Lecture I: Basic Physics
Velocity

• Velocity: Instantaneous change in position \( \vec{v} = \frac{d\vec{x}}{dt} \)

• Suppose object position \( \vec{x}_o \) and constant velocity \( \vec{v} \). After time step \( \Delta t \):
  - \( \vec{x}_o(t + \Delta t) = \vec{x}_o(t) + \vec{v}\Delta t \)
  - \( \Delta\vec{x}_o = \vec{x}_o(t + \Delta t) - \vec{x}_o(t) = \vec{v}\Delta t \).

• \( \vec{v} \) is never constant in practice
  - A function of time \( \vec{v}(t) \).
  - Position is integrated in time: \( \vec{x}_o(t) = \vec{x}_o + \int_0^t \vec{v}(s) \, ds \).

• Velocity SI units: \( m/sec \)
Acceleration

• Instantaneous change in velocity: \( \ddot{a} = \frac{d\vec{v}}{dt} \).
  • Constant acceleration: \( \Delta \vec{v} = \dot{a} \Delta t \).
  • Otherwise, integrate: \( \vec{v}(t) = \vec{v}_0 + \int_0^t \dot{a}(s) \, ds \).

• Note vector quantities!
  • Position: trajectory of a point.
  • Velocity: tangent to trajectory curve.
    • Speed: absolute value of velocity.
  • Acceleration: the change in the tangent.
Relative Quantities

- Using coordinates, our vector quantities are relative to the chosen axis system (origin + xyz direction).
- They are viewpoint dependent.
- The derivation/integration relations are invariant!
Forces

- Acceleration is induced by a force.

- Direction of force = direction of associated acceleration.

- Net force (and net acceleration): the sum of all acting forces.
Newton’s laws of motion

• In the late 17\textsuperscript{th} century, Sir Isaac Newton described three laws that govern all motion on Earth.

• ...ultimately, an approximation
  • \textbf{Small scale}: quantum mechanics.
  • \textbf{Big scale}: theories of relativity.
1st Law of Motion

• Sum of forces on an object is null ⇔ there is no change in the motion

\[ If \ F_{net} = 0, \ there \ is \ no \ change \ in \ motion \]

• With zero force sum:
  • An object at rest stays at rest.
  • A moving object perpetuates in the same velocity.

• Behavior of objects in the outer space.
2\textsuperscript{nd} Law of Motion

• Each force induces a co-directional acceleration in \textit{linear} to the mass of the object:

\[
\vec{F}_{\text{net}} = m \cdot \ddot{a}
\]

\textit{m} \textit{is the mass and} \ddot{a} \textit{the acceleration}

• Consequently:
  • More force \iff faster speed-up.
  • Same force \iff lighter objects accelerate faster than heavy objects.
3\textsuperscript{rd} Law of Motion

• Forces have consequences:

When two objects come into contact, they exert equal and opposite forces upon each other.

• All forces are actually \textit{interactions} between bodies!

What happens here?
Gravity

- **Newton’s Law of Gravitation**: the gravitation force between two masses $A$ and $B$ is:

\[
\vec{F}_g = \vec{F}_{A\rightarrow B} = -\vec{F}_{B\rightarrow A} = G \frac{m_A m_B}{r^2} \overrightarrow{u}_{AB}
\]

- $G$: gravitational constant $6.673 \times 10^{-11} \ [m^3 \text{kg}^{-1} \text{s}^{-2}]$.
- $r = |\vec{p}_A - \vec{p}_B|$: the distance between the objects.
- $\overrightarrow{u}_{AB} = \frac{\vec{p}_A - \vec{p}_B}{|\vec{p}_A - \vec{p}_B|}$: the unit direction between them.
Gravity on Earth

By applying Newton’s 2\textsuperscript{nd} law to an object with mass $m$ on the surface of the Earth, we obtain:

$$\vec{F}_{net} = \vec{F}_g = m \cdot \vec{a}$$

$$G \frac{m \cdot m_{\text{Earth}}}{r_{\text{Earth}}^2} = m \cdot a$$

$$G \frac{m_{\text{Earth}}}{r_{\text{Earth}}^2} = a \quad \text{Mass of object is canceled out!}$$

$$a = g_{\text{Earth}} = \frac{5.98 \times 10^{24}}{6.673 \times 10^{-11} \left(6.377 \times 10^6\right)^2} \approx 9.81 \text{ m/s}^2$$

http://lannyland.blogspot.co.at/2012/12/10-famous-thought-experiments-that-just.html
Gravity on Other Planets

• On Earth at altitude $h$: $a = G \frac{m_{\text{Earth}}}{(r_{\text{Earth}} + h)^2}$

• On the Moon
  • $m_{\text{moon}} = 7.35 \times 10^{22} \text{ kg}$
  • $r_{\text{moon}} = 1738 \text{ km}$
  • $g_{\text{moon}} = 1.62 \text{ m/s}^2$

• On Mars
  • $m_{\text{mars}} = 6.42 \times 10^{23} \text{ kg}$
  • $r_{\text{mars}} = 3403 \text{ km}$
  • $g_{\text{mars}} = 3.69 \text{ m/s}^2$
Weight

• Weight $\Leftrightarrow$ gravitational force

$$\vec{W} = m \cdot \vec{g}$$

• We weigh different on the moon (but have the same mass...)

• Force units: $[kg \cdot \frac{m}{sec^2}]$.
  • Denoted as **Newtons** [$N$].
Free-Body Diagram

• To get acceleration: sum forces and divide by mass (D'Alembert's principle):

\[ \vec{F}_{net} = \sum \vec{F}_i = m \cdot \vec{a} \]

• Forces add up linearly as vectors.
  • Important: when all are represented in the same axis system!

• The Free-Body Diagram includes:
  • Object shape: center of mass, contact points.
  • Applied forces: direction, magnitude, and point of application.
Normal force

- Force acting as a **reaction** to contact.
  - Direction is **normal** to the surface of contact.
  - **Magnitude** enough to cancel the weight so object doesn’t go through the plane.

Here, $\vec{F}_N = \vec{W} \cos(\alpha) = m\vec{g} \cos(\theta)$
- Related to **constraints and collision handling** (more later).
- Object slides down plane with the remainder force: $\vec{W} \sin(\theta)$. 
Friction

• Can the object stay in total equilibrium?
  • An extra tangential friction force must cancel $\vec{W} \sin(\theta)$.

• Ability to resist movement.
  • Static friction keeps an object on a surface from moving.
  • Kinetic friction slows down an object in contact.
Friction

• **Static friction**: a threshold force.
  • object will not move unless tangential force is **stronger**.

• **Kinetic friction**: when the object is moving.

• Depends on the materials in contact.
  • smoother $\iff$ less friction.

• **Coefficient of friction** $\mu$ determines friction forces:
  • Static friction: $F_s = \mu_s F_N$
  • Kinetic friction: $F_k = \mu_k F_N$
Friction

• The kinetic coefficient of friction is always smaller than the static friction.

• If the tangential force is larger than the static friction, the object moves.

• If the object moves while in contact, the kinetic friction is applied to the object.
# Friction

<table>
<thead>
<tr>
<th>Surface Friction</th>
<th>Static ($\mu_s$)</th>
<th>Kinetic ($\mu_k$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel on steel (dry)</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Steel on steel (greasy)</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>Teflon on steel</td>
<td>0.041</td>
<td>0.04</td>
</tr>
<tr>
<td>Brake lining on cast iron</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Rubber on concrete (dry)</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Rubber on concrete (wet)</td>
<td>0.30</td>
<td>0.25</td>
</tr>
<tr>
<td>Metal on ice</td>
<td>0.022</td>
<td>0.02</td>
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<tr>
<td>Steel on steel</td>
<td>0.74</td>
<td>0.57</td>
</tr>
<tr>
<td>Aluminum on steel</td>
<td>0.61</td>
<td>0.47</td>
</tr>
<tr>
<td>Copper on steel</td>
<td>0.53</td>
<td>0.36</td>
</tr>
<tr>
<td>Nickel on nickel</td>
<td>1.1</td>
<td>0.53</td>
</tr>
<tr>
<td>Glass on glass</td>
<td>0.94</td>
<td>0.40</td>
</tr>
<tr>
<td>Copper on glass</td>
<td>0.68</td>
<td>0.53</td>
</tr>
</tbody>
</table>
Fluid resistance

- An object moving in a fluid (air is a fluid) is slowed down by this fluid.
- This is called **fluid resistance**, or **drag**, and depends on several parameters, e.g.:
  - High velocity $\Leftrightarrow$ larger resistance.
  - More surface area $\Leftrightarrow$ larger resistance (“bad aerodynamics”).
Fluid resistance

• At high velocity, the drag force $F_{D_{high}}$ is **quadratic** to the relative speed $\nu$ of the object:

$$F_{D_{high}} = -\frac{1}{2} \cdot \rho \cdot \nu^2 \cdot C_d \cdot A$$

• $\rho$ is the **density** of the fluid (1.204 for air at 20°C)
• $C_d$ is the **drag coefficient** (depends on the shape of the object).
• $A$ is the **reference area** (area of the projection of the exposed shape).
Fluid resistance

• At low velocity, the drag force is approximately linearly proportional to the velocity

\[ \overrightarrow{F_{D_{low}}} \approx -b \cdot \hat{v} \]

where \( b \) depends on the properties of the fluid and the shape of the object.

• High/low velocity threshold is defined by Reynolds Number (Re).
Buoyancy

- Develops when an object is immersed in a fluid.
- A function of the volume of the object \( V \) and the density of the fluid \( \rho \):
  \[
  F_B = \rho \cdot g \cdot V
  \]
- Considers the difference of pressure above and below the immersed object.
- Directed straight up, counteracting the weight.
Springs

• React according to *Hook’s Law* on extension and compression, *i.e.* on the relative displacement.

• The relative length $l$ to the *rest length* $l_0$ determines the applied force:

$$F_k = -K(l - l_0)$$

• $K$ is the *spring constant* (in $N/m$).

• Scalar spring: two directions.
Dampers

• Without interference, objects may oscillate infinitely.
• Dampers slow down the oscillation between objects $A$ and $B$ connected by a spring.
• Opposite to the relative speed between the two objects:
  \[ \vec{F}_C = -C(\vec{v}_A - \vec{v}_B) \]
  • $C$ is the damping coefficient.
  • Resulting force applied on $A$ (opposite on $B$).
• Similar to friction or drag at low velocity!
Work

• A force \( \vec{F} \) does work \( W \) (in \( \text{Joule} = N \cdot m \)), if it achieves a displacement \( \Delta \vec{x} \) in the direction of the displacement:

\[
W = \vec{F} \cdot \Delta \vec{x}
\]

• Note dot product between vectors.
• Scalar quantity.
Kinetic energy

- The kinetic energy $E_K$ is the energy of an object in velocity:

\[ E_K = \frac{1}{2} \ m \ |\vec{v}|^2 \]

- The faster the object is moving, the more energy it has.
- The energy is a scalar (relative to speed $\nu = |\vec{v}|$, regardless of direction).
- Unit is also Joule:

\[ kg(m/sec)^2 = \left( kg \ast \frac{m}{sec^2} \right) m = N \ast m = J \]
Work-Energy theorem

• The Work-Energy theorem: net work \(\Leftrightarrow\) change in kinetic energy:

\[
W = \Delta E_K = E_K(t + \Delta t) - E_K(t)
\]

\[
i.e. \quad \vec{F} \cdot \Delta \vec{x} = \frac{1}{2} m(v(t + \Delta t)^2 - v(t)^2)
\]

• Very similar to Newton’s second law...
Potential energy

- (Gravitational) **Potential energy** is the energy ‘stored’ in an object due to **relative** height difference.
  - The amount of work that would be done if we were to set it free.

\[
E_P = m \cdot g \cdot h
\]

- Simple product of the weight \( W = m \cdot g \) and height \( h \).
- Also measured in Joules (as here \( kg \cdot \frac{m}{sec^2} \cdot m \)).

- Other potential energies exist (like a compressed spring).
Conservation of mechanical energy

- **Law of conservation**: in a closed system, energy **cannot** be created or destroyed.
  - Energy may switch form.
  - May transfer between objects.
  - Classical example: falling trades potential and kinetic energies.

\[
E_K(t + \Delta t) + E_P(t + \Delta t) = E_K(t) + E_P(t)
\]

i.e.

\[
\frac{1}{2} m v(t + \Delta t)^2 + mgh(t + \Delta t) = \frac{1}{2} m v(t)^2 + mgh(t)
\]
Conservation: Example

• A roller-coaster cart at the top of the first hill.
  • Much potential energy, but only a little kinetic energy.
  • Going down the drop: losing height, picking up speed.
  • At the bottom: almost all potential energy switched to kinetic, cart is at its maximum speed.
Conservation of Mechanical Energy

• External forces are usually applied:
  • Friction and air resistance.
  • Where does the “reduced” energy go?
  • Converted into heat and air displacements (sound waves, wind).

• We compensate by adding an extra term $E_O$ to the conservation equation:

$$E_K(t + \Delta t) + E_P(t + \Delta t) + E_O = E_K(t) + E_P(t)$$

• if $E_O > 0$, some energy is ‘lost’.

https://i.ytimg.com/vi/anb2c4Rm27E/maxresdefault.jpg
Momentum

• The linear momentum $\vec{p}$: the mass of an object multiplied by its velocity:

$$\vec{p} = m \cdot \vec{v}$$

• Heavier object/higher velocity $\Leftrightarrow$ more momentum (more difficult to stop).

• unit is $[kg \cdot m/sec]$.

• Vector quantity (velocity).
Impulse

• A change of momentum:
  \[ \vec{j} = \Delta \vec{p} \]

• Compare:
  • **Impulse** is change in **momentum**.
  • **Work** is change in **energy**.

• Unit is also \([\text{kg} \cdot \text{m/s}]\) (like momentum).

• **Impulse** \(\leftrightarrow\) force integrated over time:
  \[ \vec{j} = \int_0^t \vec{F} \, dt = m \int_0^t \vec{a} \, dt = m\Delta \vec{v}. \]
Conservation of Momentum

- **Law of conservation:** in a closed system (no external forces\(\text{\textbackslash impulses}\)), momentum **cannot** be created or destroyed.
- **Compare:** conservation of energy.
- Implied from 3\(^{\text{rd}}\) law.
  - Objects react with the same force exerted on them.
- Special case of Noether’s theorem: every physical system (With a symmetric action) has a conservation law.