HOME EXAM 2: DISCRETIZED PHYSICS

GAME PHYSICS, PERIOD 3, 2016/2017

Handout date: Friday, 24 March 2017
Submission deadline: Friday, 14 April 2017, 23:59

RULES

• Individual submissions only! Clearly state your name and student number.
• Answer all questions fully, and note the correct SI units (points will be deducted otherwise!)
• You may use any material you find online as an aid.
• It is not preventable that people use each other to aid, but the answer has to be convincingly in your own words; copying will be punishable by subdividing a single grade equally by the cardinality of a set of works that are obviously copied from each other.
• Submit a computer-edited report in PDF; it is preferable (for you) to use LaTeX, but anything will do.
• Answer concisely, but fully.
• The submission will be to a submission server that will be announced on the website.
• No unauthorized late submissions will be accepted.

1. Position-based Dynamics

An axis-aligned box falls to the ground by mere gravity, and with $CR = 1$ coefficient of restitution. The algorithm works by applying impulses to the colliding vertices. While you would expect the box to bounce, like in the first practical, it just stops. Explain why that happens, and what’s a reasonable hack to solve this. Hint: the constraint projection creates an inherently non-physical problem. Think what should happen physically.

2. Finite-Element Method

• Given a triangle mesh $M = \{V, T\}$, and a vertex-based deformation $\mathbf{u} : V \rightarrow \mathbb{R}^3$. Assume that the stiffness tensor is trivial: $C_e = I_{6 \times 6}$. Derive the expression for $B_t$ (the strain tensor per triangle), and consequently $K_t$ (the strain energy bilinear form). You should use quantities like edge lengths, and angles, and clearly mark which are what. Show that $K$ (the aggregation of $K_t$ as a big sparse matrix of $|V| \times |V|$) has the form of a Laplacian. That is that: $K\mathbf{u} = w_{ii}\mathbf{u} - \sum_{j \in N(i)} w_{ij}\mathbf{u}$ (weighted averaging of neighbors $N(i)$ to vertex $i$). Intermediately-tough optional question: what is an analytic expression for the weights $w_{ij}$?
• Do the same for a tetrahedral mesh, where the strain tensor is per tetrahedron.