1. **Clustering coefficient:**

   (a) How would you compute the (local) clustering coefficient of a node? Give an efficient algorithm and analyze its running time.

   (b) How would you compute the global clustering coefficient of a graph? Give an efficient algorithm and analyze its running time.

   (c) Consider the following graph on \( N \) vertices numbered 0, \..., \( N - 1 \): node \( i \) and \( j \) are linked if and only if \( i \) and \( j \) differ by at most \( k \) (here we treat indices modulo \( k \)). Calculate the clustering coefficient of a node and the global clustering coefficient as a function of \( k \).

2. **Clustering coefficient II:** Consider \( G(N, p) \).

   (a) Show that for large \( N \), the expected number of triangles grows like \( \langle k \rangle^3 / 6 \).

   (b) Show that for large \( N \), the expected number of triplets grows like \( N \langle k \rangle^2 / 2 \).

   (c) What is the global clustering coefficient?

3. **Random graph:** Consider a random graph with \( N = 3000 \) and \( p = 10^{-3} \).

   (a) What is the expected number of links?

   (b) In which regime is this network?

   (c) Given this \( N \), what probability \( p \) would you need to choose such that the network is at its critical point?

   (d) Given this \( p \), how big would \( N \) need to be such that it is almost surely connected?

4. **Republicans and Democrats:** Consider a network of \( N \) Republicans and \( N \) Democrats. The probability that there is a link between two members of the same party is \( p \), whereas the probability of a link between members of a different party is \( q \). A network is polarized if \( p > q \).

   (a) Calculate the average degree of a member of the Republicans in the Republican subnetwork, and within the network as a whole.

   (b) What are the smallest values of \( p \) and \( q \) such that almost surely the network consists of a single component.

   (c) Argue that, even when \( p \) is much larger than \( q \), the network still exhibits the small-world property.

5. **Diameter:**

   (a) Give an algorithm that computes the diameter of a graph. If the graph has \( N \) vertices and \( L \) edges, then your algorithm should run in \( O(NL) \) time.

   (b) Give an algorithm that computes the diameter of a tree. If the tree has \( N \) vertices, then your algorithm should run in \( O(N) \) time.