Homework I
Motion and Manipulation

Deadline: September 21, 2016, at 15:15

Note: Solutions should be handwritten. Write your name and student number on each page you hand in. Homework I consists of ten exercises. Each exercise is worth 1 point. Motivate all your answers. Show the derivations.

1: Vectors

Consider the two vectors

\[ s = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \quad t = \begin{pmatrix} 1 \\ 1 \\ \sqrt{2} \end{pmatrix}. \]

(a.) Compute \( s + 3t \).
(b.) What is the angle between \( s \) and \( t \)?
(c.) Compute \( s \times t \), and show that it is perpendicular to \( s \) and to \( t \).

2: Vectors

Consider the four vectors

\[ s = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad t = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}, \quad u = \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}, \quad v = \begin{pmatrix} -1 \\ 2 \\ -\frac{1}{3} \end{pmatrix}. \]

(a.) Are the vectors \( s \), \( t \), and \( u \) linearly independent?
(b.) Is the vector \( v \) perpendicular to \( s \) and \( t \)?

3: Matrices

Consider the two matrices

\[ A = \begin{pmatrix} 2 & 3 & 0 \\ 1 & 4 & 6 \\ 5 & 8 & 7 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 & 2 \\ 0 & 2 & -1 \\ 2 & -1 & 0 \end{pmatrix}. \]

(a.) Compute \( A + B \).
(b.) Compute \( AB \)
(c.) Compute \( AAT^T \).
4: Rotations

Let $p = (1, 2, 3)$. Compute the image of $p$ after a rotation by $\pi/4$ about the $x$-axis followed by a rotation by $\pi/2$ about the $y$-axis. Also compute the image of $p$ after a rotation by $\pi/2$ about the $y$-axis followed by a rotation by $\pi/4$ about the $x$-axis.

5: Determinants and Inverses

(a.) Compute the determinant of the matrix

$$A = \begin{pmatrix} 1 & 2 & 6 \\ 0 & 3 & -8 \\ 4 & 0 & -5 \end{pmatrix}.$$

(b.) Determine the inverse of the matrix

$$B = \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{pmatrix}.$$

6: Moore-Penrose Pseudoinverse

Compute the Moore-Penrose pseudoinverse of the matrix

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & -2 & -1 \\ 0 & 1 & 0 \end{pmatrix}.$$

7: Gaussian Elimination

Use Gaussian elimination on the augmented matrix to find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & 2 \\ 2 & 0 & 1 \end{pmatrix}.$$

8: Derivatives

Compute the derivatives of the functions

(a.) $f_1(x) = 4x^3 + 3x^2 + 2x + 1$,

(b.) $f_2(x) = 2(\cos x)^2 + \sin x$,

(c.) $f_3(x) = \frac{\sin x}{x^2}$,

(d.) $f_4(x) = e^{x \cos x}$.

9: Jacobians

Compute the Jacobian for the vector-valued function $f : \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \end{pmatrix} = \begin{pmatrix} x_1 \sin x_2 - \cos x_3 \\ x_1 + \cos x_2 \sin x_3 \end{pmatrix}.$$
10: Complex Numbers

(a.) Determine the set of complex numbers $z$ for which $zi = z$.

(b.) Use complex number multiplication to compute the image of the point $p = (2, 1)$ after a rotation by $\pi/3$ about the origin. Also compute the image of the point $p$ after a rotation by $-\pi/2$ about the origin.