Motion and Manipulation

Kinematics for Articulated Structures: the Denavit-Hartenberg Representation
Denavit-Hartenberg Representation

- Systematic assignment of coordinate frames to the links and joints of a robotic arm.
  - A joint axis connects two adjacent links
  - A link connects two successive joints
Joint Parameters

• Joint axis:
  - Revolute: rotation
  - Prismatic: slide

• Joint $k$ connects link $k-1$ and link $k$.

• Frame axis $z_{k-1}$ is aligned with joint axis $k$
Joint Parameters

- **Joint angle** $\theta_k$: angle of rotation about $z_{k-1}$ to make $x_{k-1}$ parallel with $x_k$.

- **Joint distance** $d_k$: distance of translation along $z_{k-1}$ to make $x_{k-1}$ intersect $x_k$. 

\[ \begin{align*}
\theta_k & : \text{ angle of rotation about } z_{k-1} \\
d_k & : \text{ distance of translation along } z_{k-1}
\end{align*} \]
Joint Parameters

Parameters specify relative positions and orientations of two successive links.

- Revolute joint: $\theta_k$ variable and $d_k$ fixed.

- Prismatic joint: $d_k$ variable and $\theta_k$ fixed.
Link Parameters

- Link $k$ connects joint $k$ and joint $k+1$. 

![Diagram](image.png)
Link Parameters

- **Link twist angle** $\alpha_k$: angle of rotation about $x_k$ to make $z_{k-1}$ parallel with $z_k$.

- **Link length** $a_k$: distance of translation along $x_k$ to make $z_{k-1}$ intersect $z_k$. [Diagram showing $z_{k-1}$, $a_k$, $z_k$, and $x_k$.]
Link Parameters

Parameters specify relative positions and orientations of two successive joints.

- Both $\alpha_k$ and $a_k$ are fixed.
Robotic Arm

- n-Axis robot: 4n parameters
- Per axis:
  - 3 fixed parameters
  - 1 variable parameter: joint variable
Tool or Hand

- approach vector $r_3$ aligned with tool roll axis
- sliding vector $r_2$ aligned with open-close axis
- normal vector $r_1$
Denavit-Hartenberg Algorithm

- Assigns coordinate frames to links/joints and determines joint and link parameters in a systematic way to establish that all coordinate transformations have a standard form.
Denavit-Hartenberg Algorithm

Outline:

- Joint numbering.
- Assignment of base frame.
- Assignment of frames 1 through k-1.
- Assignment of tool frame.
- Computation of joint and link parameters.
Joint Numbering

- Number the joints from 1 to n starting with the base and ending with the tool yaw, pitch, and roll, in that order.
Assignment of Base Frame

- Assign a right-handed orthonormal coordinate frame $L_0$ to the robot base, making sure that $z_0$ aligns with the axis of joint 1.
Assignment of Frame 1 through \( k-1 \)

\[
\text{for } k=1 \text{ to } n-1 \text{ do}
\]
- Align \( z_k \) with the axis of joint \( k+1 \)
- Locate the origin of \( L_k \) at the intersection of the \( z_k \) and \( z_{k-1} \) axes. If they do not intersect, use the intersection of \( z_k \) with a common normal between \( z_k \) and \( z_{k-1} \).
- Select \( x_k \) to be the orthogonal to both \( z_k \) and \( z_{k-1} \). If \( z_k \) and \( z_{k-1} \) are parallel, point \( x_k \) away from \( z_{k-1} \).
- Select \( y_k \) to form a right-handed orthonormal coordinate frame \( L_k \).
Assignment of Tool Frame

- Set the origin of $L_n$ at the tool tip. Align $z_n$ with the approach vector, $y_n$ with the sliding vector, and $x_n$ with the normal vector of the tool.
Computation of Parameters

\textbf{for} \ k=1 \ \textbf{to} \ n \ \textbf{do}

• Locate (auxiliary) point $b_k$ at the intersection of the $x_k$ and $z_{k-1}$ axes. If they do not intersect, use the intersection of $x_k$ with a common normal between $x_k$ and $z_{k-1}$.

• Compute $\theta_k$ as the angle of rotation from $x_{k-1}$ to $x_k$ measured about $z_{k-1}$.

• Compute $d_k$ as the distance from the origin of frame $L_{k-1}$ to point $b_k$ measured along $z_{k-1}$.

• Compute $a_k$ as the distance from point $b_k$ to the origin of frame $L_k$ measured along $x_k$.

• Compute $\alpha_k$ as the angle of rotation from $z_{k-1}$ to $z_k$ measured about $x_k$. 

Arm Equation

- Determine coordinate transformation from $L_k$ into $L_{k-1}$. 

![Diagram](image)
Arm Equation

- Place moving frame $M$ at $L_{k-1}$ and move towards $L_k$. Maintain transformation.
Arm Equation

- \( M \) coincides with \( L_{k-1} \).
- \( T := I \).
Arm Equation

- Rotate M by $\theta_k$ about z.
- x-axis of M is now parallel with $x_k$.
- $T := T \circ \text{Rot}_3(\theta_k) = \text{Rot}_3(\theta_k)$
Arm Equation

- Translate M by $d_k$ along z.
- x-axis of M is now collinear with $x_k$.
- $T := T \circ \text{Tran}_3(d_k e_3) = \text{Rot}_3(\theta_k) \circ \text{Tran}_3(d_k e_3)$
Arm Equation

- Translate M by $a_k$ along x.
- x-axis of M is now coincides with $x_k$.
- \[ T = T \circ \text{Tran}_1(a_k e_1) \]
  \[ = \text{Rot}_3(\theta_k) \circ \text{Tran}_3(d_k e_3) \circ \text{Tran}_1(a_k e_1) \]
Arm Equation

- Rotate M by $\alpha_k$ about x.
- M now coincides with $L_k$.
- $T := T \circ \text{Rot}_1(\alpha_k)$
  $= \text{Rot}_3(\theta_k) \circ \text{Tran}_3(d_k, e_3)$
  $\circ \text{Tran}_1(a_k, e_1) \circ \text{Rot}_1(\alpha_k)$

\[ y = y_k \]
\[ z = z_k \]

Joint $k$, Link $k$, Joint $k+1$
Coordinate Transformation Matrix

\[ T(\theta_k, d_k, a_k, \alpha_k) = \]

\[ \text{Rot}_3(\theta_k) \circ \text{Tran}_3(d_k e_3) \circ \text{Tran}_1(a_k e_1) \circ \text{Rot}_1(\alpha_k) = \]

\[ \text{Screw}_3(\theta_k, d_k) \circ \text{Screw}_1(a_k, a_k) \]

maps coordinates with respect to \( L_k \) into coordinates with respect to \( L_{k-1} \).
Kinematics for the Arm

- Joint variables \( q = (q_1 \ q_2 \ q_3 \ \cdots \ q_n)^T \)

\[
0T_n(q) = 0T_1(q_1)^1T_2(q_2)^2T_3(q_3)^3\cdots^n-1T_n(q_n) = \begin{pmatrix} R(q) & t(q) \\ 0^T & 1 \end{pmatrix}
\]

\( R(q) \) gives directions of the x-, y-, and z-axes of \( L_n \) with respect to the x-, y-, and z-axes of \( L_0 \).