something other than expected values to establish superiority for the auction.

What we do is borrow a trick from monopoly theory. Consider a seller with constant marginal costs of zero, and a linear demand curve of $p = 1 - q$. How do we know that .5 is the optimal price and quantity for the seller? If the seller chooses a quantity of .4 and a price of .6, for example, she will earn more revenue from the .4 buyers who actually purchase than if she only charges .5, so there is no direct dominance. Similarly, if a price of .4 is chosen the seller earns less from the first .5 customers, but there is that extra revenue from the last .1.

The way we establish superiority for the quantity of .5 over the alternatives is by looking at marginal revenues instead of prices. Marginal revenue equals zero at a quantity of .5. By selling .5 units, the seller earns the same positive marginal revenues on the first .4 units as a seller of only .4, plus some extra positive marginal revenues on the next .1 sales. Selling .5 is better than selling .6, because by selling .5 you earn the same positive marginal revenues on the first .5 units, and eliminate the negative marginal revenues on the last .1. By looking at something like marginal revenues we can establish the superiority of the two-bidder auction in our initial problem.[11]

Assume that bidder j receives a private signal t_j which is distributed with a density $f(t_j)$ and a cumulative density $F(t_j)$, independent of other bidders' signals, and implies a private value of $v(t_j)$. Graph value, $v(t_j)$, against "quantity," $q(t_j) \equiv 1 - F(t_j)$. In terms of our numerical example, the picture produced is an "inverse-demand curve" of $p = 1 - q$. As Figure 1 shows, the quantity of (expected)
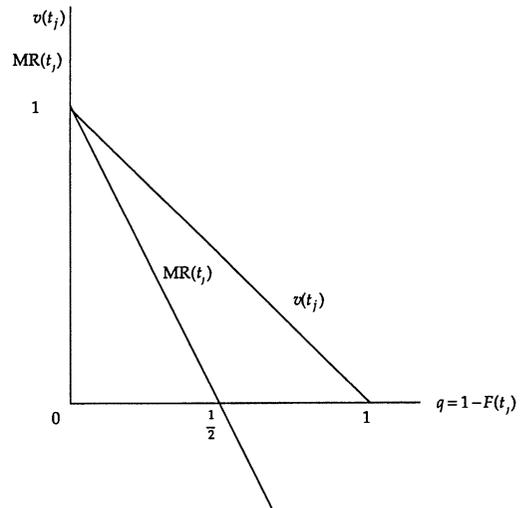

FIGURE 1. DEMAND AND MARGINAL REVENUE CURVES FOR ONE BIDDER IN OUR EXAMPLE

sales will be zero at a price of $v(t_j) = 1$, increasing linearly to one (expected) sale at a price of $v(t_j) = 0$.

Defining revenue as price times quantity, we can also derive marginal revenue in the usual way,

$$\text{MR}(t_j) \equiv \frac{d}{dq(t_j)} [v(t_j) \cdot q(t_j)]$$

$$= \frac{-1}{f(t_j)} \frac{d}{dt_j} [v(t_j)[1 - F(t_j)]],$$

and graph marginal revenue against quantity of expected sales. In our example, the marginal revenue curve from our demand curve is $\text{MR} = 1 - 2q$. Note that the sales probability that is optimal for the seller with one bidder (and the optimal take-it-or-leave-it price) can be found where MR equals zero, at $q = .5$ and $p = .5$.

Why? Just as the revenue from a take-it-or-leave-it price can be calculated by multiplying that price by the probability of sale at that price, expected revenue can also be found by taking the area under the MR curve for all the values in excess of the take-it-or-leave-it price. Obviously, the optimal take-it-or-leave-it price

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[10] The expectation of the lower of two values, one of which is known to exceed 1/2, is 11/24.

[11] The interpretation of auctions in terms of marginal revenues follows Bulow and Roberts (1989). The current paper shows how to extend this interpretation from the independent private values model to the general case.
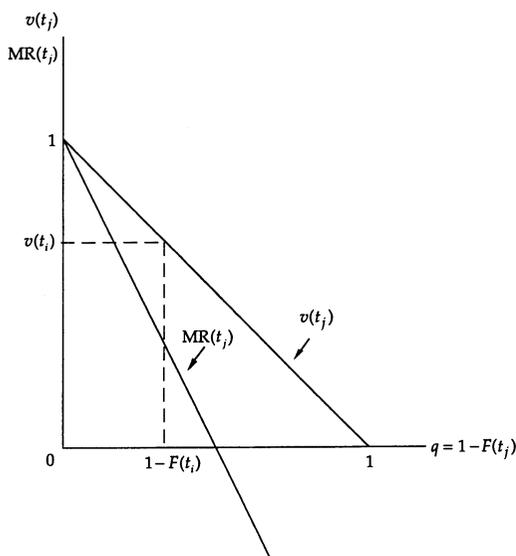
FIGURE 2. THE EXPECTATION OF MR(t_j), CONDITIONAL ON $t_j \geq t_i$, EQUALS THE AREA UNDER THE MR(t_j) CURVE FROM $q = 0$ TO $q = 1 - F(t_i)$, DIVIDED BY $1 - F(t_i)$: THIS MUST EQUAL $v(t_i)$

is where MR = 0. The seller may be thought of as receiving, in expectation, the MR of the buyer when it is positive, and zero when the buyer's MR is negative. Put slightly differently, expected revenue may be thought of as the expectation of the maximum of the MR of the bidder and zero.

Now let's move on to the problem of seller B, who holds an auction between two bidders. Assume that the "underbidder" has a value of $v(t_i)$ and the eventual winner has a value $v(t_j)$. We do not learn the value of $v(t_j)$ in the auction, but it is greater than or equal to $v(t_i)$. Consider the graph with the same demand curve and marginal revenue curve as before, and mark a point on that curve as $v(t_i)$ (see Figure 2). Contingent on the underbidder's value being $v(t_i)$, we know that the seller will earn exactly $v(t_i)$ in the auction, but it will be more useful to express the seller's winnings in terms of marginal revenue. What is the expected MR associated with the winning buyer, conditional on the selling price being $v(t_i)$? It is obviously $v(t_i)$, by the same simple math as we use to show that the average marginal revenue associated with a monopolist's customers

must be equal to her selling price. For example, in our numerical example, if $v(t_i) = .6$ the MR of the winner might be anywhere from .2 to 1, but on average it will be .6. This implies that for any $v(t_i)$, the expected revenue of the seller can be described as the expected MR of the winning bidder. Averaging over all possible values for $v(t_i)$, therefore, the seller's expected revenue equals the expected MR of the winning bidder.

Now add a conventional auction theory/monopoly theory/mechanism design assumption: assume that the MR curve is downward sloping. This implies that the buyer with the higher value, who is the one who will actually win the auction, is also the buyer with the higher MR. If the seller's expected revenue is the expected MR of the winning bidder, and the winning bidder always has the higher MR of the two bidders, then the expected revenue from the auction can be written as the expectation of the maximum of the marginal revenues of the two bidders, call it

Expected Revenue (auction with two)

$$= E\{\max(MR(t_1), MR(t_2))\}.$$

This may be compared with the expectation from the one-bidder mechanism,

Expected Revenue (optimal mechanism

with one) $= E\{\max(MR(t_1), 0)\}.$

Now compare the right-hand sides of the two expressions above in the cases where the first bidder's value exceeds the optimal reserve price, so that $MR(t_1) \geq 0$. It is obvious that contingent on that, the first expression is larger.[12] But what about if $MR(t_1) < 0$? Here

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[12] Of course, as demonstrated above, it does *not* follow that contingent on MR(t_1) $\geq$ 0,

Expected Revenue (auction with two)

> Expected Revenue (optimal mechanism with one).

Likewise, a monopolist's actual revenue from a subset of the buyers sold to does not equal the sum of the marginal revenues of these buyers.

is where our "serious-bidder" assumption, that both potential bidders have a value at least equal to the seller's value of zero, comes in to play. What is the expectation of $MR(t_2)$? Since the lowest possible value of $v(t_2)$ is zero, it must be that the expectation of $MR(t_2)$ equals zero. In demand-curve terms, if we set a price of zero, then total revenue, and therefore also the average MR of all buyers, must be zero. So if $MR(t_1) < 0$, then the second expression is zero, while the first expression is the expectation of the maximum of two terms, one of which has an expected value of zero. Again, therefore, the first expression is larger, so we have now established the auction's dominance.

The need for our serious-bidder assumption should be quite clear. Assume that there is a probability $1 - \rho$ that the second bidder values the asset below zero, and that the second bidder is otherwise drawn from the same distribution as the first bidder.[13] Then the second bidder will be worth only ρ times as much to the auction seller as if it were certain that the second bidder had a value above zero.[14] In the limit where ρ approaches zero, the extra bidder would be of virtually no use, and a reserve price would dominate. In our numerical example, we would require $\rho \geq .75$ for the auction to be at least as good as the reserve price.[15]

It is easy to extend the analysis to compare a seller with N (symmetric) bidders in an auction and a reserve price to one with $N + 1$ bidders and no reserve price. By exactly the same analysis as in the one- and two-bidder case, the expected revenue from an auction with N bidders and an optimal reserve price is equal to the expectation of the maximum of $(MR(t_1), MR(t_2), \dots, MR(t_N), 0)$ while the expected revenue from an auction with $N + 1$ bidders is equal to the expectation of the maximum of $(MR(t_1), MR(t_2), \dots, MR(t_{N+1}))$. Since the expectation of $MR(t_{N+1})$ is equal to zero, it is clear that the auction with the extra bidder yields a higher expected revenue.

We have now gone pretty far while relying on only elementary mathematics. Since it is a standard result that an auction with an optimal reserve price is an optimal mechanism if bidders are symmetric and risk neutral and have independent private values and downward sloping MRs (John G. Riley and William F. Samuelson, 1981; and Roger B. Myerson, 1981), we have already shown that, under these assumptions, an auction with $N + 1$ bidders is superior to *any* mechanism involving N buyers.

The above discussion assumed that bidders have independent private values. In fact, the argument that the expected revenue from an absolute English auction equals the expected MR of the winning bidder applies very generally. Similarly it is a very general result that the expected revenue from an English auction with an optimal reserve price equals the expectation of the maximum of the highest bidder's MR and zero. The difficulty is that in a general model bidders' values and MRs are not independent of other bidders' private signals. Conditional on the first N bidders having low MRs, the expected MR of the $(N + 1)$st bidder is also low. Furthermore since, we will show, to compute an English auction's expected revenue each bidder's MR must be calculated based on the information that the auction will reveal, a bidder's relevant MR in an N-bidder auction is different than in an $N + 1$ bidder auction. Nevertheless the method of proof outlined above can be developed to show that an extra bidder is worth more than an optimal reserve price if either bidders' values are private or bidders' signals are affiliated.[16] It then follows easily that an auction

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[13] We could also assume that there is a probability of $1 - \rho$ that the first bidder has a value below zero to maintain symmetry, but since neither sales mechanism yields any profit when the first bidder's value is below zero, we can restrict our comparison to cases where the first-bidder's value is at least zero.

[14] We assume that the seller who runs an auction can demand a minimum price of zero.

[15] Without the serious-bidder assumption, if there are enough extra bidders that in expectation the *second-highest* extra bidder has a value of at least zero (and their MRs are downward sloping), then it follows that the expectation of the highest MR of the extra bidder is at least zero, so that the extra bidders are more valuable than the reserve price in expectation, even if the seller is not allowed to insist on any minimum price, and may therefore be sometimes obligated to sell at a loss.

[16] See note 4 for an informal definition of affiliation, and the proof of Theorem 1 for the relevant implications

with $N + 1$ bidders and no reserve price is more profitable than any standard mechanism with N bidders.

II. The General Model

In our general model, bidders' private signals need not be independent, and bidders' values may be either private or common, or something intermediate.[17]

Let t_j be bidder j's private signal about the value of the asset. Without loss of generality, normalize so that $0 \le t_j \le 1 \ \forall j$, and normalize the seller's value of the object to zero. We write $\mathbf{T}$ to represent the vector $(t_1, \ldots, t_{N+1})$, $\mathbf{T}_{-j}$ to represent all of the elements of $\mathbf{T}$ other than t_j, and define $\bar{\mathbf{T}} \equiv \mathbf{T}_{-(N+1)}$, and $\bar{\mathbf{T}}_{-j}$ as all the elements of $\bar{\mathbf{T}}$ other than t_j.

We write $f(t_j | \mathbf{T}_{-j})$ for the conditional density of t_j given $\mathbf{T}_{-j}$, and $F(t_j | \mathbf{T}_{-j})$ for the probability that the jth signal is less than or equal to t_j given $\mathbf{T}_{-j}$. More generally, we write $f(x | y)$ for the conditional density of x given y, and $F(x | y)$ for this conditional distribution.

We assume $f(t_j | \mathbf{T}_{-j})$ is positive and finite for all t_j and $\mathbf{T}_{-j}$.

Let $v_j(\mathbf{T})$ be the value of the asset to bidder j as a function of the vector of signals $\mathbf{T}$, and let $\bar{v}_j(\bar{\mathbf{T}}) \equiv E_{t_{N+1}}\{v_j(\mathbf{T})\} = \int_0^1 v_j(\mathbf{T}) f(t_{N+1} | \bar{\mathbf{T}}) dt_{N+1}$ be the expectation of $v_j(\mathbf{T})$ conditional on $\bar{\mathbf{T}}$. Higher signals imply higher expected values, so $\partial v_j(\mathbf{T}) / \partial t_j > 0$, $\partial v_i(\mathbf{T}) / \partial t_j \ge 0$, and $t_j > t_i \Rightarrow v_j(\mathbf{T}) \ge v_i(\mathbf{T}) \ \forall i, j, \mathbf{T}$. In the special case of private values,

$v_i(\mathbf{T})$ is a function only of t_i, while in the special case of pure common values $v_i(\mathbf{T}) = v_j(\mathbf{T}) \ \forall i, j, \mathbf{T}$. So that seller revenue is bounded, we assume $v_j(\mathbf{T}) \le v^* < \infty \ \forall j, \mathbf{T}$.

While t_j is private information to bidder j, the functions $v_j(\mathbf{T})$ and $f(t_j | \mathbf{T}_{-j})$ are common knowledge.

We assume that all agents are risk neutral, though this is not necessary for Lemma 1 or Theorem 1.

Finally, we define

$MR_j(\mathbf{T})$

$$\equiv \frac{-1}{f(t_j | \mathbf{T}_{-j})} \frac{d}{dt_j} [v_j(\mathbf{T})[1 - F(t_j | \mathbf{T}_{-j})]]$$

and

$\overline{MR}_j(\bar{\mathbf{T}})$

$$\equiv \frac{-1}{f(t_j | \bar{\mathbf{T}}_{-j})} \frac{d}{dt_j} [\bar{v}_j(\bar{\mathbf{T}})[1 - F(t_j | \bar{\mathbf{T}}_{-j})]].$$

The interpretations of the marginal revenues MR_j and $\overline{MR}_j$ are exactly as in Section I: graphically, if we plot $v_j(\mathbf{T})$ against quantity $1 - F(t_j | \mathbf{T}_{-j})$ for any bidder, varying only t_j, we will have a downward-sloping demand curve. If we think of that graph as the demand curve of buyer j, with the quantity being the probability that the buyer would accept a take-it-or-leave-it offer at any given price if he knew the signals of all the other bidders, then the MR_j curve is just the marginal-revenue curve derived from that demand curve. Similarly, the $\overline{MR}_j$ curve is derived from the graph of $\bar{v}_j(\bar{\mathbf{T}})$ against $1 - F(t_j | \bar{\mathbf{T}}_{-j})$, for a buyer who knows the signals of all the other buyers except the $N + 1$st.[18]

We maintain the following assumptions throughout:

(A.1) *Downward-Sloping MR*: $t_j > t_i \Rightarrow MR_j(\mathbf{T}) > MR_i(\mathbf{T})$ and $\overline{MR}_j(\bar{\mathbf{T}}) > \overline{MR}_i(\bar{\mathbf{T}})$.

(A.2) *Serious Bidders*: $v_j(\mathbf{T}) \ge 0 \ \forall j, \mathbf{T}$.

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for our model. For the private values auction-with-two vs. reserve-price-with-one case the argument is easily extended to affiliated signals: the seller's revenue from the auction equals the lower of the two values, which equals the average of the values minus half their difference. Affiliation leaves the average the same but reduces the difference, so it further increases expected seller revenue above the expected revenue from a single bidder plus a reserve price. In the general case, if the extra bidder's signal is affiliated with other bidders' signals, this effectively reduces the amount of private information available to each bidder, thus reducing the "information rent" earned by the winning bidder, and so increasing the value to the seller of the additional bidder, relative to the value of a reserve price.

[17] Our model is essentially that of Milgrom and Weber (1982), although we do not always impose their affiliation assumption.

[18] Note that $\overline{MR}_j(\bar{\mathbf{T}})$ is *not* in general the expectation of $MR_j(\mathbf{T})$ unless bidders' signals are independent.

(A.3) *Symmetry*: Bidders' value functions are symmetric, so $v_i(t_1, \ldots, t_i, \ldots, t_j, \ldots) = v_j(t_1, \ldots, t_j, \ldots, t_i, \ldots) \ \forall i, j, \mathbf{T}$, bidders' signals are symmetrically distributed, and bidders choose symmetric strategies.[19]

Assumption (A.1) is a standard regularity condition in auction theory, analogous to an assumption of a downward-sloping marginal-revenue curve in monopoly theory. Assumption (A.2) ensures that every bidder is willing to make an opening offer of zero, the seller's value, in an absolute English auction. Assumption (A.3) ensures that the bidder with the highest signal always wins such an auction.[20]

A. Expected Revenue from Auctions

We now follow the strategy used in Section I to develop our main theorem. All proofs are provided in the Appendix.

LEMMA 1: *The expected revenue from an absolute English auction with N + 1 bidders equals* $E_T \{ \max(MR_1(\mathbf{T}), MR_2(\mathbf{T}), \ldots, MR_{N+1}(\mathbf{T})) \}$.

LEMMA 2: *The expected revenue from an English auction with N risk-neutral bidders followed, after the N − 1 low bidders have quit, by an optimally chosen take-it-or-leave-it offer to the remaining bidder, equals* $E_{\bar{T}} \{ \max(\overline{MR}_1(\bar{\mathbf{T}}), \overline{MR}_2(\bar{\mathbf{T}}), \ldots, \overline{MR}_N(\bar{\mathbf{T}}), 0) \}$.

The proofs of these two lemmas straightforwardly follow the arguments of Section I; as with independent private values, the optimal take-it-or-leave-it final offer is the maximum of the price at which the last losing bidder quits, and the price at which the winner's marginal revenue would equal zero.[21] In the general case, however, each bidder's marginal revenue depends on all other bidders' signals, so the optimal final offer can only be determined after all the losing bidders' signals have been inferred from the prices at which they quit the auction. This can explain why it is common for a seller to announce a reserve price only at the end of the auction.[22]

THEOREM 1: *Expected revenue from an absolute English auction with N + 1 bidders exceeds expected revenue from an English auction with N bidders followed by a take-it-or-leave-it offer to the last remaining bidder if either (i) bidders' values are private; or (ii) bidders' signals are affiliated.*[23]

Just as for our independent private values example, the proof considers separately the

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[19] That is, in an absolute English auction each bidder's equilibrium strategy is to drop out of the bidding at the price he would just be willing to pay given the actual signals of the bidders who have already dropped out (in equilibrium their signals can be inferred from where they dropped out) and assuming all the remaining active bidders (whose signals he does not know) have signals equal to his own. (To see this, observe that if all other bidders follow this rule, a bidder is happy (unhappy) to find himself the winner at any price below (above) this stopping price; in the special case of pure private values each bidder just drops out at his own value.) Note that Sushil Bikhchandani and Riley (1993) show that there may be other (asymmetric) equilibria.

[20] If bidder signals are negatively correlated, then (A.1) is less likely to hold than with independent signals. (A.2) is less likely to apply in a common-values setting than with private values. For further discussion of the importance of the assumptions see sections 9 and 10 of our working paper, Bulow and Klemperer (1994a).

[21] Strictly, bidders with very low signals may be indifferent about participating since they might know that they would never meet the seller's take-it-or-leave-it price. However, the seller can induce all bidders to participate at an arbitrarily small cost in expected revenue by committing to foregoing the take-it-or-leave-it offer with a probability approaching zero and to always accepting the highest bid in this event. Note also that, strictly speaking the rule by which the seller's final take-it-or-leave-it offer will be determined must be precommitted to before the bidding. Otherwise there is in theory the possibility of other symmetric equilibria that are less profitable for the seller. For example, it is a sequential equilibrium that every bidder drops out at a certain price; if any bidder stays, that bidder is believed to have the highest possible signal and is offered a very high final price.

[22] Of course, a seller should also not commit to a reserve price until the end of the auction. Many auction houses seem to commit to secret reserve prices before auctions, but there are often further subsequent negotiations if an object is unsold at its reserve price.

[23] Note that independent signals are affiliated.

cases in which the highest of the first N bidders, say bidder j, has a positive or negative $\overline{MR}_j$, that is his value exceeds or does not exceed the optimal reserve price (i.e. take-it-or-leave-it offer) that would be set contingent on the other $N - 1$ of the first N signals. As before, when there would be no sale the expectation over t_{N+1} of MR_{N+1} equals bidder $N + 1$'s lowest possible value, which equals or exceeds zero by the "serious bidder" assumption. When there would be a sale, affiliation implies that the expectation (over t_j and t_{N+1}) of MR_j is greater than or equal to the expectation (over t_j) of $\overline{MR}_j$, contingent on the other $N - 1$ signals and on a sale. With either sale or no sale, then, the expectation of the maximum of MR_j and MR_{N+1} exceeds in expectation the maximum of $\overline{MR}_j$ and 0.

The difference between MR_j and $\overline{MR}_j$, which means that affiliation reinforces our result that auctions beat negotiations, is exactly the difference that implies that with three or more bidders an open ascending English auction is more profitable than a sealed-bid second-price auction (see Milgrom and Weber, 1982).[24]

B. Auctions versus Optimal Mechanisms

Lemma 3 extends to general value functions Myerson's (1981) theorem, that with independent signals and risk-neutral bidders, any two mechanisms that always result in the same winning bidder are revenue equivalent. (Myerson considers only common values in which players' values are additive functions of signals.) We also reinterpret Myerson's "virtual utilities" as marginal revenues.[25]

LEMMA 3: *With independent signals and N risk-neutral bidders, the expected revenue from any sales mechanism equals the expectation of the marginal revenue of the winning bidder, provided any bidder with the lowest-possible signal expects zero surplus; the marginal revenue of the winning bidder is $\overline{MR}_j(\boldsymbol{T})$ if j is the winner and is taken to be zero if the good is retained by the seller.*

Clearly no sales procedure with voluntary participation can earn greater profits than one in which bidders with the lowest possible signals expect zero surplus.[26] A corollary of Lemma 3, therefore, is that the mechanism of Lemma 2—which always sells to the bidder j for whom $\overline{MR}_j(\boldsymbol{T})$ is largest if that value is greater than zero and makes no sale otherwise—is optimal with risk-neutral bidders and independent signals, under our assumptions (A.1)–(A.3):

THEOREM 2: *With independent signals and N risk-neutral bidders, an optimal mechanism for a risk-neutral seller is an English auction followed by an optimally-chosen take-it-or-leave-it offer to the last remaining bidder.*

Theorems 1 and 2 together imply the main point of our paper:

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[24] In the sealed-bid auction with three bidders the bidder with the second-highest signal, who determines the price, bids his expected value assuming that he is tied with the highest signal, and estimates the distribution of the third signal based on this assumption. This bid equals the lowest-possible expected value of the winner, say j, that is, equals the expectation (over t_j) of $\overline{MR}_j$. In an open auction the second-highest bidder chooses his dropout price by assuming that he is tied with the highest signal and based on the actual third signal which he infers assuming equilibrium behaviour—see note 19. His final bid therefore equals the lowest-possible actual value of the winner, that is, the expectation (over t_j) of MR_j, and affiliation implies the expectation of this bid exceeds the sealed bid. In our context, the expectation (over t_j) of $\overline{MR}_j$ equals the lowest expected value j could have, and if j has the lowest-possible signal he will estimate the distribution of t_{N+1} based on this. However the expectation (over t_j and t_{N+1}) of MR_j equals the expectation (over t_{N+1}) of the lowest value j could have given the actual t_{N+1}. Affiliation implies the distribution of the actual t_{N+1} stochastically dominates the distribution of t_{N+1} contingent on j having the lowest possible signal, so the expected MR_j exceeds the expected $\overline{MR}_j$.

[25] Special cases of Lemma 3 and Theorem 2 have been independently obtained by Fernando Branco (1994) and Lopomo (1995).

[26] No sales procedure can give any type of any bidder a negative surplus, and giving the lowest type a positive surplus would require raising all other types' surpluses. See the proof of Lemma 3.

COROLLARY: *With independent signals and risk-neutral bidders, an absolute English auction with $N + 1$ bidders is more profitable in expectation than any negotiation with N bidders.*

Of course, to the extent that it is unrealistic to expect a seller to be able to commit as firmly as is needed for the optimal mechanism, and to compute the optimal reserve price, the Corollary's statement about the auction's superiority is conservative.

When buyers' signals are nonindependent, the mechanism described in Theorem 2 is not optimal, and a seller who can choose any mechanism can generally extract all bidders' surplus (see Jacques Cremer and Richard McLean, 1985; McAfee et al., 1989; and McAfee and Philip J. Reny, 1992). When a seller can extract all surplus from N bidders, it is not hard to show that this will always dominate an absolute auction with $N + 1$ bidders. However, Lopomo (1995) has shown that with affiliated signals and risk-neutral bidders expected revenue from the mechanism of Theorem 2 is higher than from any other mechanism in which (i) losers do not pay and (ii) in equilibrium the winner, if anyone, is the bidder with the highest signal and his payments are weakly increasing in his own signal for any realization of other bidders' signals.[27] Thus it remains true that an absolute auction with $N + 1$ bidders is better than any standard mechanism for selling to N bidders.

C. Negotiations Followed by an Auction

A final question is: if a seller has $N + 1$ risk-neutral bidders with independent signals, can she benefit by first negotiating with N of the bidders only, reserving the right to hold an absolute auction among all $N + 1$ bidders if the negotiations failed to produce a sale? The answer is no.

The reason is that if the seller has the option of resorting to the auction, it will be obvious to all that if negotiations fail, there will be an auction. Viewing the two-stage process as a whole, then, the seller is constrained to choose among mechanisms that always lead to a sale. But clearly any optimal mechanism that always sells must always sell to the buyer with the highest signal. Therefore, it will not be optimal to sell in the negotiation stage unless it is certain that the buyer's signal is greater than or equal to the signal of the $(N + 1)$th bidder. Therefore the seller should insist on a price in the negotiation phase that will only be accepted when a buyer gets the maximum signal of 1, which occurs with probability zero.

Therefore, under our assumptions, the seller should not accept any high "lock-up" bid that a buyer may be willing to offer in return for not holding an auction with an additional buyer.[28]

D. Multiple Units

Our model extends easily to a seller with K goods to sell and $N \geq K$ symmetric bidders each interested in buying one unit. With independent signals, the optimal sales mechanism is to sell to the K bidders with the highest signals, provided $\overline{MR}_i(\overline{T}) \geq 0$ for K or more bidders. Otherwise, sell only to those bidders for whom $\overline{MR}_i(\overline{T}) \geq 0$.[29] The optimal mechanism yields expected revenue equal to the expected sum of the K highest values among $\overline{MR}_1(\overline{T}), \dots, \overline{MR}_N(\overline{T})$ and K zeros. It is not

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[27] So, for example, the mechanism of Theorem 2 is better than any of the English, Dutch, first-price sealed bid or second-price sealed bid auctions together with a reserve price. See Vijay Krishna and John Morgan (1994) for an analysis of auctions in which all bidders pay.

[28] This result would be unaffected by other bidders having costs of entering the auction. However, the presence of such costs can explain why bidders may jump bid to deter competitors from entering; see Michael J. Fishman (1988) and Hirshleifer and Ivan P. L. Png (1989). See also Christopher Avery (1993), Kent Daniel and Hirshleifer (1993), and Nils Henrik von der Fehr (1994) for related discussion.

[29] Optimal negotiation is in general more complex than in the single-unit case since determining any bidder's MR requires knowing all other bidders' signals. One way to achieve optimal negotiation is (i) ask each bidder i to independently report his signal t_i (in equilibrium all reports will be honest), and let $\hat{t}$ be the $(K + 1)$st highest signal reported; (ii) for each i who reports a signal in the top K signals, compute $\tilde{t}_i$ such that $\overline{MR}_i(t_1, \dots, t_{i-1}, \tilde{t}_i, t_{i+1}, \dots, t_N) = 0$ and sell to this bidder if $t_i \geq \tilde{t}_i$, at the maximum of $\overline{v}_i(t_1, \dots, t_{i-1}, \hat{t}, t_{i+1}, \dots, t_N)$ and $\overline{v}_i(t_1, \dots, t_{i-1}, \tilde{t}_i, t_{i+1}, \dots, t_N)$. See our paper, Bulow and Klemperer (1994b), for a further analysis of multiple-unit auctions.
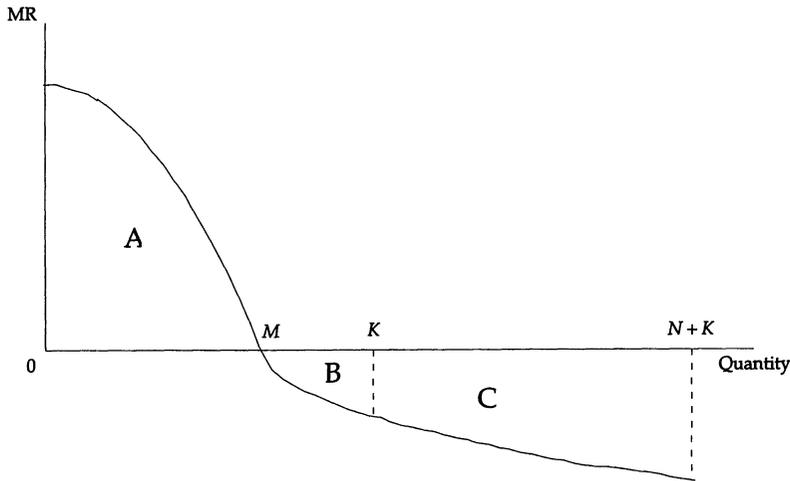
FIGURE 3. MARGINAL REVENUE IF DEMAND IS EXPANDED

hard to extend our earlier arguments to show that this is less than the expected revenue from an absolute auction with $N + K$ bidders with independent signals (in which the final K bidders pay the price at which the last excess bidder quits).

The analogue of this argument in traditional price theory is informative. Consider an industry with total capacity K at some constant marginal cost c which we normalize to zero. Demand at a price of zero is $N \geq K$. The industry has the ability to do one of two things: invest in a monitoring program which will enable it to collude perfectly, or invest in an advertising campaign which will proportionally increase demand by a factor of $(N + K)/N$. In the latter case, the industry will be perfectly competitive and will sell K units. Which option is more profitable?

Assuming that the industry has a downward sloping MR curve, the answer is that increasing demand and remaining competitive is more valuable than colluding. The result follows directly from our auction-theory model, with K units and independent private values. However, if N and K large enough that there is no aggregate uncertainty about valuations, the argument can be made even more simply.

Figure 3 shows marginal revenue for the proportionately-expanded demand curve. Collusive profits on this demand curve equal area

[A] (the integral of marginal revenue up to the monopoly quantity M), so collusion on the original demand curve would yield profits of $(N/(N + K))[A]$. Competitive profits on the expanded demand curve are the integral of marginal revenues up to K, that is, [A] − [B]. However, downward-sloping MR implies $[B] \leq ((K − M)/((N + K) − M))([B] + [C])$, and total marginal revenue equals zero at price zero so $([B] + [C]) = [A]$. So competitive profits equal $[A] − [B] \geq [A] − ((K − M)/((N + K) − M))[A] = (N/(N + K − M))[A]$, which exceeds collusive profits.

III. Conclusion

A simple competitive auction with $N + 1$ bidders will yield a seller more expected revenue than she could expect to earn by fully exploiting her monopoly selling position against N bidders.

When a company is approached by a potential buyer or buyers, its options may be either to negotiate or to put the company up for auction. Our analysis implies that if the board expects at least one extra serious bidder to appear in an auction, then it should generally not negotiate and should directly begin an auction.

Of course, institutional considerations may make any given situation more complex. For example, if allowing many bidders access to

confidential financial information would cause the company's value to be diminished to the eventual buyer, then one might wish to restrict bidding.

But remember that our analysis assumed that a seller could negotiate optimally, making credible commitments of the sort that might not be possible in real life, and we also as-

sumed that bidders had no bargaining power in a negotiation. We therefore believe that our basic result does not overstate the efficacy of auctions relative to negotiations. Certainly a firm that refused to negotiate with a potential buyer, and instead put itself up for auction, should be presumed to have exercised reasonable business judgment.

APPENDIX

Write $(x, \mathbf{T}_{-j})$ for $(t_1, \ldots, t_{j-1}, x, t_{j+1}, \ldots, t_{N+1})$, that is, for the vector $\mathbf{T}$ but with the jth element replaced by x, and write $(x, \overline{\mathbf{T}}_{-j})$ for the vector $\overline{\mathbf{T}}$ with the jth element replaced by x.

PROOF OF LEMMA 1:
If bidder j has the highest signal and bidder i has the second-highest signal, then bidder j will win the auction at the price $v_i(t_i, \mathbf{T}_{-j})$, that is, the value i would have, if j's signal were t_i.[30] But by symmetry, $v_i(t_i, \mathbf{T}_{-j})$ equals $v_j(t_i, \mathbf{T}_{-j})$, and

$$v_j(t_i, \mathbf{T}_{-j}) = \frac{1}{1 - F(t_j|\mathbf{T}_{-j})|_{t_j=t_i}} \int_{t_i}^{1} MR_j(\mathbf{T}) \cdot f(t_j|\mathbf{T}_{-j}) dt_j = E_{t_j}\{MR_j(\mathbf{T})|t_j \geq t_i, \mathbf{T}_{-j}\}$$

which is to say that the sales price equals the expected MR of the winning bidder, contingent on all the other signals. Because the winning bidder has the highest MR, the result follows.

PROOF OF LEMMA 2:
As in the absolute auction, the next-to-last bidder i leaves at price $\overline{v}_i(t_i, \overline{\mathbf{T}}_{-j})$ equals $\overline{v}_j(t_i, \overline{\mathbf{T}}_{-j})$. Let the seller choose a take-it-or-leave-it offer for the last bidder, j, of $\overline{v}_j(\hat{t}, \overline{\mathbf{T}}_{-j})$, where $\hat{t} \geq t_i$. (The seller infers $\overline{\mathbf{T}}_{-j}$ from the points where the low bidders quit.) If ex post $t_j \geq \hat{t}$ then the seller will receive

$$\overline{v}_j(\hat{t}, \overline{\mathbf{T}}_{-j}) = \frac{1}{1 - F(t_j|\overline{\mathbf{T}}_{-j})|_{t_j=\hat{t}}} \int_{\hat{t}}^{1} \overline{MR}_j(\overline{\mathbf{T}}) \cdot f(t_j|\overline{\mathbf{T}}_{-j}) dt_j = E_{t_j}\{\overline{MR}_j(\overline{\mathbf{T}})|t_j \geq \hat{t}, \overline{\mathbf{T}}_{-j}\}.$$

If $t_j < \hat{t}$, then the seller will receive zero. That is, revenue equals, in expectation, $\overline{MR}_j(\overline{\mathbf{T}})$ when $t_j \geq \hat{t}$, and zero when $t_j < \hat{t}$. Since $\overline{MR}_j(\overline{\mathbf{T}})$ is increasing in t_j, the seller maximizes expected profit by choosing $\hat{t}$ so that $\overline{MR}_j(\hat{t}, \overline{\mathbf{T}}_{-j}) = 0$ if $\overline{MR}(t_i, \overline{\mathbf{T}}_{-j}) < 0$, and chooses $\hat{t} = t_i$ otherwise. Since the winning bidder has the highest $\overline{MR}$, the result follows.

PROOF OF THEOREM 1:
Conditional on any $\overline{\mathbf{T}}_{-j}$, and on the jth signal being the highest of the first N signals, let $\hat{t}$ be such that $\overline{v}_j(\hat{t}, \overline{\mathbf{T}}_{-j})$ is the seller's optimal take-it-or-leave-it final offer (computed as in the proof of Lemma 2) when selling to the N bidders.
  If $t_j < \hat{t}$,

$$\max(\overline{MR}_1(\overline{\mathbf{T}}), \ldots, \overline{MR}_N(\overline{\mathbf{T}}), 0) = 0 \leq v_{N+1}(0, \overline{\mathbf{T}}) = \int_{0}^{1} MR_{N+1}(\mathbf{T}) f(t_{N+1}|\overline{\mathbf{T}}) dt_{N+1} = E_{t_{N+1}}\{MR_{N+1}(\mathbf{T})\}.$$

If $t_j \geq \hat{t}$, $\max(\overline{MR}_1(\overline{\mathbf{T}}), \ldots, \overline{MR}_N(\overline{\mathbf{T}}), 0) = \overline{MR}_j(\overline{\mathbf{T}})$, so conditional on $t_j \geq \hat{t}$ and $\overline{\mathbf{T}}_{-j}$, the expectation of $\max(\overline{MR}_1(\overline{\mathbf{T}}), \ldots, \overline{MR}_N(\overline{\mathbf{T}}), 0)$

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[30] See note 19 for a full description of the equilibrium-bidding strategies.

$$= \frac{1}{1 - F(t_j | \overline{\mathbf{T}}_{-j})|_{t_j = \hat{t}}} \int_{\hat{t}}^{1} \overline{\mathrm{MR}}_j(\overline{\mathbf{T}}) f(t_j | \overline{\mathbf{T}}_{-j}) dt_j$$

$$= \overline{v}_j(\hat{t}, \overline{\mathbf{T}}_{-j}).$$

However, the expectation of $\mathrm{MR}_j(\mathbf{T})$ conditional on $t_j \geq \hat{t}$ and $\overline{\mathbf{T}}_{-j}$

$$= \int_{t_{N+1}=0}^{1} \int_{t_j=0}^{1} \mathrm{MR}_j(\mathbf{T}) f(t_j, t_{N+1} | t_j \geq \hat{t}, \overline{\mathbf{T}}_{-j}) dt_j dt_{N+1}$$

$$= \int_{t_{N+1}=0}^{1} \left(\int_{t_j=0}^{1} \mathrm{MR}_j(\mathbf{T}) f(t_j | t_j \geq \hat{t}, \overline{\mathbf{T}}_{-j}, t_{N+1}) \right) f(t_{N+1} | t_j \geq \hat{t}, \overline{\mathbf{T}}_{-j}) dt_j dt_{N+1}$$

$$= \int_{t_{N+1}=0}^{1} \left(\int_{t_j=\hat{t}}^{1} \mathrm{MR}_j(\mathbf{T}) \frac{f(t_j | \mathbf{T}_{-j})}{1 - F(t_j | \mathbf{T}_{-j})|_{t_j=\hat{t}}} dt_j \right) f(t_{N+1} | t_j \geq \hat{t}, \overline{\mathbf{T}}_{-j}) dt_{N+1}$$

$$= \int_{t_{N+1}=0}^{1} v_j(\hat{t}, \mathbf{T}_{-j}) f(t_{N+1} | t_j \geq \hat{t}, \overline{\mathbf{T}}_{-j}) dt_{N+1}$$

$$\geq \int_{t_{N+1}=0}^{1} v_j(\hat{t}, \mathbf{T}_{-j}) f(t_{N+1} | t_j = \hat{t}, \overline{\mathbf{T}}_{-j}) dt_{N+1} = \overline{v}_j(\hat{t}, \overline{\mathbf{T}}_{-j}).$$

The inequality applies if signals are affiliated, because then the distribution of t_{N+1} conditional on $t_j \geq \hat{t}$ and $\overline{\mathbf{T}}_{-j}$ stochastically dominates the distribution of t_{N+1} conditional on $t_j = \hat{t}$ and $\overline{\mathbf{T}}_{-j}$. (With independent signals the inequality holds with equality.) With private values, $v_j(\hat{t}, \mathbf{T}_{-j})$ is independent of t_{N+1}, so the inequality always holds with equality.

So conditional on any lowest $N - 1$ of the first N signals, $\overline{\mathbf{T}}_{-j}$, and either on any $t_j < \hat{t}$ or on $t_j \geq \hat{t}$, the expectation of $\max(\overline{\mathrm{MR}}_1(\overline{\mathbf{T}}), \dots, \overline{\mathrm{MR}}_N(\overline{\mathbf{T}}), 0)$ is (weakly) less than the expectation of $\max(\mathrm{MR}_1(\mathbf{T}), \dots, \mathrm{MR}_{N+1}(\mathbf{T}))$. Since the inequalities are strict for a set of $\overline{\mathbf{T}}_{-j}$ that occurs with positive probability,

$$E_{\overline{\mathbf{T}}} \{ \max(\overline{\mathrm{MR}}_1(\overline{\mathbf{T}}), \overline{\mathrm{MR}}_2(\overline{\mathbf{T}}), \dots, \overline{\mathrm{MR}}_N(\overline{\mathbf{T}}), 0) \} < E_{\mathbf{T}} \{ \max(\mathrm{MR}_1(\mathbf{T}), \mathrm{MR}_2(\mathbf{T}), \dots, \mathrm{MR}_{N+1}(\mathbf{T})) \}$$

and the result follows by Lemmas 1 and 2.

The above proof assumed risk-neutral bidders. If bidders are risk averse the expected revenue from the absolute auction is unchanged, but the expected revenue from the N-bidder mechanism is reduced, increasing the advantage of the absolute auction.

PROOF OF LEMMA 3:
Let $p_i(\overline{\mathbf{T}})$ be the probability that i will receive the object, in equilibrium, let $S_i(t_i)$ be the equilibrium expected surplus to bidder i, and since we have independent signals, write $f(t_i)$ and $F(t_i)$ for $f(t_i | \overline{T}_{-i})$ and $F(t_i | \overline{T}_{-i})$. For $p_i(\cdot)$ to be an equilibrium, it must be incentive compatible. In particular, the ith bidder, with signal t_i', cannot gain by deviating to the strategy he would use if he had signal t_i so, with independent signals,

$$S_i(t_i') \geq S_i(t_i) + E_{\overline{\mathbf{T}}_{-i}} \{ (\overline{v}_i(t_i', \overline{\mathbf{T}}_{-i}) - \overline{v}_i(t_i, \overline{\mathbf{T}}_{-i})) p_i(\overline{\mathbf{T}}) \}.$$

So $S_i(t_i)$ has derivative $dS_i(t_i)/dt_i = E_{\overline{\mathbf{T}}_{-i}} \{ (\partial \overline{v}_i(\overline{\mathbf{T}})/\partial t_i) p_i(\overline{\mathbf{T}}) \} \equiv H_i(t_i)$ and $S_i(t_i) = S_i(0) + \int_0^{t_i} H_i(t) dt$. So $E_{t_i} \{ S_i(t_i) \} = S_i(0) + \int_0^1 \int_0^{t_i} H_i(t) dt f(t_i) dt_i = S_i(0) + \int_0^1 (1 - F(t_i)) H_i(t_i) dt_i$ (integrating by parts) $= S_i(0) + E_{t_i} \{ ((1 - F(t_i))/f(t_i)) H_i(t_i) \}$.

But expected seller profits can be written as the expected value of the good to the winning bidder, $E_{\overline{\mathbf{T}}} \{ \sum_{i=1}^{N} (\overline{v}_i(\overline{\mathbf{T}}) p_i(\overline{\mathbf{T}})) \}$, less the expected surplus of the N bidders, $\sum_{i=1}^{N} E_{t_i} \{ S_i(t_i) \}$. So expected profits are

$$E_{\overline{\mathbf{T}}} \left\{ \sum_{i=1}^{N} \left[\left(\overline{v}_i(\overline{\mathbf{T}}) - \frac{1 - F(t_i)}{f(t_i)} \frac{\partial \overline{v}_i(\overline{\mathbf{T}})}{\partial t_i} \right) p_i(\overline{\mathbf{T}}) - S_i(0) \right] \right\} = E_{\overline{\mathbf{T}}} \left\{ \sum_{i=1}^{N} (\overline{\mathrm{MR}}_i(\overline{\mathbf{T}}) p_i(\overline{\mathbf{T}}) - S_i(0)) \right\}.$$

REFERENCES

Avery, Christopher. "Strategic Jump Bidding and English Auctions." Discussion paper, Kennedy School of Government, Harvard University, 1993.

Bikhchandani, Sushil and Riley, John G. "Equilibria in Open Auctions." Unpublished manuscript, University of California, Los Angeles, September 1993.

Branco, Fernando. "Common Value Auctions with Independent Types." Discussion paper, Universidade Catolica Portuguesa and Banco de Portugal, July 1994.

Bulow, Jeremy I. and Klemperer, Paul D. "Auctions vs. Negotiations." National Bureau of Economic Research (Cambridge, MA) Working Paper No. 4608, January 1994a.

_____. "Rational Frenzies and Crashes." *Journal of Political Economy*, February 1994b, *102*(1), pp. 1–23.

Bulow, Jeremy I. and Roberts, D. John. "The Simple Economics of Optimal Auctions." *Journal of Political Economy*, October 1989, *97*(5), pp. 1060–90.

Choper, Jesse H.; Coffee, John C., Jr. and Morris, C. Robert, Jr. *Cases and Materials on Corporations*, 3rd Ed. Boston: Little, Brown, 1989.

Cramton, Peter C. "Sequential Bargaining Mechanisms," in Alvin Roth, ed., *Game theoretic models of bargaining*. New York: Cambridge University Press, 1985, pp. 149–80.

Cremer, Jacques and McLean, Richard. "Optimal Selling Strategies under Uncertainty for a Discriminatory Monopolist when Demands Are Interdependent." *Econometrica*, March 1985, *53*(2), pp. 345–61.

Daniel, Kent and Hirshleifer, David. "A Theory of Costly Sequential Bidding." Manuscript, Graduate School of Business, University of Chicago, 1993.

Engelbrecht-Wiggans, Richard. "Optimal Auctions Revisited." *Games and Economic Behavior*, April 1993, *5*(2), pp. 227–39.

Fishman, Michael J. "A Theory of Pre-emptive Takeover Bidding." *Rand Journal of Economics*, Spring 1988, *19*(1), pp. 88–101.

Grossman, Sanford J. and Hart, Oliver D. "Takeover Bids, the Free-Rider Problem, and the Theory of the Corporation." *Bell Journal of Economics*, Spring 1980, *11*(1), pp. 42–64.

Harris, Milton and Raviv, Artur. "Corporate Control Contests and Capital Structure." *Journal of Financial Economics*, 1988, *20*(1/2), pp. 55–86.

Hirshleifer, David and Png, Ivan P. L. "Facilitation of Competing Bids and the Price of a Takeover Target." *Review of Financial Studies*, 1989, *2*(4), pp. 587–606.

Hirshleifer, David and Titman, Sheridan. "Share Tendering Strategies and the Success of Hostile Takeover Bids." *Journal of Political Economy*, April 1990, *98*(2), pp. 295–324.

Holmstrom, Bengt and Nalebuff, Barry. "To the Raider Goes the Surplus? A Reexamination of the Free-Rider Problem." *Journal of Economics and Management Strategy*, November 1992, *1*(1), pp. 37–62.

Krishna, Vijay and Morgan, John. "An Analysis of the War of Attrition and the All-Pay Auction." Discussion Paper, Pennsylvania State University, September 1994.

Lopomo, Giuseppe. "Optimality and Robustness of the English Auction." New York University, Stern School of Business Working Paper No. EC-95-03, January 1995.

McAfee, R. Preston and McMillan, John. "Auctions with Entry." *Economics Letters*, 1987a, *23*(4), pp. 343–47.

_____. "Auctions and Bidding." *Journal of Economic Literature*, June 1987b, *25*(2), pp. 699–738.

McAfee, R. Preston; McMillan, John and Reny, Philip J. "Extracting the Surplus in the Common-Value Auction." *Econometrica*, November 1989, *57*(6), pp. 1451–59.

McAfee, R. Preston and Reny, Philip J. "Correlated Information and Mechanism Design." *Econometrica*, March 1992, *60*(2), pp. 395–421.

McAfee, R. Preston and Vincent, Daniel. "Updating the Reserve Price in Common Value Auctions." *American Economic Review*, May 1992, *82*(2), pp. 512–18.

Milgrom, Paul. "Auction Theory," in Truman F. Bewley, ed., *Advances in economic theory*. New York: Cambridge University Press, 1987.

Milgrom, Paul and Weber, Robert J. "A Theory of Auctions and Competitive Bidding."

Econometrica, September 1982, *50*(5), pp. 1089–122.

Myerson, Roger B. "Optimal Auction Design." *Mathematics of Operations Research*, February 1981, *6*(1), pp. 58–73.

Riley, John G. and Samuelson, William F. "Optimal Auctions." *American Economic Review*, June 1981, *71*(3), pp. 381–92.

Shleifer, Andrei and Vishny, Robert W. "Greenmail, White Knights, and Shareholders' Interest." *Rand Journal of Economics*, Autumn 1986, *17*(3), pp. 293–309.

———. "Value Maximization and the Acquisition Process." *Journal of Economic Perspectives*, Winter 1988, *2*(1), pp. 7–20.

von der Fehr, Nils Henrik. "Predatory Bidding in Sequential Auctions." *Oxford Economic Papers*, 1994, *46,* pp. 345–56.

Wang, Ruqu. "Auctions versus Posted-Price Selling." *American Economic Review*, September 1993, *83*(4), pp. 838–51.