Homework 2
Motion and Manipulation

Deadline: October 18, 2017, at 15:15

Note: Solutions should be handwritten. Write your name and student number on each page you hand in. Homework 2 consists of ten exercises. Each exercise is worth 1 point. Motivate all your answers. Show the derivations.

1: Configuration Space

Determine the configuration space for a system of two ball-shaped entities \( A_1 \) and \( A_2 \) moving in contact in a three-dimensional Euclidean workspace.

2: Configuration Space

Consider a rotating and translating polyhedral entity \( A \) in a three-dimensional Euclidean space and let \( v \) be one of the vertices on its boundary. Determine the configuration space for \( A \) when its vertex \( v \) is constrained to move in the plane \( z = 0 \).

3: Configuration Space

Consider a system of two square entities \( A_1 \) and \( A_2 \) translating in a two-dimensional Euclidean workspace.
(a.) Determine the configuration space of the system when \( A_1 \) and \( A_2 \) move independently.
(b.) Determine the configuration space of the system when \( A_1 \) and \( A_2 \) must touch at all times.

4: Minkowski Sums

Consider a two-dimensional Euclidean space. What is the area of the Minkowski sum of a square with side length 3 and disk with radius 1?

5: Minkowski Sums and Configuration Space Obstacles

Consider a two-dimensional Euclidean space. Let \( p_1 = (0, 0) \), \( p_2 = (4, 0) \), \( p_3 = (4, 4) \), \( p_4 = (2, 4) \), \( p_5 = (2, 2) \), and \( p_6 = (0, 2) \). Let \( B \) be the non-convex object bounded by the edges \( p_1 p_2 \), \( p_2 p_3 \), \( p_3 p_4 \), \( p_4 p_5 \), \( p_5 p_6 \), and \( p_6 p_1 \). Let \( A \) be the line segment with endpoints \( (0, 0) \) and \( (3, 3) \)
(a.) Determine \( A \oplus B \) and list its vertices.
(b.) Now assume that \( A \) is a moving entity that is only allowed to translate and \( B \) is an obstacle. Determine the configuration space obstacle \( C_{obs} \) corresponding to all placements at which \( A \) intersects \( B \). List the vertices of \( C_{obs} \).
6: Minkowski Sums and Configuration Space Obstacles

When does the configuration space obstacle corresponding to all placements at which a translating entity \( A \) intersects an obstacle \( B \) equal the Minkowski sum \( A \oplus B \)?

7: Minkowski Sums

Consider a two-dimensional Euclidean space. Let \( p_1 = (1, 0) \), \( p_2 = (5, 0) \), \( p_3 = (2, 1) \), \( p_4 = (3, 3) \), and \( p_5 = (1, 3) \). Let \( A \) be the non-convex object bounded by the edges \( p_1p_2 \), \( p_2p_3 \), \( p_3p_4 \), \( p_4p_5 \), and \( p_5p_1 \).

(a.) Let \( B \) be the triangle with corners \((1, 1)\), \((2, 0)\), and \((2, 2)\). Determine the Minkowski sum \( A \oplus B \), and list its vertices.

(b.) Let \( B' \) be the triangle with corners \((2, 3)\), \((3, 2)\), and \((3, 4)\). How do the vertices of the Minkowski sum \( A \oplus B' \) relate to those of the Minkowski sum \( A \oplus B \)?

8: Configuration Space Obstacles

Consider a three-dimensional Euclidean space with a box-shaped obstacle \( B \) with corners \((0, 0, 0)\), \((2, 0, 0)\), \((0, 3, 0)\), \((2, 3, 0)\), \((0, 0, 4)\), \((2, 0, 4)\), \((0, 3, 4)\), and \((2, 3, 4)\) and a tetrahedral translating entity \( A \) with corners \((0, 0, 0)\), \((1, 0, 0)\), \((0, 1, 0)\), and \((0, 0, 1)\). Determine the configuration space obstacle \( C_{\text{obs}} \) corresponding to all placements at which \( A \) intersects \( B \). List the vertices of \( C_{\text{obs}} \).

9: Minkowski Sums

Let \( -X = \{ -x \mid x \in X \} \). For two arbitrary sets \( A \) and \( B \) in two-dimensional Euclidean space prove that if \((0, 0) \in A \oplus (-B)\) then also \((0, 0) \in B \oplus (-A)\).

10: Free Space

Construct a situation in which the free space of a translating unit square entity \( A \) moving among four unit square obstacles \( B_1, \ldots, B_4 \) consists of two disconnected components.