Final Test  
Motion and Manipulation  

November 9, 2018  
13:30-16:00  

Note: It is not allowed to use pocket calculators or consult books, notes, slides, etc. Fill out your name and student number on each page you hand in. The test consists of nine exercises. Motivate all your answers!

1: Kinematics I (0.5 + 1.0)

We are given a fixed orthonormal frame $F = \{f_1, f_2, f_3\}$ and a mobile orthonormal frame $M = \{m_1, m_2, m_3\}$. Initially the frames $M$ and $F$ coincide. We rotate $M$ by $-\pi/2$ radians about $f_2$ and then by $\pi/2$ radians about $f_3$.

(a.) Determine the transformation matrix that maps mobile $M$ coordinates into fixed $F$ coordinates.

(b.) The above composition of two rotations is equivalent to a single rotation. Determine the axis and the angle of this rotation.

2: Kinematics II (1.0 + 0.5)

(a.) Use quaternions to determine the image of the point $p = (0, 1, 1)$ after a rotation by an angle of $\pi/3$ about the line through the origin with direction vector $(1, 0, \sqrt{3})^T$.

(b.) We are given a fixed orthonormal frame $F$ and a mobile orthonormal frame $M$. Initially the frames $M$ and $F$ coincide. We rotate $M$ by $\pi/3$ about the line through the origin with direction vector $(1, 0, \sqrt{3})^T$ (given with respect to $F$) and then rotate $M$ by $\pi/2$ about the line through the origin with direction vector $(1, 1, 0)^T$ (also given with respect to $F$). Determine the quaternion corresponding to the composition of these rotations.

3: Kinematics for Linkages (0.5 + 0.5)

Let $L_{k-1} = \{x_{k-1}, y_{k-1}, z_{k-1}\}$ and $L_k = \{x_k, y_k, z_k\}$ be two subsequent frames along a robotic arm. Consider the joint and link parameters that are associated with the transformation between both frames: joint angle, joint distance, link length, and link twist angle.

(a.) Which parameter specifies the rotation about $x_k$ to make $z_{k-1}$ parallel to $z_k$?

(b.) Which parameter specifies the length of translation along $z_{k-1}$ to make $x_{k-1}$ intersecting with $x_k$?

4: Inverse Kinematics (1.0)

Consider a line segment $S$ of length 2 moving in two-dimensional Euclidean space and denote its endpoints by $a$ and $b$. The segment $S$ can rotate freely while its endpoint $a$ is constrained to the $x$-axis. Let $d$ be the $x$-coordinate of $a$ and let $\theta$ be the orientation of $S$ (given by the
Consider the inverse kinematics problem of finding the values of \( d \) and \( \theta \) that place the tip \( b \) at the point \((3, 1)\). Although it is easy to determine an analytic solution to this inverse kinematics problem we consider the use of the iterative solution method. Perform one iteration of the iterative solution method to find improved values \( d^{(1)} \) and \( \theta^{(1)} \) for the joint variables \( d \) and \( \theta \) respectively, using initial values \( d^{(0)} = 2 \) and \( \theta^{(0)} = 0 \).

5: Minkowski Sums (0.5 + 0.5)

Let \( s_0 \) be the line segment with endpoints \((0, 0)\) and \((4, 0)\) and \( s_1 \) be the line segment with endpoints \((0, 0)\) and \((2, 4)\). Define \( L_0 = s_0 \cup s_1 \). Let \( t_0 \) be the line segment with endpoints \((0, 1)\) and \((1, 3)\) and \( t_1 \) be the line segment with endpoints \((0, 1)\) and \((2, 1)\). Define \( L_1 = t_0 \cup t_1 \).

(a.) Construct the Minkowski sum \( L_0 \oplus L_1 \).
(b.) Construct the Minkowski sum \( L_0 \oplus (-L_1) \).

6: Short Questions (0.5 + 0.5)

(a.) Determine the configuration space for a system of two robots \( A_1 \) and \( A_2 \) moving in a three-dimensional Euclidean workspace, where \( A_1 \) is a sphere and \( A_2 \) is cube rotating and translating while keeping a given corner in contact with \( A_1 \).

(b.) Explain the difference between the integral and derivative term in the control command of a PID-controller.

7: Plücker Coordinates (1.0)

Determine the Plücker coordinates of the line \( \ell \) through the points \((2, 0, -1)\) and \((-1, 7, -3)\). What is the distance between the line \( \ell \) and the origin \( O \)?

8: Form Closure Grasps (1.0)

Let \( O \) be the convex object with vertices \((0, 0)\), \((0, 2)\), \((10, 0)\), and \((4, 6)\). Let \( p_1 = (0, 1) \), \( p_2 = (3, 5) \), and \( p_3 = (6, 4) \). Let \( e \) be the edge of \( O \) connecting the vertices \((0, 0)\) and \((10, 0)\). Let \( p_x = (x, 0) \) be a point on \( e \) (with \( x \in (0, 10) \)). What is the range of values for \( x \) such that frictionless point contacts at \( p_1, p_2, p_3, \) and \( p_x \) put \( O \) in form closure? Justify your answer.

9: Force Closure Grasps (1.0)

The boundary of the convex semi-algebraic object

\[
C = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 - 25 \leq 0 \} \cap \{(x, y) \in \mathbb{R}^2 | y - 4 \leq 0 \} \cap \{(x, y) \in \mathbb{R}^2 | y - 4 \leq 0 \}
\]

consists of circular arcs and line segments. Let \( a_1 = (-4, -3) \) and \( a_2 = (4, -3) \). Choose two points \( a_3 \) and \( a_4 \) along the boundary of \( C \) such that frictionless point fingers at \( a_1, a_2, a_3, \) and \( a_4 \) put \( C \) in force closure. Use wrench analysis to justify your answer.