1. **Clustering coefficient:**

   (a) How would you compute the (local) clustering coefficient of a node? Give an efficient algorithm and analyze its running time.
   
   **Trivial in** $O(N^2)$ **worst case.**

   (b) How would you compute the global clustering coefficient of a graph? Give an efficient algorithm and analyze its running time.
   
   **Trivial in** $O(N^3)$ **worst case.**

   (c) Consider the following graph on $N$ vertices numbered $0,\ldots,N-1$: node $i$ and $j$ are linked if and only if $i$ and $j$ differ by at most $k$ (here we treat indices modulo $N$). Calculate the clustering coefficient of a node and the global clustering coefficient as a function of $k$.
   
   **Local:** $\frac{3}{2} \cdot \frac{k-1}{2k-1} \approx \frac{3}{4}$.
   
   **Global:** $\frac{3\binom{N}{2}}{\binom{2k}{2}} \approx \frac{3}{4}$.

2. **Clustering coefficient II:** Consider $G(N,p)$.

   (a) Show that for large $N$, the expected number of triangles grows like $\langle k \rangle^3 / 6$.
   
   How many triangles can there potentially be in the whole graph? What is the probability they indeed turn into a triangle?

   (b) Show that for large $N$, the expected number of triplets grows like $N\langle k \rangle^2 / 2$.
   
   How many triplets can have their center on single a node? What is the probability they indeed turn into a triplet?

   (c) What is the global clustering coefficient?

3. **Random graph:** Consider a random graph with $N = 3000$ and $p = 10^{-3}$.

   (a) What is the expected number of links?
   
   $\approx 4500$

   (b) In which regime is this network?
   
   Supercritical.

   (c) Given this $N$, what probability $p$ would you need to choose such that the network is at its critical point?
   
   $\approx 1/3$

   (d) Given this $p$, how big would $N$ need to be such that it is almost surely connected?
   
   $N > 9118$

4. **Republicans and Democrats:** Consider a network of $N$ Republicans and $N$ Democrats. The probability that there is a link between two members of the same party is $p$, whereas the probability of a link between members of a different party is $q$. A network is polarized if $p > q$.

   (a) Calculate the average degree of a member of the Republicans in the Republican subnetwork, and within the network as a whole.
   
   $(N - 1)p$ **and** $(N - 1)p + Nq$. 

(b) What are the smallest values of \( p \) and \( q \) such that almost surely the network consists of a single component?

What is the probability needed so that Democrats and Republicans each form a single component almost surely? Then it suffices that there is at least one edge between Democrats and Republicans.

(c) Argue that, even when \( p \) is much larger than \( q \), the network still exhibits the small-world property.

Look at the answer for b. Democrats and Republicans each likely have small world. Again, it suffices that there is at least one edge between Democrats and Republicans.

5. Diameter:

(a) Give an algorithm that computes the diameter of a graph. If the graph has \( N \) vertices and \( L \) edges, then your algorithm should run in \( O(NL) \) time.

BFS from each node.

(b) Give an algorithm that computes the diameter of a tree. If the tree has \( N \) vertices, then your algorithm should run in \( O(N) \) time.

Iteratively remove leaves.