Signal Processing Basics

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Illustration of various stages of biomedical signal processing and analysis
Signal

• Conveys information about the behaviour or attributes of some phenomenon measured quantity that varies with time (or position)

• Any quantity exhibiting variation in time or variation in space (e.g. image) is a signal that
  – may provide information about a system, or
  – convey a message between observers

• IEEE Transactions on Signal Processing: the term "signal" includes speech, audio, image, video, communication, geophysical, sonar, radar and medical signals
Some examples

• Most physiological processes are accompanied by signals of several types that reflect their nature and activities:
  – electrical, in the form of potential or current, and
  – physical, in the form of pressure or temperature

• Continuous signal:
  – With respect to value
  – With respect to time
  – It could have a limited (countable) number of interruptions
    – Voltage, temperature, speed, speech signal, etc.

• Discrete-time signal: daily average temperature, discretized continuous signals

• Need a good understanding of a system of interest to observe the corresponding signals and assess the state of the system
Signal data acquisition

Signal Source

→ Transducers
→ Isolation
→ Amplifiers & filters
→ A/D conversion

Pattern recognition, classification, & diagnosis decision

Analysis of events of waves; feature extraction

Detection of events & components

Filtering to remove artifacts

Signal analysis

Signal processing

Feedback
Example

- Let us observe a simple body temperature measuring

- Sensed easily, in a relative and qualitative manner, via the palm of one’s hand.
- Objective or quantitative measurement of temperature
  requires an instrument, such as a thermometer

- Continuous or discrete?
Continuous signal

• A single measurement $x$ of temperature is a scalar:
  – represents the thermal state of the body at a particular or single instant of time $t$

• If we record the temperature continuously, we obtain a signal as a function of time: expressed in continuous-time or analog form as $x(t)$. 
Digital signal

- When the temperature is measured at discrete points of time, it may be expressed in discrete-time form as $x(nT)$ or $x(n)$,
  - $n$: index or measurement sample number of the array of values,
  - $T$: uniform interval between the time instants of measurement (time period)
- A discrete-time signal that can take values from a limited list of quantized levels is called a digital signal.
Temperature signal

<table>
<thead>
<tr>
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<th>°C</th>
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<tbody>
<tr>
<td>08:00</td>
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<tr>
<td>10:00</td>
<td>33.3</td>
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<td>12:00</td>
<td>34.5</td>
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<tr>
<td>14:00</td>
<td>36.2</td>
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</tr>
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<td>18:00</td>
<td>37.5</td>
</tr>
<tr>
<td>20:00</td>
<td>38.0</td>
</tr>
<tr>
<td>22:00</td>
<td>37.8</td>
</tr>
<tr>
<td>24:00</td>
<td>38.0</td>
</tr>
</tbody>
</table>
Another example: blood pressure

• Each measurement consists of two values
  – the systolic pressure and the diastolic pressure.
• Units: mm of Hg in clinical practice (not Pa)
• A single BP measurement:
  – a vector $\mathbf{x} = [x_1, x_2]^T$ with two components:
    • $x_1$ indicating the systolic pressure and
    • $x_2$ indicating the diastolic pressure.
• When BP is measured at a few instants of time:
  – an array of vector values $\mathbf{x}(n)$, or a function of time $\mathbf{x}(t)$.
• A digital signal is superior to an analog signal because it is more robust to noise and can easily be recovered, corrected and amplified.
Signal data acquisition

Signal Source

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Signal processing
What is Signal Processing?

Signals may have to be transformed in order to

• amplify or filter out embedded information
• detect patterns
• prepare the signal to survive a transmission channel
• undo distortions contributed by a transmission channel
• compensate for sensor deficiencies
• find information encoded in a different domain.

To do so, we also need:

• methods to measure, characterize, model, and simulate signals.
• mathematical tools that split common channels and transformations into easily manipulated building blocks.
Digital Signal Processing (DSP)

Advantages:
• noise is easy to control after initial quantization
• highly linear (with limited dynamic range)
• complex algorithms fit into a single chip
• flexibility, parameters can be varied in software
• digital processing in insensitive to component tolerances, aging, environmental conditions, electromagnetic inference

But
• discrete time processing artifacts (aliasing, delay)
• can require significantly more power (battery, cooling)
• digital clock and switching cause interference
Digital Signal Processing

- Communication Theory
- Numerical Analysis
- Probability and Statistics
- Analog Signal Processing

- Analog Electronics
- Digital Electronics
- Decision Theory
Signal presentation

- Time domain
  - Complex to solve
- Frequency domain

See the Fourier transform handout
Complex Wave Patterns

- Sound waves occupying the same space combine to form a new wave of a different shape.
- Harmonically related waves add together and can create any complex wave pattern.
- Harmonically related waves have frequencies that are multiples of a basic frequency.

**Note:** All frequency waves do not have to start at zero, they can be “out of phase”. The amount of shift in degrees is called their phase angle.
Time Domain: A composite wave summing different frequencies

Frequency Domain: Split time domain into component frequencies
Generation of discrete-time signal

In practice, discrete-time signal can often arise from periodic sampling of an analog signal.

\[ x = x_a[nT], \quad -\infty < n < \infty \]
Generation of digital signal

Analog signal → Sampling → Quantizing → Encoding → 111100

Quantized signal
Sampling

- Analog signal is sampled every $T_s$ seconds (sampling interval).
- $f_s = 1/T_s$ is called the **sampling rate** (sampling frequency).
- In general, there are 3 sampling methods:
  - Ideal: an impulse at each sampling instant
  - Natural: a pulse of short width with varying amplitude
  - Flattop: sample and hold, like natural but with single amplitude value
- The outcome of sampling is a signal with analog (non integer) values
- Discretization in time-domain
Sampling methods

a. Ideal sampling

b. Natural sampling

c. Flat-top sampling
Ideal sampling

\[ x_c(t) \]

\[ t \]

\[ -3T \quad -2T \quad -T \quad 0 \quad T \quad 2T \quad 3T \quad 4T \]

\[ x(n) \]

\[ n \]

\[ -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

\[ x_c(t) \]

\[ t \]

\[ -8T \quad -4T \quad -2T \quad 0 \quad 2T \quad 4T \quad 8T \quad 10T \]

\[ x(n) \]

\[ n \]

\[ -6 \quad -4 \quad -2 \quad 0 \quad 2 \quad 4 \quad 6 \quad 8 \]
We want to restore $x_c(t)$ from $x(n)$. What condition has to be placed on the sampling rate?
Sampling

- As sampling rate increases, sampled waveform looks more and more like the original
- Many applications (e.g. communication systems) care more about frequency content in the waveform and not its shape
According to the Nyquist theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.
Shannon Sampling Theorem

• A continuous-time signal $x(t)$ with frequencies no higher than $f_{\text{max}}$ can be reconstructed from its samples $x[n] = x(n T_s)$ if the samples are taken at a rate $f_s$ which is greater than $2f_{\text{max}}$. 
  
  Nyquist rate $= 2f_{\text{max}}$
  
  Nyquist frequency $= f_s/2$.

• What happens if $f_s = 2f_{\text{max}}$?

• Consider a sinusoid $\sin(2\pi f_{\text{max}} t)$
  
  Use a sampling period of $T_s = 1/f_s = 1/2f_{\text{max}}$.
  
  Sketch: sinusoid with zeros at $t = 0, 1/2f_{\text{max}}, 1/f_{\text{max}}, \ldots$
Nyquist sampling rate for low-pass and bandpass signals

- **Low-pass signal**: Nyquist rate = $2 \times f_{\text{max}}$

- **Bandpass signal**: Nyquist rate = $2 \times f_{\text{max}}$
What if?

• We oversample?
  – Oversampling allows us to re-create the original signal, but it comes with a cost - redundancy.

• Under-sample?
  – Sampling below the Nyquist rate does not produce a signal that looks like the original one.
Example

a. Sampling at Nyquist rate: $T_s = \frac{T}{2}$

b. Oversampling (above Nyquist rate): $T_s = \frac{T}{4}$

c. Undersampling (below Nyquist rate): $T_s = \frac{3T}{4}$

Any similar example that is known to you?
• The “backward rotation” of the wheels of a forward-moving car in a movie.
• This can be explained by under-sampling.
• A movie is filmed at 24 frames per second. If a wheel is rotating more than 12 times per second, the under-sampling creates the impression of a backward rotation.
POTS example

• Telephone companies digitize voice by assuming a maximum frequency of 4000 Hz.
• The sampling rate therefore is 8000 samples per second. What is the sampling period?

• Where does 4 KHz come from?
• A complex bandpass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

• Solution
  – We cannot find the minimum sampling rate in this case because we do not know where the bandwidth starts or ends. We therefore do not know the maximum frequency in the signal.
Quantization

• Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.
• The amplitude values are infinite between the two limits (continuous).
• We need to map the infinite amplitude values onto a finite set of known values.
• This is achieved by dividing the distance between min and max into \( L \) zones, each of height \( \Delta \).

\[ \Delta = \frac{(\text{max} - \text{min})}{L} \]
Quantization Levels

- The midpoint of each zone is assigned a value from 0 to L-1 (resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint.
Quantization Zones

• Assume we have a voltage signal with amplitudes $V_{\text{min}} = -20V$ and $V_{\text{max}} = +20V$.
• Voltage range is 40V.
• Use L=8 quantization levels.
• Zone width $\Delta = (20 - -20)/8 = 5V$
• The 8 zones are: -20 to -15, -15 to -10, -10 to -5, -5 to 0, 0 to +5, +5 to +10, +10 to +15, +15 to +20
• The midpoints are: -17.5, -12.5, -7.5, -2.5, 2.5, 7.5, 12.5, 17.5
Quantization and encoding of a sampled signal

<table>
<thead>
<tr>
<th>Quantization codes</th>
<th>Normalized amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>19.7</td>
</tr>
<tr>
<td>6</td>
<td>16.2</td>
</tr>
<tr>
<td>5</td>
<td>7.5</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-6.1</td>
</tr>
<tr>
<td>2</td>
<td>-5.5</td>
</tr>
<tr>
<td>1</td>
<td>-11.3</td>
</tr>
<tr>
<td>0</td>
<td>-9.4</td>
</tr>
</tbody>
</table>

Normalized quantized values:

-1.50 1.50 3.50 3.50 2.50 -1.50 -2.50 -1.50

Normalized error:

-0.38 0 +0.26 -0.44 +0.30 -0.40 -0.24 +0.38 -0.30

Quantization code:

2 5 7 7 6 2 1 2 2

Encoded words:

010 101 111 111 110 010 001 010 010
• Any signal other than that of interest is interference, artefact, or noise.
• Sources of noise depend on the signal type, and could be numerous, e.g. instrumentation, or the environment of the experiment.
• Interference that arises from a random process such as thermal noise in electronic devices is random noise.
• A random process is characterized by the probability density function (PDF) representing the probabilities of occurrence of all possible values of a random variable.
Random noise

• Random process $\xi$ characterized by PDF $p(\xi)$.
• Mean $\mu_\xi$: first-order moment of the PDF
  \[
  \mu_\xi = E[\xi] = \int_{-\infty}^{\infty} \xi p(\xi) d\xi,
  \]
• where $E[\ ]$ represents the statistical expectation operator.
• Common to assume mean of a random noise process $= 0$. 
Statistical measures

- Mean-squared (MS) value, i.e. second-order moment,

\[ E[\xi^2] = \int_{-\infty}^{\infty} \xi^2 p(\xi) d\xi \]

- Variance, second central moment

\[ \sigma_{\xi}^2 = E[(\xi - \mu_{\xi})^2] = \int_{-\infty}^{\infty} [(\xi - \mu_{\xi})^2] p(\xi) d\xi \]

- Square root of variance is standard deviation (SD), \( \sigma_{\xi} \)

\[ \sigma_{\xi}^2 = E[\xi^2] - (\mu_{\xi})^2 \]

- When is MS= variance?
• When the values of a random process form a time series or a function of time, we have a random signal or a stochastic process $\xi(t)$.

• Then, the statistical measures have physical meanings:
  – mean = DC component;
  – MS = average power;
  – square root of MS = root mean-squared or RMS value = average noise magnitude.

• See handouts for more information
• When a signal $x(t)$ is observed with random noise, the measured signal $y(t)$ may be treated as a realization of another random process $y$.

• In most cases the noise is additive:

$$y(t) = x(t) + \xi(t)$$
Filtering

• A filter is a signal processing system, algorithm, or method, in hardware or software, used to modify a signal.

• A signal may be filtered to remove undesired components, noise, or artefacts, and to enhance desired components.

• Filters may be categorized as
  – linear or nonlinear,
  – stationary or nonstationary,
  – Fixed (time-invariant) or adaptive (time-variant),
  – active or passive,
  – statistical or deterministic.
Filtering

• A fundamental characteristic of a linear time-invariant (LTI) or shift-invariant (LSI) filter is its impulse response:
  – output of the system when the input is a Dirac delta or impulse function.

• How does Dirac function look like?
The delta function is also defined in the following terms:

\[
\int_{T_1}^{T_2} x(t) \delta(t - t_0) \, dt = \begin{cases} 
  x(t_0) & \text{if } T_1 < t_0 < T_2, \\
  0 & \text{otherwise,}
\end{cases}
\]

where \( x(t) \) is a function that is continuous at \( t_0 \).

This is known as the **sifting property** of the delta function, because the value of the function \( x(t) \) at \( t_0 \) of the delta function is sifted or selected from all of its values.
• Consider a continuous-time signal, $x(t)$, processed by an LSI system
• An LSI system is completely characterized or specified by its impulse response, $h(t)$, which is the output of the system when the input is a delta function.
• The output of the system, $y(t)$, is given by the convolution of the input, $x(t)$, with the impulse response, $h(t)$:
• This is a linear function

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) \, d\tau$$
How about discrete time?

- Principle is the same, but we handle the samples.
- Two important points:
  - $h(-k)$ represents a reversal in time of $h(k)$;
  - $h(n - k)$ represents a shift of the reversed signal $h(-k)$ by $n$ samples.
- Multiplication of $h(n-k)$ by $x(k)$ can be viewed as scaling.
- The summation represents accumulation of the results or integration of $x(k) h(n - k)$ over the interval from $k = 0$, the origin of time, to $n$, the present instant of time.

$$y(n) = \sum_{k=0}^{n} x(k) h(n - k).$$
<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>x(n)</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>1</td>
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<tr>
<td>x(n-1)</td>
<td>4</td>
<td>5</td>
<td>3</td>
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<tr>
<td>x(n-2)</td>
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<td>5</td>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

$x - N_1$, $h - N_2$ samples

How many samples in a result?
• The output ceases to exist (or has only zero values) when the shift exceeds a certain amount such that $x(k)$ and $h(n - k)$ do not overlap in time any more.

• Linear convolution of two discrete-time signals with $N_1$ and $N_2$ samples leads to a result with $N_1 + N_2 - 1$ samples.
Example
• The noisy signal was filtered by computing the mean of each sample and the preceding 10 samples:

\[ y(n) = \frac{1}{11} \sum_{k=0}^{10} x(n-k) \]

where \( n=10, 11, \ldots, N-1, N \) – number of samples.

• This is a moving average (MA) filter,

• The average values of the input signal are computed in a “moving” temporal window and used to define the output signal.
DSP Applications

communication systems
modulation/demodulation, channel
equalization, echo cancellation
consumer electronics
perceptual coding of audio and
video on DVDs, speech synthesis,
speech recognition
Music
synthetic instruments, audio
effects, noise reduction
medical diagnostics
Magnetic-resonance and
ultrasonic imaging, computer
tomography, ECG, EEG, MEG, AED,
audiology
Geophysics
seismology, oil exploration

astronomy
VLBI, speckle interferometry
experimental physics
sensor data evaluation
aviation
radar, radio navigation
security
steganography, digital
watermarking, biometric
identification, visual surveillance
systems, signal intelligence,
electronic warfare
engineering
control systems, feature
extraction for pattern recognition
Eye movements and tracking
– Noton and Stark (1971)

• showed that participants tend to fixate identifiable regions of interest, containing “informative details”;
• coined term “scanpath” describing eye movement patterns

An example

Instructions
1: examine at will
2: estimate wealth
3: estimate ages
4: guess previous activity
5: remember clothing
6: remember position
7: time since last visit
The eye—“the world’s worst camera”

– suffers from numerous optical imperfections...
– ...endowed with several compensatory mechanisms
Human visual field

- **The retina** is a light sensitive structure inside of the eye responsible for transforming light into signals, which are later converted into an image by the visual cortex in the brain.

- **The fovea** is a section of the retina that contains a high density of both kinds of light receptor cells found in the eye, i.e. Cone and Rod cells.
Eye movements

• Eye movements are mainly used to reposition the fovea

• Five main classes of eye movements:
  – Saccadic (saccades and fixations)
  – Smooth pursuit
  – Vergence
  – Vestibular
  – Physiological nystagmus (“micro saccades”)

• Other types of movements are e.g. adaptation, accommodation
Saccades

- Rapid eye movements between fixations used to reposition fovea
- Voluntary and reflexive
- Range in duration from 10 ms – 100 ms
- Effectively blind during transition
- Deemed *ballistic* (pre-programmed) and *stereotyped* (reproducible)
Fixations

• Probably the most important type of eye movement for attentional applications
  – 90% viewing time is devoted to fixations
  – duration: 150 ms – 600 ms
Applications

- Ergonomics and Human Factors
- Marketing and Advertising
- Websites, Virtual reality
- Displays, HCI
- Psychology, Psychophysics, Neuroscience
Marketing and Advertising

• Applications range from assessing ad effectiveness (copy testing) in various media (print, images, video, etc.) to disclosure research (visibility of fine print)

• Examples:
  – eye movements over print media (e.g., magazines)
  – eye movements over TV ads, web pages, etc.
Scanpaths over magazine ads
Websites

- Most people view websites in a “F” shaped flow.
  - First they scan the page at the top, from left to right.
  - Then the eyes go back to the left and down the page.
  - They again scan to the right and back along the same pattern.
Psychology and Neuroscience

- Applications range from basic research in vision science to investigation of visual exploration in aesthetics (e.g., perception of art).

- **Examples:**
  - psychophysics: spatial acuity, contrast, sensitivity
  - perception: reading, natural scenery, ...
  - neuroscience: cognitive load, with fMRI an ERP
  - psycholinguistics
Techniques and Equipment

• Two broad applications of eye movement monitoring/recording techniques:
  – measuring position of eye relative to the head
  – measuring orientation of eye in space, or the “point of regard” (POR)—used to identify fixated elements in a visual scene

• The most widely used apparatus for measuring the POR is the video-based corneal reflection eye-tracker
Techniques

• First method for objective eye movement measurements using corneal reflection reported in 1901
• Techniques using contact lenses to improve accuracy developed in 1950s (invasive)
• Remote (non-invasive) trackers rely on visible features of the eye (e.g., pupil)
• Fast image processing techniques have facilitated real-time video-based systems
Electro-oculography

- Relies on measurement of skin’s potential differences, using electrodes placed around the eye.
- Most widely used method some 30 years ago (still used today).
- Similar to electro-mechanical motion-capture.
- Measures eye movements relative to head position.
- Not generally suitable for POR measurement (unless head is also tracked).
Head-mounted video-based eye tracker

- most suitable for (graphical) interactive systems, e.g., VR
- Monocular and binocular systems
SMI

TOBII

ISCAN

ASL head-mounted

ISCAN child head-mounted
POR Method

- A light source is used to cause reflection patterns on the cornea and pupil of the test person.
- A camera will then be used to capture an image of the eye.
- The direction of the gaze is then calculated using the angles and distances.
Eye-Tracking in Reading
Preliminaries

• Cooper (1974), Tanenhaus et al. (1995)

• The Mind-Eye hypothesis
  • Relationship between eye fixations and the meaning of concurrently spoken sentence
  • Using this relationship as a research tool in cognitive psychology and psycholinguistics
  • Applications:
    – Speech perception and memory
    – Language processing


Examples of Eye Movements while Reading

• A gaze replay, recorded at 300Hz using the Tobii TX300 eye tracker, of a participant in a reading study: https://www.youtube.com/watch?v=VBTZNydhUh0w

• This may be something to show in your project
Eye- Movement
Parameters

• Studying eye movements *per se* to learn about reading
• Using eye movements in reading as a means to infer cognitive processes (e.g., language processing)
Eye-Movement Parameters

- Saccade latency: 150-175 ms;
- Perceptual span: ~4 symbols to the left, ~15 symbols to the right;
- Skipping words: 2-3-letter words are skipped 75%, 8-letter words are never skipped;
- Regressions: 10-letter spaces happen because of problems in understanding text