Motion and Manipulation

Kinematics for Articulated Structures:
the Denavit-Hartenberg Representation

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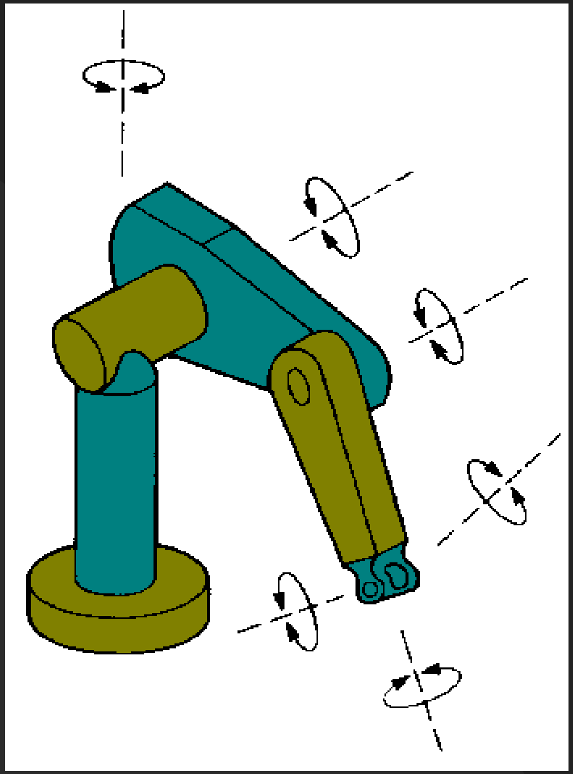
Denavit-Hartenberg Representation

- Systematic assignment of coordinate frames to the links and joints of a robotic arm.
  - A joint axis connects two adjacent links
  - A link connects two successive joints



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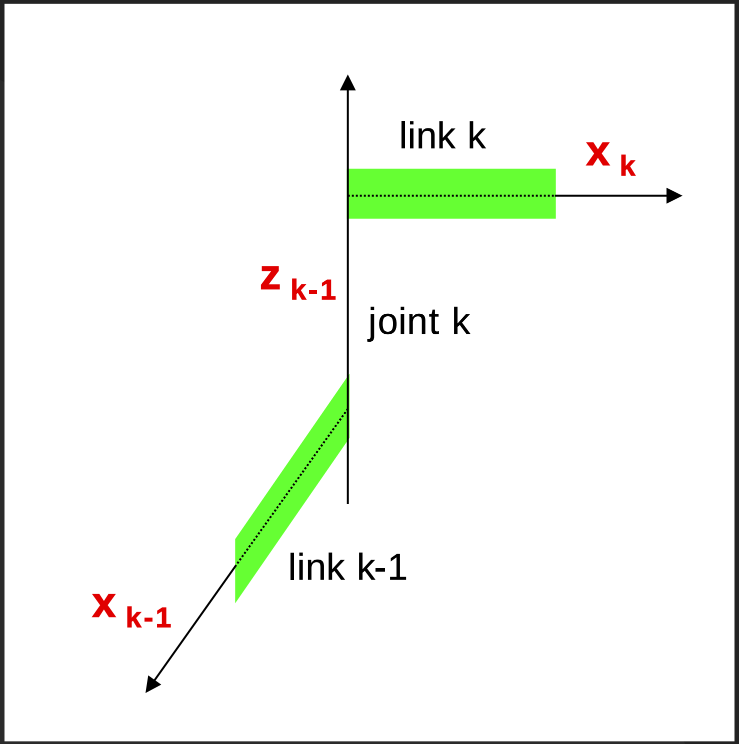
Joint Parameters

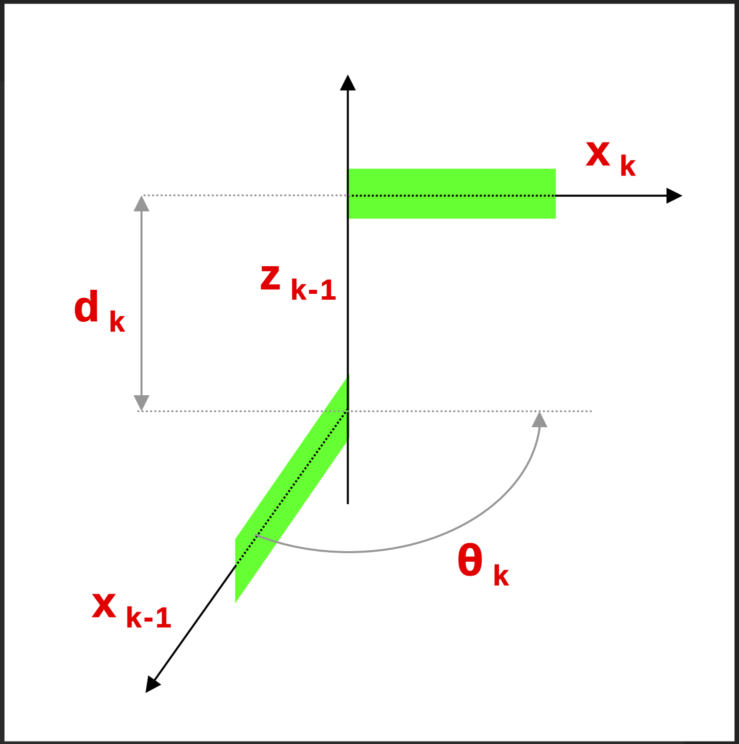
- Joint axis:
  - Revolute: rotation
  - Prismatic: slide
- Joint k connects link k-1 and link k.
- Frame axis z_{k-1} is aligned with joint axis k



Joint Parameters

- Joint angle θ_k: angle of rotation about z_{k-1} to make x_{k-1} parallel with x_k.

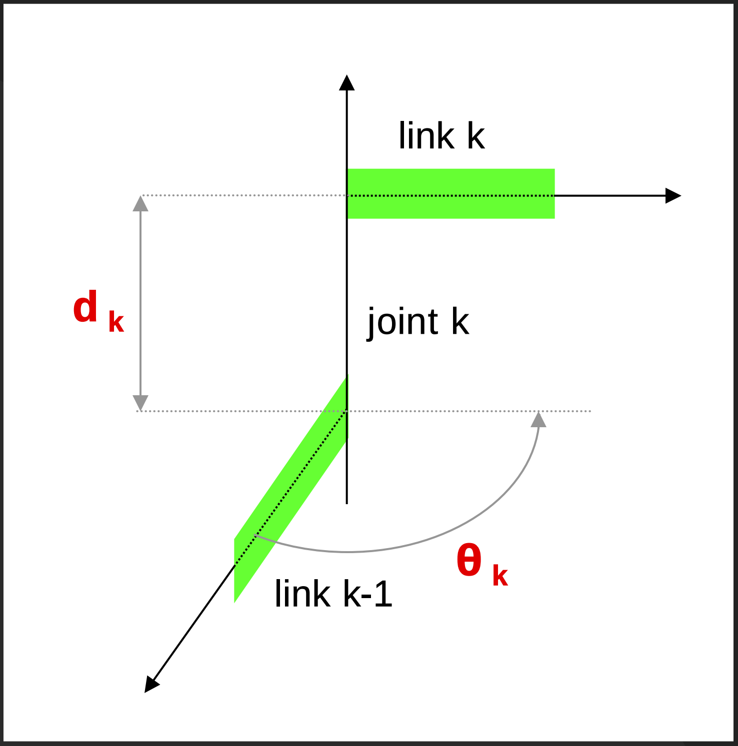
- Joint distance d_k: distance of translation along z_{k-1} to make x_{k-1} intersect x_k.



Joint Parameters

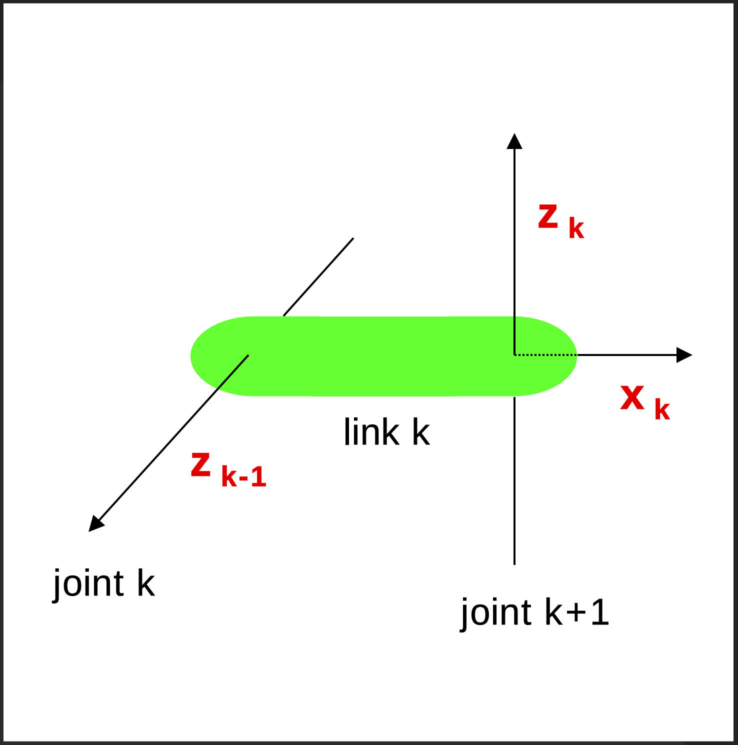
Parameters specify relative positions and orientations of two successive links.

- Revolute joint: θ_k variable and d_k fixed.
- Prismatic joint: d_k variable and θ_k fixed.



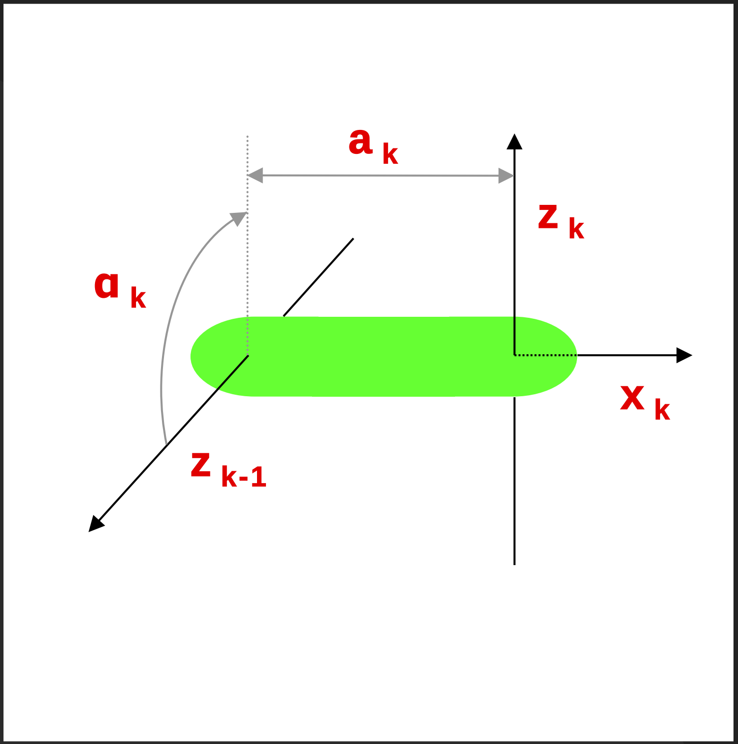
Link Parameters

- Link k connects joint k and joint k+1.



Link Parameters

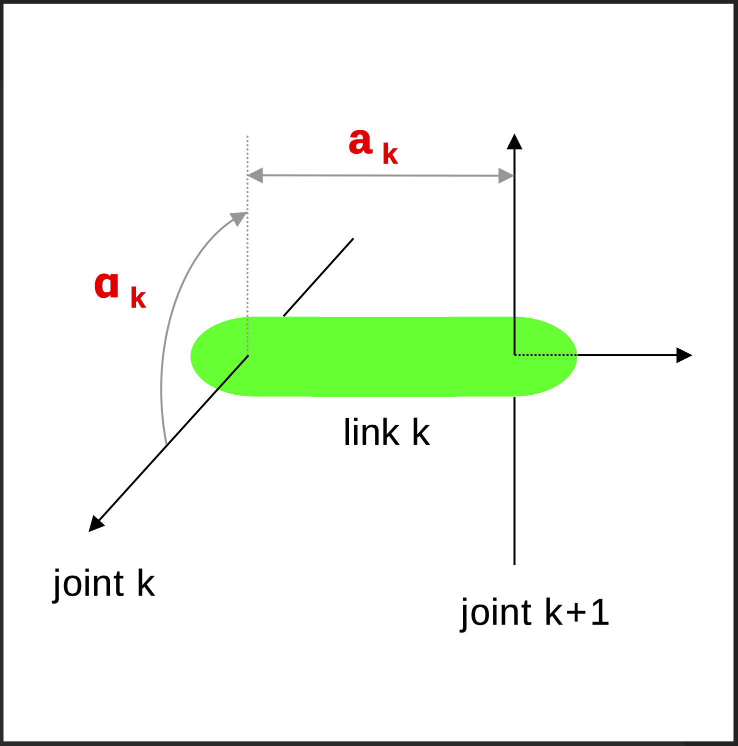
- Link twist angle α_k: angle of rotation about x_k to make z_{k-1} parallel with z_k.
- Link length a_k: distance of translation along x_k to make z_{k-1} intersect z_k.



Link Parameters

Parameters specify relative positions and orientations of two successive joints.

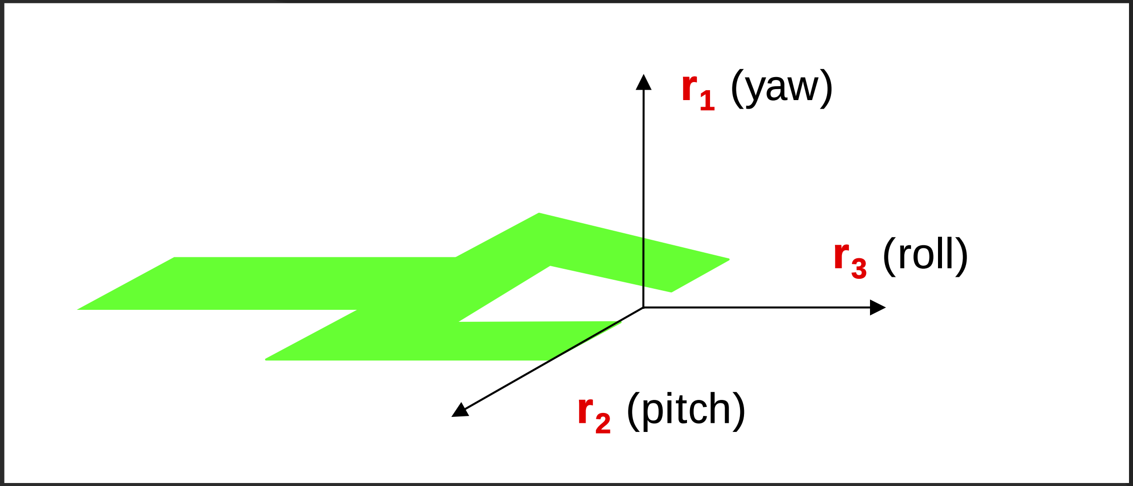
- Both α_k and a_k are fixed.



Robotic Arm

- n-Axis robot: 4n parameters
- Per axis:
  - 3 fixed parameters
  - 1 variable parameter: joint variable

Tool or Hand



- approach vector r_3 aligned with tool roll axis
- sliding vector r_2 aligned with open-close axis
- normal vector r_1

Denavit-Hartenberg Algorithm

- Assigns coordinate frames to links/joints and determines joint and link parameters in a systematic way to establish that all coordinate transformations have a standard form.

Denavit-Hartenberg Algorithm

Outline:

- Joint numbering.
- Assignment of base frame.
- Assignment of frames 1 through k-1.
- Assignment of tool frame.
- Computation of joint and link parameters.

Joint Numbering

- Number the joints from 1 to n starting with the base and ending with the tool yaw, pitch, and roll, in that order.

Assignment of Base Frame

- Assign a right-handed orthonormal coordinate frame L_0 to the robot base, making sure that z_0 aligns with the axis of joint 1.

Assignment of Frame 1 through k-1

for k=1 **to** n-1 **do**

- Align z_k with the axis of joint k+1
- Locate the origin of L_k at the intersection of the z_k and z_{k-1} axes. If they do not intersect, use the intersection of z_k with a common normal between z_k and z_{k-1}.
- Select x_k to be the orthogonal to both z_k and z_{k-1}. If z_k and z_{k-1} are parallel, point x_k away from z_{k-1}.
- Select y_k to form a right-handed orthonormal coordinate frame L_k.

Assignment of Tool Frame

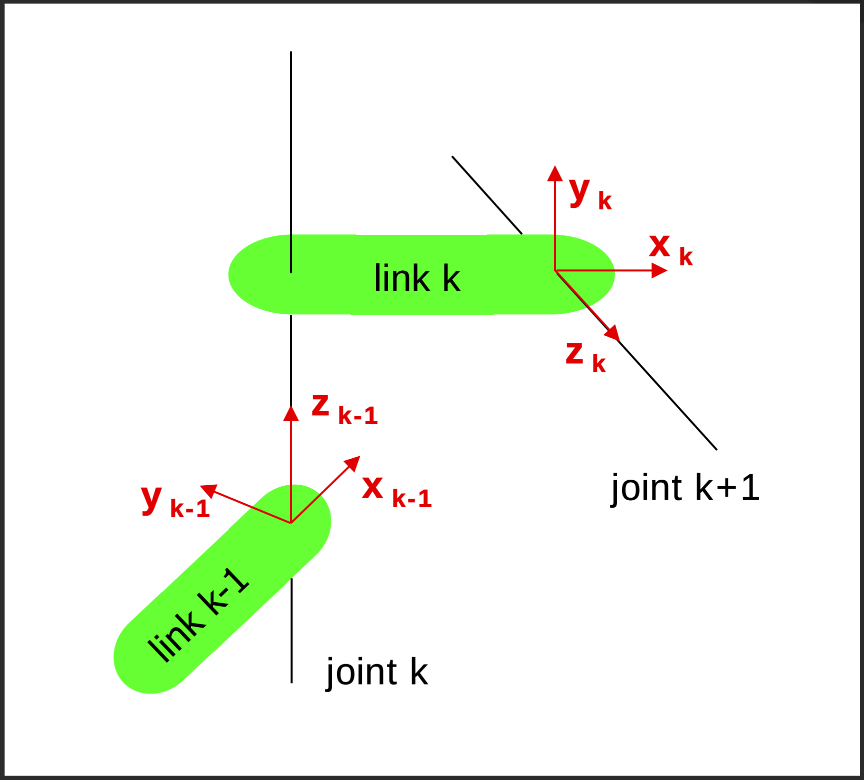
- Set the origin of L_n at the tool tip. Align z_n with the approach vector, y_n with the sliding vector, and x_n with the normal vector of the tool.

Computation of Parameters

for k=1 **to** n **do**

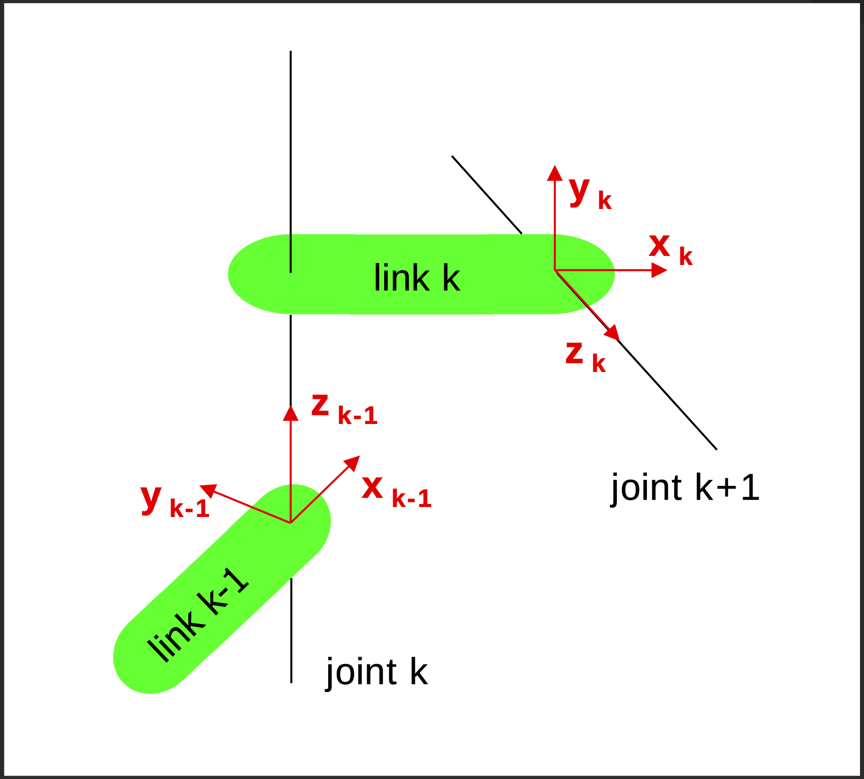
- Locate (auxiliary) point b_k at the intersection of the x_k and z_{k-1} axes. If they do not intersect, use the intersection of x_k with a common normal between x_k and z_{k-1}.
- Compute θ_k as the angle of rotation from x_{k-1} to x_k measured about z_{k-1}.
- Compute d_k as the distance from the origin of frame L_{k-1} to point b_k measured along z_{k-1}.
- Compute a_k as the distance from point b_k to the origin of frame L_k measured along x_k.
- Compute α_k as the angle of rotation from z_{k-1} to z_k measured about x_k.

Arm Equation



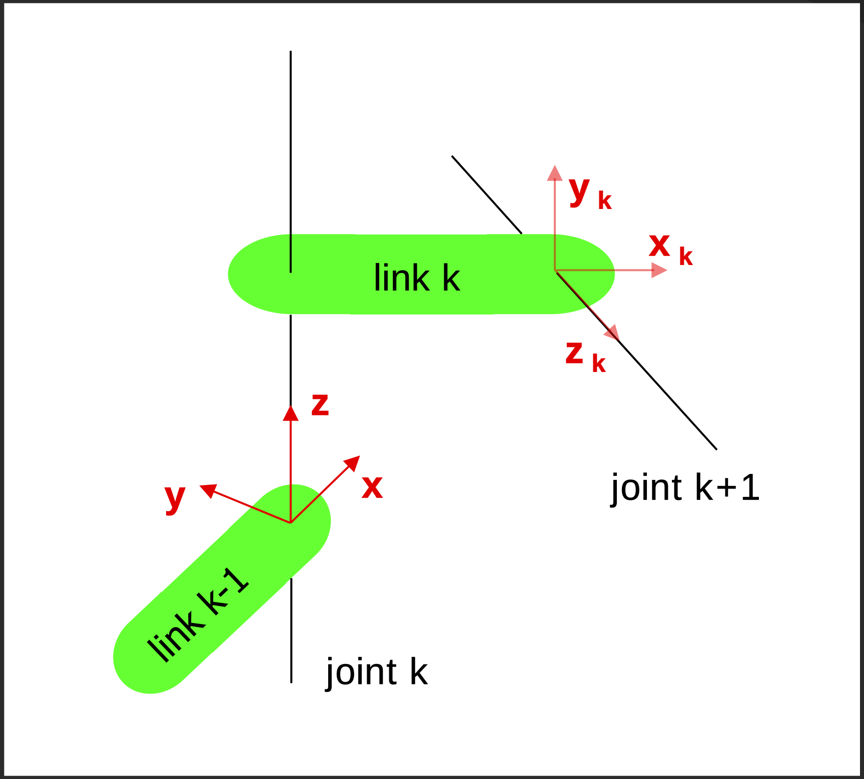
- Determine coordinate transformation from L_k into L_{k-1}.

Arm Equation



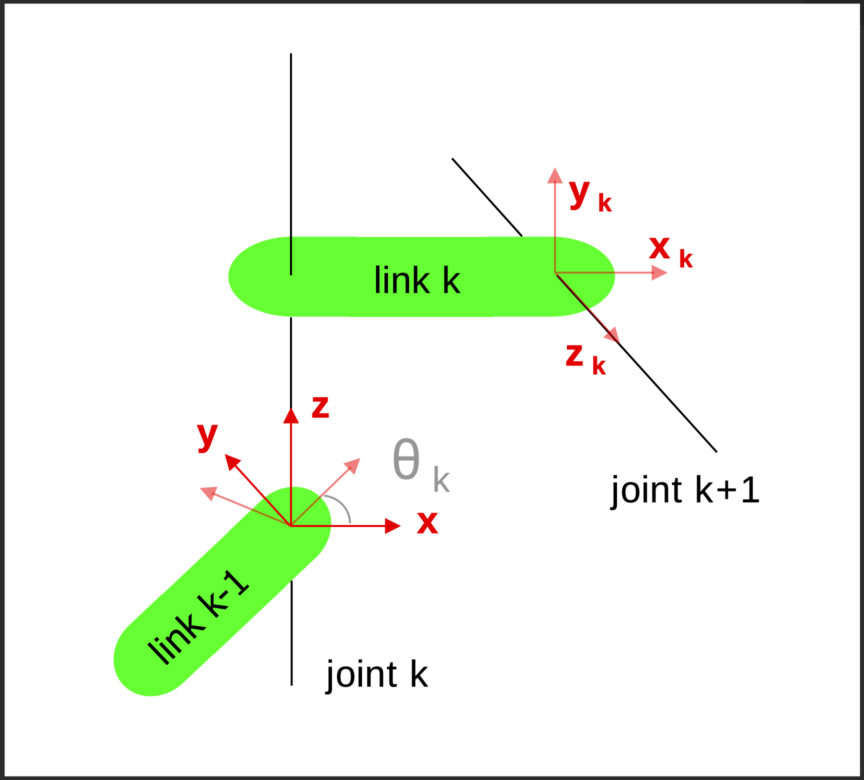
- Place moving frame M at L_{k-1} and move towards L_k. Maintain transformation.

Arm Equation



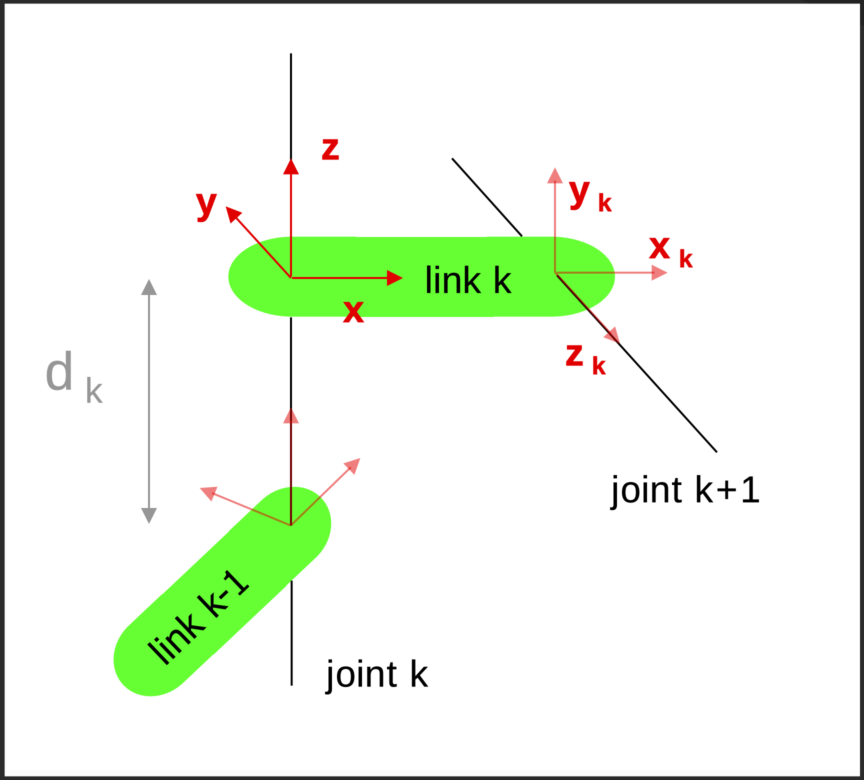
- M coincides with L_{k-1}.
- T:=I.

Arm Equation



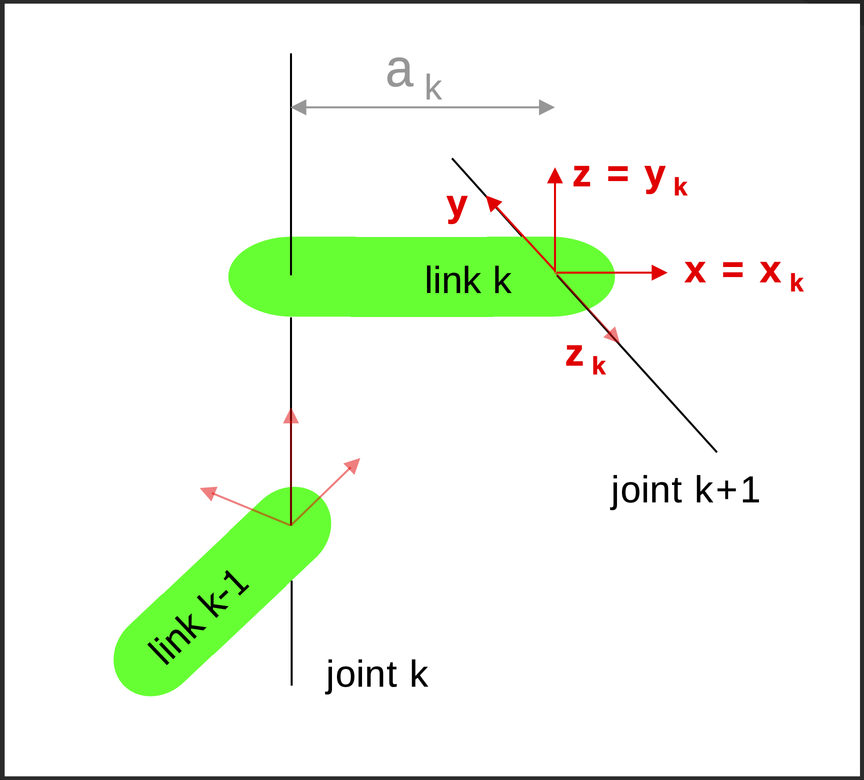
- Rotate M by θ_k about z.
- x-axis of M is now parallel with x_k.
- $T := T \odot Rot_3(\theta_k)$
  $= Rot_3(\theta_k)$

Arm Equation



- Translate M by d_k along z.
- x-axis of M is now collinear with x_k.
- $T := T \odot Tran_3(d_k e_3)$
  $= Rot_3(\theta_k) \odot Tran_3(d_k e_3)$

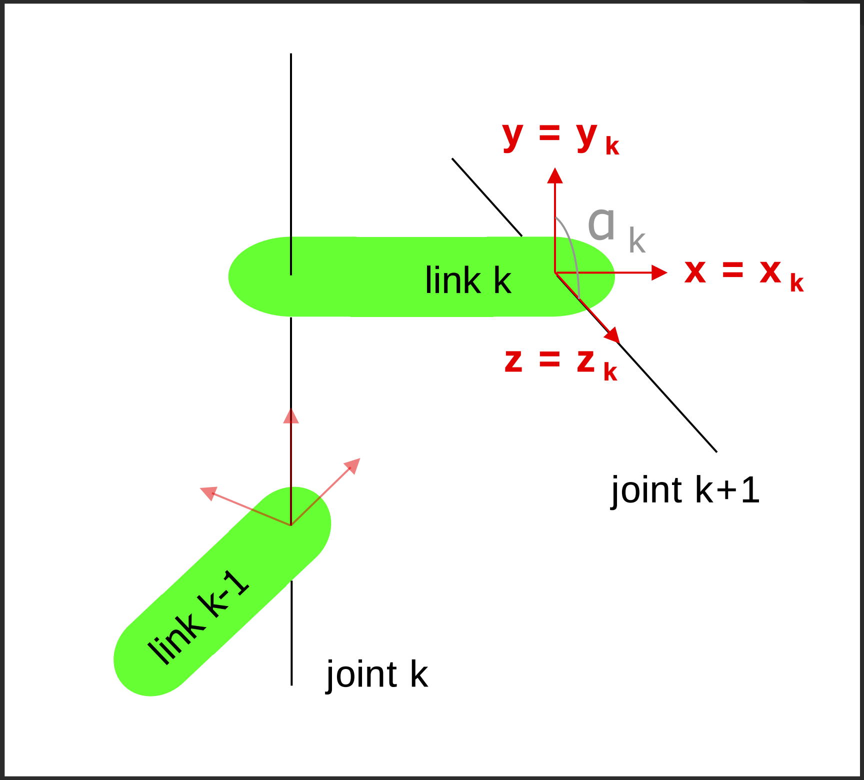
Arm Equation



- Translate M by a_k along x.
- x-axis of M is now coincides with x_k.
- $T := T \circ Tran_1(a_k e_1)$

  $= Rot_3(\theta_k) \circ Tran_3(d_k e_3) \circ Tran_1(a_k e_1)$

Arm Equation



- Rotate M by α_k about x.
- M now coincides with L_k.
- $T := T \odot Rot_1(\alpha_k)$

  $= Rot_3(\theta_k) \odot Tran_3(d_k\ e_3)$

  $\odot Tran_1(a_k\ e_1) \odot Rot_1(\alpha_k)$

Coordinate Transformation Matrix

$T(\theta_k, d_k, a_k, \alpha_k) =$

$Rot_3(\theta_k) \circ Tran_3(d_k\, e_3) \circ Tran_1(a_k\, e_1) \circ Rot_1(\alpha_k) =$

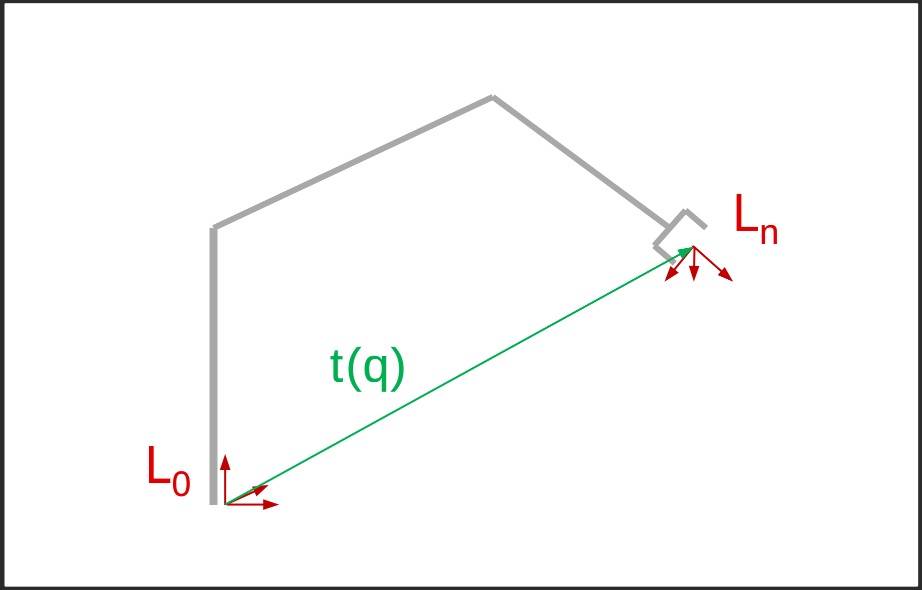
$Screw_3(\theta_k, d_k) \circ Screw_1(\alpha_k, a_k)$

maps coordinates with respect to L_k into coordinates with respect to L_{k-1}.

Kinematics for the Arm

- Joint variables $q = (q_1 \quad q_2 \quad q_3 \quad \cdots \quad q_n)^T$

$$ {}^0T_n(q) = {}^0T_1(q_1)\,{}^1T_2(q_2)\,{}^2T_3(q_3) \cdots {}^{n-1}T_n(q_n) = \begin{pmatrix} R(q) & t(q) \\ 0^T & 1 \end{pmatrix} $$



R(q) gives directions of the x-, y-, and z-axes of L_n with respect to the x-, y-, and z-axes of L_0