Signal presentation

- Time domain
  - Complex to solve

- Frequency domain

See the Fourier transform handout
Complex Wave Patterns

• Signal waves occupying the same space combine to form a new wave of a different shape.
• Harmonically related waves add together and can create any complex wave pattern.
• Harmonically related waves have frequencies that are multiples of a basic frequency.

Note: All frequency waves do not have to start at zero, they can be “out of phase”. The amount of shift in degrees is called their phase angle.
Generation of digital signal

Analog signal → Sampling → Quantizing → Encoding → 11...1100 (Digital data)

Quantized signal
Sampling

• Analog signal is sampled every $T_s$ seconds (sampling interval)
• $f_s = 1/T_s$ is called the sampling rate (sampling frequency).
• Discretization in time-domain
According to the Nyquist theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.
Quantization and encoding of a sampled signal

### Quantization codes

<table>
<thead>
<tr>
<th>Quantization codes</th>
<th>Normalized amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>4D</td>
</tr>
<tr>
<td>6</td>
<td>3D</td>
</tr>
<tr>
<td>5</td>
<td>2D</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-D</td>
</tr>
<tr>
<td>1</td>
<td>-2D</td>
</tr>
<tr>
<td>0</td>
<td>-3D</td>
</tr>
<tr>
<td></td>
<td>-4D</td>
</tr>
</tbody>
</table>

### Normalized quantized values

<table>
<thead>
<tr>
<th>Time</th>
<th>-1.22</th>
<th>1.50</th>
<th>3.24</th>
<th>3.94</th>
<th>2.20</th>
<th>-1.10</th>
<th>-2.26</th>
<th>-1.88</th>
<th>-1.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized quantized values</td>
<td>-1.50</td>
<td>1.50</td>
<td>3.50</td>
<td>3.50</td>
<td>2.50</td>
<td>-1.50</td>
<td>-2.50</td>
<td>-1.50</td>
<td>-1.50</td>
</tr>
</tbody>
</table>

### Normalized error

<table>
<thead>
<tr>
<th>Time</th>
<th>-1.22</th>
<th>1.50</th>
<th>3.24</th>
<th>3.94</th>
<th>2.20</th>
<th>-1.10</th>
<th>-2.26</th>
<th>-1.88</th>
<th>-1.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized error</td>
<td>-0.38</td>
<td>0</td>
<td>+0.26</td>
<td>-0.44</td>
<td>+0.30</td>
<td>-0.40</td>
<td>-0.24</td>
<td>+0.38</td>
<td>-0.30</td>
</tr>
</tbody>
</table>

### Quantization code

<table>
<thead>
<tr>
<th>Time</th>
<th>2</th>
<th>5</th>
<th>7</th>
<th>7</th>
<th>6</th>
<th>2</th>
<th>1</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
</table>

### Encoded words

<table>
<thead>
<tr>
<th>Time</th>
<th>010</th>
<th>101</th>
<th>111</th>
<th>111</th>
<th>110</th>
<th>010</th>
<th>001</th>
<th>010</th>
<th>010</th>
</tr>
</thead>
</table>
Speech Signal Sampling

- The analog speech signal captures pressure variations in air that are produced by the speaker
  - The same function as the ear
- The analog speech input signal from the microphone is \textit{sampled} periodically at some fixed \textit{sampling rate}

![Diagram of analog speech signal with sampling points]

Voltage

Analog speech signal

Sampling points

Time
• What remains after sampling is the value of the analog signal at *discrete time points*
• This is the *discrete-time signal*
Speech Signal Sampling

• The analog speech signal has many *frequencies*
  – The human ear can perceive frequencies in the range 50 Hz-15 kHz (more if you’re young)

• The information about what was spoken is carried in all these frequencies

• But most of it is in the range 150Hz-5kHz
Frequency-intensity range of speech and music within the audibility of the human auditory system. Adapted from Limb (2011).
Ideally, one would sample the speech signal at a sufficiently high rate to retain all perceivable components in the signal
  – > 30kHz

For practical reasons, lower sampling rates are often used, however
  – Save bandwidth / storage
  – Speed up computation
• Audio hardware typically supports several standard rates
  – *E.g.* 8, 16, 11.025, or 44.1 KHz
  – CD recording employs 44.1 KHz per channel

• 8KHz sampling rate for telephone speech and 16KHz for wideband speech
  – Telephone signal is *narrowband*
  – Good microphones provide a *wideband* speech signal
    • 16KHz sampling can represent audio frequencies up to 8 KHz
Digitization

- Each sampled value is *digitized* (*quantized* and *encoded*) into one of a set of fixed discrete levels
  - Each analog voltage value is *mapped* to the nearest discrete level
  - Since there are a fixed number of discrete levels, the mapped values can be represented by a number; *e.g.* 8-bit, 12-bit or 16-bit

- Digitization can be *linear* (uniform) or *non-linear* (non-uniform)
Linear Coding

• Linear coding (aka *pulse-code modulation* or PCM) splits the input analog range into some number of uniformly spaced levels
• The number of discrete levels determines number of bits needed to represent a quantized signal value; *e.g.*:
  – 4096 levels need a 12-bit representation
  – 65536 levels require 16-bit representation
Linear Coding

- Example PCM quantization: 16 and 64 levels
- Since an entire analog range is mapped to a single value, quantization leads to *quantization error*
  - Average error can be reduced by increasing the number of discrete levels
Non-Linear Coding

Converts non-uniform segments of the analog axis to uniform segments of the quantized axis

- Spacing between adjacent segments on the analog axis is chosen based on the relative frequencies of sample values in that region
- Sample regions of high frequency are more finely quantized
Signal Quality

• The quality of the final digitized signal depends critically on all the other components:
  – The microphone quality
  – Environmental quality – the microphone picks up not just the subject’s speech, but all other ambient noise
  – The electronics performing sampling and digitization
    • Poor quality electronics can severely degrade signal quality
  – Proper setting of the recording level
    • Too low a level underutilizes the available signal range, increasing susceptibility to noise
    • Too high a level can cause clipping
    • Clipping example
• Figures below show energy at various frequencies in a signal as a function of time
  – Called a spectrogram

• Different instances of a sound will have the same generic spectral structure
Systems

- Process inputs and produce outputs.
- For the most part, we will take an input-output perspective.
Examples

- An RLC circuit
- An edge detection algorithm for medical images.
- A DSP system in a cell phone
System

Interconnections

• Doing stuff to inputs to produce outputs.
  – Transformations, shifting, scaling, inverting
Noise

• Any signal other than that of interest is interference, artefact, or noise.
• Sources of noise depend on the signal type, and could be numerous, e.g. instrumentation, or the environment of the experiment.
• Interference that arises from a random process such as thermal noise in electronic devices is random noise.
• A random process is characterized by the probability density function (PDF) representing the probabilities of occurrence of all possible values of a random variable.
Random noise

- Random process $\xi$ characterized by PDF $p(\xi)$.
- Mean $\mu_\xi$: first-order moment of the PDF
  \[ \mu_\xi = E[\xi] = \int_{-\infty}^{\infty} \xi p(\xi) d\xi, \]
  where $E[\ ]$ represents the statistical expectation operator.
- Common to assume mean of a random noise process $= 0$. 
Statistical measures

- Mean-squared (MS) value, i.e. second-order moment,

\[ E[\xi^2] = \int_{-\infty}^{\infty} \xi^2 p(\xi) d\xi \]

- Variance, second central moment

\[ \sigma^2 = E[(\xi - \mu_\xi)^2] = \int_{-\infty}^{\infty} [(\xi - \mu_\xi)^2] p(\xi) d\xi \]

- Square root of variance is standard deviation (SD), \( \sigma_\xi \)

\[ \sigma^2 = E[\xi^2] - (\mu_\xi)^2 \]

- When is MS = variance?

13-Sep-18
• When the values of a random process form a time series or a function of time, we have a random signal or a stochastic process $\xi(t)$.

• Then, the statistical measures have physical meanings:
  – mean = DC component;
  – $MS$ = average power;
  – square root of $MS$ = root mean-squared or RMS value = average noise magnitude.

• See handouts for more information
• When a signal $x(t)$ is observed with random noise, the measured signal $y(t)$ may be treated as a realization of another random process $y$.

• In most cases the noise is additive:

$$y(t) = x(t) + \xi(t)$$
Filtering

- A filter is a signal processing system, algorithm, or method, in hardware or software, used to modify a signal.

- A signal may be filtered to remove undesired components, noise, or artefacts, and to enhance desired components.

- Filters may be categorized as
  - linear or nonlinear,
  - stationary or nonstationary,
  - Fixed (time-invariant) or adaptive (time-variant),
  - active or passive,
  - statistical or deterministic.
Filtering

• A fundamental characteristic of a linear time-invariant (LTI) or shift-invariant (LSI) filter is its **impulse response**: 
  – output of the system when the input is a Dirac delta or impulse function.

• How does Dirac function look like?
Delta function - sifting

- The delta function is also defined in the following terms

\[
\int_{T_1}^{T_2} x(t) \delta(t - t_0) \, dt = \begin{cases} 
  x(t_0) & \text{if } T_1 < t_0 < T_2, \\
  0 & \text{otherwise},
\end{cases}
\]

where \( x(t) \) is a function that is continuous at \( t_0 \).

- This is known as the sifting property of the delta function, because the value of the function \( x(t) \) at \( t_0 \) of the delta function is sifted or selected from all of its values.
• Consider a continuous-time signal, \( x(t) \), processed by an LSI system.

• An LSI system is completely characterized or specified by its impulse response, \( h(t) \), which is the output of the system when the input is a delta function.

• The output of the system, \( y(t) \), is given by the convolution of the input, \( x(t) \), with the impulse response, \( h(t) \):

\[
y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) \, d\tau.
\]

• This is a linear function.
How about discrete time?

- Principle is the same, but we handle the samples

- Two important points:
  - $h(-k)$ represents a reversal in time of $h(k)$;
  - $h(n - k)$ represents a shift of the reversed signal $h(-k)$ by $n$ samples.

- Multiplication of $h(n-k)$ by $x(k)$ can be viewed as scaling.

- The summation represents accumulation of the results or integration of $x(k) h(n - k)$ over the interval from $k = 0$, the origin of time, to $n$, the present instant of time

\[ y(n) = \sum_{k=0}^{n} x(k) h(n - k). \]
Example

n  0  1  2  3  4  5  6  7
x(n)  4  5  3  1
x(n-1)  4  5  3  1
x(n-2)  4  5  3  1

x – N₁, h – N₂ samples

How many samples in a result?
• The output ceases to exist (or has only zero values) when the shift exceeds a certain amount such that \( x(k) \) and \( h(n - k) \) do not overlap in time any more.

• Linear convolution of two discrete-time signals with \( N_1 \) and \( N_2 \) samples leads to a result with \( N_1 + N_2 - 1 \) samples.
Example
• The noisy signal was filtered by computing the mean of each sample and the preceding 10 samples:

\[ y(n) = \frac{1}{11} \sum_{k=0}^{10} x(n - k) \]

where \( n = 10, 11, \ldots, N-1, N \) – number of samples.
• This is a moving average (MA) filter,
• The average values of the input signal are computed in a “moving” temporal window and used to define the output signal.
DSP Applications

communication systems
modulation/demodulation, channel equalization, echo cancellation
consumer electronics
perceptual coding of audio and video on DVDs, speech synthesis, speech recognition
Music
synthetic instruments, audio effects, noise reduction
medical diagnostics
Magnetic-resonance and ultrasonic imaging, computer tomography, ECG, EEG, MEG, AED, audiology
Geophysics
seismology, oil exploration

astronomy
VLBI, speckle interferometry
experimental physics
sensor data evaluation
aviation
radar, radio navigation
security
steganography, digital watermarking, biometric identification, visual surveillance systems, signal intelligence, electronic warfare
engineering
control systems, feature extraction for pattern recognition