Motion and Manipulation

Immobilization based on motion analysis: form closure and caging
Grasping and Fixturing
Fixtures

- Bulky
- Complex
- Multilateral
- Dedicated
- Expensive
- Long Lead time
- Black Art
Fundamental Problems in Immobilization

- **Analysis**: Does a given placement of fingers immobilize a part?
- **Existence**: How many fingers are necessary or sufficient to immobilize a part?
- **Synthesis**: Determine one/some/all placement(s) of the fingers such that a part is immobilized.
Immobility Analysis

- Based on an analysis of velocities and/or motions (today)
- Based on an analysis of forces and moments (next lecture)
Immobilization

• Form Closure: Analysis of Instantaneous Velocities [1870s]

• Force Closure: Analysis of Forces and Moments [1970s]
  – 4 fingers sufficient for most 2D parts
  – 7 fingers sufficient for most 3D parts

• 2nd Order Immobility: Mobility Analysis in Configuration Space [1990s]
  – 3 fingers sufficient for most 2D parts
  – 4 fingers sufficient for most 3D parts
Kinematics

- Study of motion of objects leaving the causes out of consideration.

- Aim: specify placements (for example position and orientation) of robots or moving and movable objects in a game or a virtual environment. Here we focus on rigid objects.
Standard Approach

- (Rigidly) attach frame to moving object. Position and orientation of object specified by position and orientation of that (moving) frame with respect to fixed frame.

fixed (world) frame

coordinates of object with respect to moving frame are fixed.

moving (object) frame
Displacements and Rigid Bodies

• A displacement is a change of configuration that does not change the distance between any pair of points, nor does it change the handedness of the system.

• A rigid body is a system that is capable of displacements only.
Rotations and Translations

- A **rotation** is a displacement that leaves at least one point fixed.

- A **translation** is a displacement for which all points move equal distances along parallel lines.
For any displacement $D$ in Euclidean space and any point $O$, displacement $D$ can be expressed as the composition of a translation with a rotation about $O$.

- $D = T \circ R$, for some translation $T$ and rotation $R$
- $D = R' \circ T'$, for some rotation $R'$ and translation $T'$
Displacements: Summary

Basis of representation of placements:
• Any displacement in $\mathbb{R}^d$ is the composition of a translation and a rotation about a selected point $O$ (origin).

Here:
• Any displacement in $\mathbb{R}^2$ is a translation or a rotation about some point in the plane.

And then even:
• Any displacement in $\mathbb{R}^2$ is a rotation about some point in the projective plane.
Projective Plane

Projective plane $\mathbb{P}^2$
- Euclidean points
- Ideal points, at infinity

‘Formalization’ by lines through $O$ in $\mathbb{R}^3$
- Euclidean points $(x,y)$ represented by lines $(x,y,1)^T$
- Ideal points by lines parallel to $z=1$
Displacements

- A planar displacement is completely determined by the motion of any two points.
Displacements

Rotation about C.
Displacements

Translation along \( t \).
Displacements

Rotation about $C$!
Instantaneous Velocity Centers

So even:

- Any displacement in $\mathbb{R}^2$ is a rotation about some point (provided that points at infinity are incorporated).

- Velocity, direction of motion: completely determined by
  - rotation center, and
  - sign
    - $+$, or counterclockwise
    - $-$, or clockwise
Given

- polygonal part P with n vertices
- frictionless point fingers to immobilize the part
Instantaneous Velocity Centers

Analysis by Reuleaux, from 1876, in 2D only:
- based on first-order (velocity) considerations; fingers constrain possible directions of motion of the part.
Instantaneous Velocity Centers

- Form closure: no infinitesimal motions are possible.

Constraints imposed by contacts should exclude all motions.
Half-Plane Analysis

Consider all velocity centers

- Eliminate possible centers of clockwise rotation
- Eliminate possible centers of counterclockwise rotation
Instantaneous Velocity Centers

• Eliminate possible centers of clockwise rotation
Instantaneous Velocity Centers

- Eliminate possible centers of counterclockwise rotation
Form Closure Analysis

- No possible centers of clockwise rotation
- No possible centers of counterclockwise rotation

- Four point contacts are necessary and sufficient for form closure (in the plane)
Four Points Suffice

- Polygonal part without parallel edges

Largest enclosed circle touches the object along edges only
Four Points Suffice

- Polygonal part without parallel edges

Replace one point by two on either side of the original one
Four Points Suffice

General condition if two contacts are along the same edge:

Form closure if and only if

- the three normal directions positively span the plane
- the other two normals intersect inside the slab defined by the two parallel ones
Four Points Suffice

- Polygonal part without parallel edges

Largest enclosed circle touches the object in at least one vertex and neither of the edges incident to the vertex is tangent to the circle.
Four Points Suffice

- Polygonal part without parallel edges

Three points are sufficient
Four Points Suffice

- Polygonal part without parallel edges

Largest enclosed circle touches the object in at least one vertex and one of the edges incident to the vertex is tangent to the circle.
Four Points Suffice

- Polygonal part without parallel edges

Move the point on the tangent edge away from the vertex
Four Points Suffice

- Constructive proofs involving the vertices with maximum distance also exist. These also work for parts with parallel edges [Nguyen 1988]
Exercise

Analysis of a three-finger grasp
Exercise

Half-plane analysis uses sufficient, but not necessary condition
Immobilization

More precise analysis, taking into account curvature of part/finger motions (more than a century later)

• Czyzowicz, Stojmenovic, and Urrutia: intuitive geometric analysis, mostly for planar cases

• Rimon and Burdick: thorough general analysis in configuration space
Second-Order Considerations

- Analysis [Czyzowicz, Stojmenovic, Urrutia 1991] of mobility in the space of the part, taking into account curvature effects
  - 3 contacts for most polygons in 2D

Parallel edges: 4 contacts may be necessary
2\textsuperscript{nd} Order Immobility Grasp Analysis

Regard
- part as the robot,
- fingers as the obstacles.

Part is immobilized if and only if the current placement is an isolated point in free configuration space \cite{Rimon and Burdick 1995}
2nd Order Immobility Grasp Existence

- 3 points suffice to immobilize most 2D parts.
- 4 points suffice to immobilize most 3D parts.

Any misplacement of a finger generally destroys immobilization.
Imprecision and Form Closure Grasps

- 4 fingers sufficient for form closure in 2D

Sufficiently small misplacement of fingers along the part boundary is allowed, but no guaranteed lower bound on allowed perturbations.
2nd Order Immobility Grasps

- 3 points suffice to immobilize most 2D parts.
- 4 points suffice to immobilize most 3D parts.

Any misplacement of a finger generally destroys immobilization.
Caging

- Three-finger 2nd order immobility grasp

immobilizing grasp

misplaced fingers: no immobilization
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers
Caging

- Rigid motion of the fingers forces part to move along
Caging

- Fingers cage a part if there exists no motion that takes the part from its current placement to a remote placement without colliding with a finger.

If the current placement lies in a bounded component of free configuration space then the part is caged.
Relation to Motion Planning

• Analysis of a caging grasp: is there a path to a remote placement?

• Synthesis of caging grasps: determine all placements of fingers that prevent the existence of a path to a remote placements.
Relation to Motion Planning

- Analysis of a caging grasp: is there a path to a remote placement?
  (For the part or the fingers...!)

- Synthesis of caging grasps: determine all placements of fingers that prevent the existence of a path to a remote placements.
Relation to Motion Planning

• Analysis of a caging grasp: is there a path to a remote placement?

• Synthesis of caging grasps: determine all placements of fingers that prevent the existence of a path to a remote placement.

Caging is a very challenging problem!
Two-Finger Caging Grasps

- Given a polygon P with n vertices, find all placements of two point (or disk fingers) that cage P.

Set of grasps can be empty (even for non-convex P) but also have quadratic complexity (in n).

- Computation time: $O(n^2 \log n)$
Three-Finger Caging Grasps

- Given polygon \( P \) with \( n \) vertices and placements of two base fingers, determine the caging set = all placements of a third finger such that the three fingers cage \( P \).

- Computation time: \( O(n^6 \log^2 n)! \)

\( a \quad b \quad d \)

\( a \quad b \)