Lecture I: Basic Physics
Velocity

• Velocity: Instantaneous change in position \( \vec{v} = \frac{d\vec{x}}{dt} \)

• Suppose object position \( \vec{x}_o \) and constant velocity \( \vec{v} \). After time step \( \Delta t \):
  • \( \vec{x}_o(t + \Delta t) = \vec{x}_o(t) + \vec{v}\Delta t \)
  • \( \Delta \vec{x}_o = \vec{x}_o(t + \Delta t) - \vec{x}_o(t) = \vec{v}\Delta t \).

• \( \vec{v} \) is never constant in practice
  • A function of time \( \vec{v}(t) \).
  • Position is integrated in time: \( \vec{x}_o(t) = \vec{x}_o + \int_0^t \vec{v}(s) \, ds \).

• Velocity SI units: \( \frac{m}{sec} \)
Acceleration

- **Instantaneous** change in velocity: \( \vec{a} = \frac{d\vec{v}}{dt} \).
  - Constant acceleration: \( \Delta \vec{v} = \vec{a} \Delta t \).
  - Otherwise, integrate: \( v(t) = V + \int_{0}^{t} a(s) \, ds \).

- **Note vector quantities!**
  - Position: trajectory of a point.
  - Velocity: tangent to trajectory curve.
    - Speed: absolute value of velocity.
  - Acceleration: the change in the tangent.
Relative Quantities

• Using coordinates, our vector quantities are relative to the chosen axis system (origin + xyz direction)
• They are viewpoint dependent.
• The derivation/integration relations are invariant!
Forces

• Acceleration is induced by a force.

• Direction of force = direction of associated acceleration.

• Net force (and net acceleration): the sum of all acting forces.
Newton’s laws of motion

• In the late 17\textsuperscript{th} century, Sir Isaac Newton described three laws that govern all motion on Earth.

• ...ultimately, an approximation
  • Small scale: quantum mechanics.
  • Big scale: theories of relativity.
1st Law of Motion

• Sum of forces on an object is null ⇔ there is no change in the motion

\[ F_{net} = 0, \text{there is no change in motion} \]

• With zero force sum:
  • An object at rest stays at rest.
  • A moving object perpetuates in the same velocity.

• Behavior of objects in the outer space.
2\textsuperscript{nd} Law of Motion

- Each force induces a co-directional acceleration in linear to the mass of the object:

\[
\vec{F}_{\text{net}} = m \cdot \vec{a}
\]

- Consequently:
  - More force \(\Leftrightarrow\) faster speed-up.
  - Same force \(\Leftrightarrow\) lighter objects accelerate faster than heavy objects.
3rd Law of Motion

- Forces have consequences:

\[
\text{When two objects come into contact, they exert equal and opposite forces upon each other.}
\]

- All forces are actually \textit{interactions} between bodies!

What happens here?
Gravity

- **Newton’s Law of Gravitation:** the gravitation force between two masses $A$ and $B$ is:

\[
\vec{F}_g = \vec{F}_{A\rightarrow B} = -\vec{F}_{B\rightarrow A} = G \frac{m_A m_B}{r^2} \overrightarrow{u}_{AB}
\]

$G$: gravitational constant $6.673 \times 10^{-11} \ [m^3 kg^{-1} s^{-2}]$.

$r = |\vec{p}_A - \vec{p}_B|$: the distance between the objects.

$\overrightarrow{u}_{AB} = \frac{\vec{p}_A - \vec{p}_B}{|\vec{p}_A - \vec{p}_B|}$: the unit direction between them.
Gravity on Earth

• By applying Newton’s 2\textsuperscript{nd} law to an object with mass \(m\) on the surface of the Earth, we obtain:

\[
\vec{F}_{\text{net}} = \vec{F}_g = m \cdot \vec{a}
\]

\[
G \frac{m \cdot m_{\text{Earth}}}{r_{\text{Earth}}^2} = m \cdot a
\]

\[
G \frac{m_{\text{Earth}}}{r_{\text{Earth}}^2} = a \quad \text{Mass of object is canceled out!}
\]

\[
a = g_{\text{Earth}} = 6.673 \times 10^{-11} \frac{5.98 \times 10^{24}}{(6.377 \times 10^6)^2} \approx 9.81 \text{ m/s}^2
\]
Gravity on Other Planets

- On Earth at altitude $h$: $a = G \frac{m_{\text{Earth}}}{(r_{\text{Earth}} + h)^2}$

- On the Moon
  - $m_{\text{moon}} = 7.35 \times 10^{22} \text{ kg}$
  - $r_{\text{moon}} = 1738 \text{ km}$
  - $g_{\text{moon}} = 1.62 \text{ m/s}^2$

- On Mars
  - $m_{\text{mars}} = 6.42 \times 10^{23} \text{ kg}$
  - $r_{\text{mars}} = 3403 \text{ km}$
  - $g_{\text{mars}} = 3.69 \text{ m/s}^2$
Weight

• Weight $\Leftrightarrow$ gravitational force

\[
\vec{W} = m \cdot \vec{g}
\]

• We weigh different on the moon (but have the same mass...)

• Force units: $[kg \cdot \frac{m}{sec^2}]$.
  • Denoted as Newtons $[N]$. 
Free-Body Diagram

• To get acceleration: sum forces and divide by mass (D'Alembert's principle):

\[ \vec{F}_{net} = \sum \vec{F}_i = m \cdot \vec{a} \]

• Forces add up linearly as vectors.
  • Important: when all are represented in the same axis system!

• The Free-Body Diagram includes:
  • Object shape: center of mass, contact points.
  • Applied forces: direction, magnitude, and point of application.

https://www2.southeastern.edu/Academic/Faculty/rallain/plab193/files/a8312fbf3bde4804309096169ad22bd5-46.html
Normal force

• Force acting as a reaction to contact.
  • Direction is normal to the surface of contact.
  • **Magnitude** enough to cancel the weight so object doesn’t go through the plane.

![Normal force diagram](image)

• Here, $F_N = \vec{W} \cos(\alpha) = m\vec{g} \cos(\theta)$
• Related to **constraints and collision handling** (more later).
• Object slides down plane with the remainder force: $\vec{W} \sin(\theta)$. 
Friction

• Can the object stay in total equilibrium?
  • An extra tangential friction force must cancel \( \vec{W} \sin(\theta) \).

• Ability to resist movement.
  • Static friction keeps an object on a surface from moving.
  • Kinetic friction slows down an object in contact.
Friction

• **Static friction**: a threshold force.
  • object will not move unless tangential force is stronger.

• **Kinetic friction**: when the object is moving.

• Depends on the materials in contact.
  • smoother $\leftrightarrow$ less friction.

• **Coefficient of friction** $\mu$ determines friction forces:
  • Static friction: $F_s = \mu_s F_N$
  • Kinetic friction: $F_k = \mu_k F_N$
Friction

• The kinetic coefficient of friction is always smaller than the static friction.

• If the tangential force is larger than the static friction, the object moves.

• If the object moves while in contact, the kinetic friction is applied to the object.
### Friction

<table>
<thead>
<tr>
<th>Surface Friction</th>
<th>Static ($\mu_s$)</th>
<th>Kinetic ($\mu_k$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel on steel (dry)</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Steel on steel (greasy)</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>Teflon on steel</td>
<td>0.041</td>
<td>0.04</td>
</tr>
<tr>
<td>Brake lining on cast iron</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Rubber on concrete (dry)</td>
<td>1.0</td>
<td>0.9</td>
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<tr>
<td>Rubber on concrete (wet)</td>
<td>0.30</td>
<td>0.25</td>
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<tr>
<td>Metal on ice</td>
<td>0.022</td>
<td>0.02</td>
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<tr>
<td>Steel on steel</td>
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<td>0.57</td>
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<tr>
<td>Aluminum on steel</td>
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<td>0.47</td>
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<tr>
<td>Copper on steel</td>
<td>0.53</td>
<td>0.36</td>
</tr>
<tr>
<td>Nickel on nickel</td>
<td>1.1</td>
<td>0.53</td>
</tr>
<tr>
<td>Glass on glass</td>
<td>0.94</td>
<td>0.40</td>
</tr>
<tr>
<td>Copper on glass</td>
<td>0.68</td>
<td>0.53</td>
</tr>
</tbody>
</table>
Fluid resistance

- An object moving in a fluid (air is a fluid) is slowed down by this fluid.

- This is called **fluid resistance**, or **drag**, and depends on several parameters, e.g.:
  - High velocity $\Leftrightarrow$ larger resistance.
  - More surface area $\Leftrightarrow$ larger resistance (“bad aerodynamics”).

![Diagram of fluid resistance](image-url)
Fluid resistance

- At high velocity, the drag force $F_{D_{\text{high}}}$ is quadratic to the relative speed $\nu$ of the object:

$$F_{D_{\text{high}}} = -\frac{1}{2} \cdot \rho \cdot \nu^2 \cdot C_d \cdot A$$

- $\rho$ is the density of the fluid (1.204 for air at 20°C)
- $C_d$ is the drag coefficient (depends on the shape of the object).
- $A$ is the reference area (area of the projection of the exposed shape).
Fluid resistance

• At low velocity, the drag force is approximately linearly proportional to the velocity

\[ \overrightarrow{F_{D_{low}}} \approx -b \cdot \overrightarrow{v} \]

where \( b \) depends on the properties of the fluid and the shape of the object.

• High/low velocity threshold is defined by Reynolds Number (Re).
Buoyancy

• Develops when an object is immersed in a fluid.
• A function of the volume of the object $V$ and the density of the fluid $\rho$:
  \[ F_B = \rho \cdot g \cdot V \]
• Considers the difference of pressure above and below the immersed object.
• Directed straight up, counteracting the weight.
Springs

• React according to *Hook’s Law* on extension and compression, *i.e.* on the relative displacement.

• The relative length \( l \) to the rest length \( l_0 \) determines the applied force:

\[
F_k = -K(l - l_0)
\]

• \( K \) is the *spring constant* (in \( N/m \)).
• Scalar spring: two directions.
Dampers

- Without interference, objects may oscillate infinitely.
- **Dampers** slow down the oscillation between objects \( A \) and \( B \) connected by a spring.
- Opposite to the relative speed between the two objects:
  \[
  \vec{F}_C = -C(\vec{v}_A - \vec{v}_B)
  \]
  - \( C \) is the damping coefficient.
  - Resulting force applied on \( A \) (opposite on \( B \)).
- Similar to friction or drag at low velocity!
Work

• A force $\vec{F}$ does work $W$ (in Joule = $N \cdot m$), if it achieves a displacement $\Delta \vec{x}$ in the direction of the displacement:

$$W = \vec{F} \cdot \Delta \vec{x}$$

• Note dot product between vectors.
• Scalar quantity.
Kinetic energy

- The kinetic energy $E_K$ is the energy of an object in velocity:

\[ E_K = \frac{1}{2} m |\vec{v}|^2 \]

- The faster the object is moving, the more energy it has.

- The energy is a scalar (relative to speed $\nu = |\vec{v}|$, regardless of direction).

- Unit is also Joule:

\[ kg(m/sec)^2 = \left( kg \times \frac{m}{sec^2} \right) m = N \times m = J \]
Work-Energy theorem

• The Work-Energy theorem: net work \( \Leftrightarrow \) change in kinetic energy:

\[
W = \Delta E_K = E_K(t + \Delta t) - E_K(t)
\]

\[\text{i.e.} \]

\[
\vec{F} \cdot \Delta \vec{x} = \frac{1}{2} m (\nu(t + \Delta t)^2 - \nu(t)^2)
\]

• Very similar to Newton’s second law...
Potential energy

• (Gravitational) **Potential energy** is the energy ‘stored’ in an object due to **relative** height difference.
  
  • The amount of work that would be done if we were to set it free.

\[
E_P = m \cdot g \cdot h
\]

• Simple product of the weight \( W = m \cdot g \) and height \( h \).
• Also measured in Joules (as here \( kg \cdot \frac{m}{sec^2} \cdot m \)).

• Other potential energies exist (like a compressed spring).
Conservation of mechanical energy

- **Law of conservation:** in a closed system, energy cannot be created or destroyed.
  - Energy may switch form.
  - May transfer between objects.
  - Classical example: falling trades potential and kinetic energies.

\[
E_K(t + \Delta t) + E_P(t + \Delta t) = E_K(t) + E_P(t)
\]

\[
i.e.
\frac{1}{2}mv(t + \Delta t)^2 + mgh(t + \Delta t) = \frac{1}{2}mv(t)^2 + mgh(t)
\]
Conservation: Example

- A roller-coaster cart at the top of the first hill.
  - Much potential energy, but only a little kinetic energy.
  - Going down the drop: losing height, picking up speed.
  - At the bottom: almost all potential energy switched to kinetic, cart is at its maximum speed.
Conservation of Mechanical Energy

- External forces are usually applied:
  - Friction and air resistance.
  - Where does the “reduced” energy go?
  - Converted into heat and air displacements (sound waves, wind).
- We compensate by adding an extra term $E_O$ to the conservation equation:
  \[ E_K(t + \Delta t) + E_P(t + \Delta t) + E_O = E_K(t) + E_P(t) \]
- if $E_O > 0$, some energy is ‘lost’.

https://i.ytimg.com/vi/anb2c4Rm27E/maxresdefault.jpg
Momentum

• The **linear momentum** $\vec{p}$: the mass of an object multiplied by its velocity:

$$\vec{p} = m \cdot \vec{v}$$

• Heavier object/higher velocity $\iff$ more momentum (more difficult to stop).

• unit is $[kg \cdot m/sec]$.

• **Vector** quantity (velocity).
Impulse

• A change of momentum:
  \[ \vec{J} = \Delta \vec{p} \]

• Compare:
  • **Impulse** is change in **momentum**.
  • **Work** is change in **energy**.

• Unit is also \([\text{kg} \cdot \text{m/s}]\) (like momentum).

• Impulse \(\Leftrightarrow\) force integrated over time:
  \[ \vec{J} = \int_0^t \vec{F} \, dt = m \int_0^t \vec{a} \, dt = m \Delta \vec{v}. \]
Conservation of Momentum

• **Law of conservation:** in a closed system (no external forces/impulses), momentum **cannot** be created or destroyed.

• **Compare:** conservation of energy.

• Implied from 3\(^{rd}\) law.
  • Objects react with the same force exerted on them.

• Special case of Noether’s theorem: every physical system (With a symmetric action) has a conservation law.