Final Test
Motion and Manipulation

November 7, 2012
14:00-16:30

Note: It is not allowed to use pocket calculators or consult books, notes, slides, etc. Fill out your name and student number on each page you hand in. The test consists of **seven** exercises. **Motivate all your answers!**

1: Kinematics I (1.0)

We are given a fixed orthonormal frame $F = \{f^1, f^2, f^3\}$ and a mobile orthornormal frame $M = \{m^1, m^2, m^3\}$. Initially the frames M and F coincide. We rotate M by an angle $\pi/3$ about a line through the origin with global direction vector $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\sqrt{2})$. Determine the rotation matrix R that maps coordinates with respect to M to coordinates with respect to F.

2: Kinematics II (1.0)

We are given a fixed orthonormal frame $F = \{f^1, f^2, f^3\}$ and a mobile orthornormal frame $M = \{m^1, m^2, m^3\}$. Initially the frames M and F coincide. We rotate M about f^1 by $\pi/3$ radians. How far should we subsequently translate M along m^2 to establish that the point p with coordinates $(0, 0, 1)$ with respect to M has a y-coordinate equal to 0 with respect to F?

3: Configuration Space Obstacles (1.0)

Let $p_1 = (1, 1)$, $p_2 = (5, 1)$, $p_3 = (5, 2)$, $p_4 = (2, 2)$, $p_5 = (2, 5)$, and $p_6 = (1, 5)$. Let O be the non-convex obstacle bounded by the edges p_1p_2, p_2p_3, p_3p_4, p_4p_5, p_5p_6, and p_6p_1. We are given a two-dimensional Euclidean workspace with the obstacle O and a triangular robot A with vertices $(0, 0)$ (its reference point), $(2, 1)$, and $(-1, 1)$. The robot A is only allowed to translate. Construct the configuration-space obstacle C_{obs} corresponding to all placements at which A intersects O, and list its vertices.

4: Kinematics for Linkages (2.0)

Consider the three-axis robot on the separate sheet. Use the Denavit-Hartenberg algorithm to attach coordinate frames to its axes of motion and its hand. Use these frames to determine the joint angle θ_i, the joint distance d_i, the link length a_i, and the link twist angle α_i for each of the axes $i = 1, 2, 3$. Clearly indicate which parameters are variable.

5: Short Questions (0.5 + 0.5 + 0.5 + 0.5)

Give short answers to each of the following questions.

(a.) Determine the configuration space for a system of two cylindric robots A_1 and A_2 moving in contact in a three-dimensional Euclidean workspace.

(b.) Describe how the GJK-algorithm iteratively improves its estimate of the distance from the origin O to a convex polygon C.

(c.) Describe how a PI-controller adjusts the process it controls.

(d.) Construct a convex polygonal object O with four vertices and choose a center of mass inside O such that O has exactly *two* stable orientations when being pushed by a jaw.

6: Plücker and Screw Coordinates (1.0)

Determine the Plücker coordinates of the line ℓ through the points $(1, 2, 1)$ and $(3, 3, 1)$. What are the screw coordinates for the screw with pitch 7 about ℓ?

7: Grasping (1.0 + 1.0)

Let $p_1 = (0, 0)$, $p_2 = (0, -3)$, $p_3 = (2, -3)$, $p_4 = (2, 2)$, $p_5 = (-3, 2)$, and $p_6 = (-3, 0)$. Let N be the non-convex object bounded by the edges p_1p_2, p_2p_3, p_3p_4, p_4p_5, p_5p_6, and p_6p_1. Assume that one frictionless point finger is placed at $a_1 = p_1$.

(a.) Place a second and third frictionless finger at $a_2 = (2, -1)$ and $a_3 = (1, 2)$. Show that a_1, a_2, and a_3 do not put N in *form closure*. Which instantaneous rotations remain possible?

(b.) Choose two points a_4 and a_5 such that frictionless fingers at a_1, a_4, and a_5 put N in *force closure*. Justify your answer.