Assignment 5 Fuzzy Systems
Development of Knowledge Systems,
December 2008

Getting started

Read:

- The Artificial Intelligence: Chapter 4.1 to 4.6
- Lecture notes.

Hand in: Nothing.

a. What is the normal range of truth values in Boolean logic? And in fuzzy logic?
b. Would it be correct to call fuzzy logic multi-valued?
c. Define a linguistic variable and its values. Give an example and draw the fuzzy set for that variable. How can a hedge modify this variable, and its fuzzy set?
d. The following fuzzy function was used to calculate membership values for the set healthy. A membership value of 1 is healthy; a membership value of 0 is not healthy; a membership value between 0 and 1 is the degree of membership in the healthy set.

\[
\text{healthy}(x) = \begin{cases} 
0, & \text{if } \text{bmi}(x) < 18 \\
\frac{(\text{bmi}(x)-18)}{2}, & \text{if } 18 \leq \text{bmi}(x) \leq 20 \\
1, & \text{if } 20 < \text{bmi}(x) < 25 \\
\frac{(27-\text{bmi}(x))}{2}, & \text{if } 25 \leq \text{bmi}(x) \leq 27 \\
0, & \text{if } \text{bmi}(x) > 27 
\end{cases}
\]

BMI values that range from 20 to 25 are members of the healthy set (1). BMI values greater than 27 or less then 18 are not members of the healthy set (0). BMI values close to the healthy range (20 to 25) are a value between 0 and 1. For example, a BMI of 19.6 is 0.8 degree of membership in the healthy set.

1. Draw the graphic for the healthy set, representing the values, healthy and unhealthy.
2. What is the degree of membership to the fuzzy set healthy of John who has a BMI of 26.2? And to the fuzzy set unhealthy?
3. Calculate now your BMI (weight in kg/(length in m)^2). What is your degree of membership for the healthy set?
Hand in Exercises
Exercise 1. Fuzzy sets and probabilities
Hand-in: everything.

a. You are assigned the task of identifying images in an overhead reconnaissance photograph. The two fuzzy sets representing a car image and a truck image are defined as:

Car = \{0.5 / truck, 0.4 / motor, 0.3 / boat, 0.9 / car, 0.1 / house\}
Truck = \{1 / truck, 0.1 / motor, 0.4 / boat, 0.4 / car, 0.2 / house\}

Find the following:

1. Car \cup Truck  
3. not(Car)  
5. Car \cup not(Car)
2. Car \cap Truck  
4. Car \cap not(Truck)  
6. Car \cap not(Car)

b. The sides of each die in a pair of dice are numbered from 1 to 6. When the dice are cast, a "high number" is defined according to the following membership function:

High number = \{1/12, 0.95/11, 0.8/10, 0.6/9, 0.4/8, 0.2/7, 0.1/6, 0.1/5, 0.0/4, 0/3, 0/2\}

For a pair of "fair dice" the probability P(x) of getting any number from 12 to 2 is

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What is the probability of throwing a "high number"?
Exercise 2. Mamdani rule
Hand-in: everything.

Consider the following fuzzy expert system:

RULE1: IF temperature is hot or warm, THEN the swimming pool is crowded.
RULE2: IF temperature is cold, THEN the swimming pool is quiet.

The membership function in the universes of discourse are given as

\[
\mu_{Hot}(x) = \begin{cases} 
0, & \text{if } x < 15 \\
\frac{x-15}{10}, & \text{if } 15 \leq x \leq 25 \\
1, & \text{if } x > 25 
\end{cases}
\]

\[
\mu_{Warm}(x) = \begin{cases} 
-(\frac{x-25}{10}), & \text{if } 15 \leq x \leq 25 \\
\frac{x-7}{8}, & \text{if } 7 \leq x < 15 \\
0, & \text{if } x < 7 \text{ or } x > 25
\end{cases}
\]

\[
\mu_{Cold}(x) = \begin{cases} 
1, & \text{if } x < 7 \\
1 - \frac{x-7}{8}, & \text{if } 7 \leq x \leq 15 \\
0, & \text{if } x > 15 
\end{cases}
\]

\[
\mu_{Crowd}(x) = \begin{cases} 
0, & \text{if } x < 100 \\
\frac{x-100}{400}, & \text{if } 100 \leq x \leq 500 \\
1, & \text{if } 500 \leq x \leq 800 
\end{cases}
\]

\[
\mu_{Quiet}(x) = \begin{cases} 
1, & \text{if } x < 100 \\
-(\frac{x-500}{400}), & \text{if } 100 \leq x \leq 500 \\
0, & \text{if } 500 \leq x \leq 800
\end{cases}
\]

a. Which are the linguistic variables and the linguistic values?
b. Draw the graphics for the membership functions of temperature and number of customers in the swimming pool.
c. Suppose the temperature is 21 degrees. Apply the Mamdani implication rule to determine the expected number of clients in the swimming pool. Describe carefully each of the steps.
Exercise 3. Fuzzy inference and defuzzification

Hand-in: everything.

Consider a simple controller using an error signal, \( e \), and a change in error signal, \( de \), as inputs and 4 rules:

- **RULE 1:** IF \( e = P \) AND \( de = P \) THEN \( x = N \)
- **RULE 2:** IF \( e = P \) AND \( de = N \) THEN \( x = 0 \)
- **RULE 3:** IF \( e = N \) AND \( de = P \) THEN \( x = 0 \)
- **RULE 4:** IF \( e = N \) AND \( de = N \) THEN \( x = P \)

There are two fuzzy sets as values of the fuzzy input variables \( e \) and \( de \): the \( P \) (positive) and \( N \) (negative). The output fuzzy variable has 3 values: \( P \) (positive), \( 0 \) (zero), \( N \) (negative) with membership functions shown in figure 1. Assume that the input variables have the following membership values in the input fuzzy sets:

\[
\mu_N(e) = 0.4; \mu_P(e) = 0.6 \quad \text{and} \quad \mu_N(de) = 0.2; \mu_P(de) = 0.8
\]

**Figure 1: Output Variable Membership Functions.**

**Figure 2: Overall Implied Output Membership Function.**

a. Use Mamdani inference to show that the overall implied fuzzy output set is as shown by the heavy red line in the graph of figure 2. Demonstrate graphically.
b. Defuzzify using the centroid technique.
c. Use a zero-order Sugeno model to calculate the value of the output. Demonstrate graphically.
d. Compare the results of the two methods of inference.