Welcome!

Lecture 12 - “Probability Theory”

\[ I(x, x') = g(x, x') \left[ \epsilon(x, x') + \int_\mathcal{S} \rho(x, x', x'') I(x', x'') dx'' \right] \]
Previously in Advanced Graphics...
\[ L_0(p, \omega_i) \approx \frac{\text{lights}}{N} \sum_{i=1}^{N} f_r(p, \omega_o, P) L_d(p, P) V(p \leftrightarrow P) \]

Sampling a light

\[ L_0(p, \omega_i) \approx \frac{2\pi}{N} \sum_{i=1}^{N} f_r(p, \omega_o, \omega_i) L_d(p, \omega_i) \cos \theta_i \]

Sampling the hemisphere

\[ E(f(X)) \approx \frac{B - A}{N} \sum_{i=1}^{N} f(X) \]

pdf: \( f = 1 \), \( p(X) > 0 \) if \( f(X) > 0 \)
Today’s Agenda: **Monte Carlo**

- Sampling an Area Light with One Ray
- Sampling Multiple Area Lights with One Ray
- Difficult Cases: Spherical Lights, Occluded Lights
- The Random Walk
- Random Walk with Next Event Estimation
- Digest
- P3 Topics
Lights

Case 1: Point Light

Situation:
- surface point: location \( p \), normal \( \mathbf{N} \);
- point light: location \( e \), intensity \( I \) (in Watt, or joule per second);
- distance between \( p \) and \( e \): \( d \).
- unit vector from \( p \) to \( e \): \( \mathbf{L} \).

Flux leaving \( e \): \( I \) joules per second.
Flux arriving at a sphere, radius \( r \), surface \( 4\pi r^2 \) around \( e \): \( I \)
(Ir)radiance arriving on that sphere: \( \frac{I}{4\pi r^2} \) (W/m\(^2\))

Flux arriving per steradian: \( \frac{I}{4\pi} \)

Steradians for a unit area surface patch at location \( p \): \( \sim \frac{\mathbf{N} \cdot \mathbf{L}}{d^2} \)

This is the solid angle of the unit area surface patch as seen from \( e \), or: The area of the patch projected on the unit sphere around \( e \).

Light arriving at \( p \) from a point light at distance \( d \):

\[
I \frac{\mathbf{N} \cdot \mathbf{L}}{4\pi d^2} \text{ per unit surface area.}
\]

This is the *projected radiance*, i.e. *irradiance* from the light at \( e \) arriving at point \( p \).

The contribution of multiple lights is summed.
Case 2: Area Light

**Situation:**
- surface point, location $p$, normal $\vec{N}_p$;
- single-sided area light $e$, intensity $I$, area $A$, normal $\vec{N}_e$.

Steradians for the area light, as seen from $p$:
$$A \cdot \vec{N}_e \cdot \vec{N}_e \cdot L \cdot d^2 \approx 0 \ldots 2\pi \text{ (approximately)}.$$

The **radiance** arriving from $e$ at $p$ is:
$$I \cdot SA \approx \frac{A(\vec{N}_e \cdot \vec{L})}{d^2}.$$

The **irradiance** (joules per second per unit area) from $e$ at $p$ is:
$$I \cdot SA \cdot N_p \cdot L \approx \frac{A(\vec{N}_e \cdot \vec{L})}{d^2}.$$
Lights

Sampling an Area Light

The irradiance (joules per second per unit area) is:

\[ I = \frac{A_{\text{visible}} (N_e \cdot -L) (N_p \cdot L)}{d^2} \]

Here, \( A_{\text{visible}} \) is the visible area of the light source. \( A_{\text{visible}} \) may be smaller than \( A \) in the presence of occluders.

We send 1 million rays to the light source. \( N \) rays reach the light source. The visible area is estimated as:

\[ A_{\text{visible}} = A \frac{N}{1,000,000} \]

Now, we send a single ray to the light source. The probability of a ray reaching the light source is \( \rho \). Now, \( A_{\text{visible}} = A \rho \).

For this single ray, the answer is usually wrong. However, on average the answer is correct.
Today’s Agenda: *Monte Carlo*

- Sampling an Area Light with One Ray
- Sampling Multiple Area Lights with One Ray
- Difficult Cases: Spherical Lights, Occluded Lights
- The Random Walk
- Random Walk with Next Event Estimation
- Digest
- P3 Topics
Lights

Sampling Multiple Lights

“To sample $N$ lights with a single ray, chose a random light, and multiply whatever the ray returns by $N$.”

Situation: two lights.

Mental steps:

- If the lights would have been point lights, we would have sampled both and summed the results.
- We know that we can sample an area light with a single ray. So, we sample both using a single ray, and sum the results.
- Using one ray, we could sample alternating lights. Since each light is now sampled in half the cases, we should increase the result we get each time by 2.
- Or, we can sample a randomly selected light. On average, each light is again sampled in half of the cases, so we scale by 2.
- In other words, we scale by $1/50\% = 2$, where 50% is the probability of selecting a light.

Generalized:

If we have $N$ lights, and we sample each with a probability $\rho_i$, we scale the contribution by $\frac{1}{\rho_i}$ to get an unbiased sample of the set of $N$ lights.

Any $\rho_i$ is valid, as long as $\sum \rho_i = 1$ and $\rho_i > 0$ unless we know the sample will yield 0.
Today’s Agenda: *Monte Carlo*

- Sampling an Area Light with One Ray
- Sampling Multiple Area Lights with One Ray
- Difficult Cases: Spherical Lights, Occluded Lights
- The Random Walk
- Random Walk with Next Event Estimation
- Digest
- P3 Topics
**Lights**

**Sampling a Spherical Light**

"Any $\rho_i$ is valid, as long as $\sum \rho_i = 1$ and $\rho_i > 0$ unless we know that the sample will yield 0."

**Situation: spherical light source.**

**Starting point: selecting $N=10$ points on the sphere, uniformly, so $\rho_i=1/N=0.1$.**

- Now, suppose we skip points on one hemisphere.
- Skipping points means $\rho_i = 0$.
- To ensure that $\sum \rho_i = 1$: double $\rho_i$ for remaining points, to 0.2.

Before: $E = \frac{1}{N} \sum_{i=1}^{N} \frac{V_i=0.5}{\rho_i=0.1}$ after: $\frac{1}{N} \sum_{i=1}^{N} \frac{V_i=1}{\rho_i=0.2}$

**Similar situation: when evaluating the Lambertian BRDF, we skip the hemisphere below the surface. We account for this by reducing the domain to $2\pi$.**
Lights

Sampling Occluded Lights

Situation 1:

*We have no information about occlusion.*

**NEE probes each light with 50% probability.**

- samples are scaled up by $1/50\% = 2$;
- rays to light 2 always yields 0;
- ➔ point $p$ receives energy from light 1 in 50% of the cases, but the light is multiplied by 2.

Situation 2:

*We know light 2 is occluded.*

**NEE probes light 1 with 100% probability.**

- samples are scaled by 1;
- ➔ point $p$ receives energy from light 1 in 100% of the cases, multiplier is 1.

The only difference between situation 1 and 2 is variance: in situation 1, we get twice the energy each time we sample light 1, but it gets sampled in only 50% of the cases.

In situation 2, we get a much more even amount of energy for each sample.
Today’s Agenda: *Monte Carlo*

- Sampling an Area Light with One Ray
- Sampling Multiple Area Lights with One Ray
- Difficult Cases: Spherical Lights, Occluded Lights
- The Random Walk
- Random Walk with Next Event Estimation
- Digest
- P3 Topics
The Random Walk

How much light gets transported to the eye?
1. The light that $p$ emits towards the eye (typically: nothing); plus:
2. The light that $p$ reflects towards the eye.

$$L_o = \int_{\Omega} f_r(p, \theta_o, \theta_i) \cos \theta_i L_i$$
The Random Walk

How much light gets transported to the eye?

1. The light that $p$ emits towards the eye (typically: nothing);
2. The light that $p$ reflects towards the eye:
   a) That is: the light that $q$, $r$, $s$ emit towards $p$, plus
   b) The light that $q$, $r$, $s$ (and all other scene surface points) reflect towards $p$.

Regarding 2b:

- The further away a point, the lower the probability that a random ray from $p$ strikes it.
- The probability is also proportional to $\vec{N}_{s,p,q} \cdot -\vec{L}$.
- At $p$, we scale by $\vec{N}_p \cdot \vec{L}$ to compensate for the fact that we sample radiance, while in fact we need irradiance for the BRDF.
Sampling the Hemisphere using a Single Ray

The light being reflected towards the eye is the light arriving from all directions over the hemisphere, scaled by the BRDF: \( \int_{\Omega} f_r(p, \theta_o, \theta_i) E_i \).

Sampling the integral using a single random ray: \( \text{Scale up by } 2\pi \).
Walk

Random Walk

Point $p$ reflects what point $q$ reflects, which is what point $r$ emits.
Random Walk

If we leave the scene, the path returns no energy.
Today’s Agenda: *Monte Carlo*

- Sampling an Area Light with One Ray
- Sampling Multiple Area Lights with One Ray
- Difficult Cases: Spherical Lights, Occluded Lights
- The Random Walk
- Random Walk with Next Event Estimation
- Digest
- P3 Topics
Importance-Sampling the Hemisphere

How much light gets transported to the eye?

1. The light that $p$ emits towards the eye (typically: nothing); plus:
2. The light that $p$ reflects towards the eye.
3. In practice: we have no idea. But we can guess.

Reflected energy:
Light (radiance) coming from all directions, reflected in a single direction; i.e:

$$L_0 = \int_{\Omega} f_r(p, \theta_o, \theta_i) \cos \theta_i L_i$$
By the way...

There exists a class of estimators known as zero variance estimators.

Example 1:

\[
f(x) = 1, \quad x \in [0,1].
\]

\[
\int_0^1 f(x) = 1 = E(f(X)) \approx \frac{1}{N} \sum_{i=1}^{N} f(X_i)
\]

Even at \(N=1\) we reach the expected value.
By the way...

There exists a class of estimators known as zero variance estimators.

Example 2:

Point \( x \) is illuminated solely by a uniform blue sky.

\[
E(f(X)) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{f(X)}{p(X)}
\]

\[
f(x) = C \cos \theta
\]

\[
p(x) = \frac{\cos \theta}{\pi}
\]

And again, at \( N=1 \) we reach the expected value.
By the way...

There exists a class of estimators...

Example 3:

Point $x$ is illuminated solely by a rectangular light source.

The noise on the ceiling is caused by randomly sampling the area of the light: $\cos \theta$ will have variance.

The noise on the floor is caused by shadow rays sometimes not reaching the light, which will have variance.
By the way...

There exists a class of estimators...

Example 3:

Point $x$ is illuminated solely by a rectangular light. The noise on the floor is caused by shadow rays sometimes not reaching the light, which will have variance.
Next Event Estimation

At each vertex, we sample the light source using an explicit light ray.
Next Event Estimation

Why does this work?

“The light arriving via point $p$ is the light reflected by point $p$, plus the light emitted by point $p$."

And thus:

The light reflected by point $p$ is the light arriving at $p$ originating from light sources (1), plus the light reflected towards $p$ (2).

1: Direct light at point $p$.

2: Indirect light at point $p$. 
Next Event Estimation

If we send out a ray in a random direction over the hemisphere of \( p \), this ray may return two types of illumination:

1: Direct: the ray hit the light source;
2: Indirect: the ray missed the light source.

If we ignore all random rays that hit a light source (as in: terminate them, return 0), we remove the direct light arriving at \( p \).

If we sample just the lights, we remove the indirect light arriving at \( p \).

Since the contributions show no overlap, we can sample them individually, using two rays, and sum the result.
Next Event Estimation

Direct and indirect illumination can be sampled separately, as long as we guarantee that there will not be overlap.

This works for any point, not just the primary hit:

E.g., the light that point $q$ reflects towards point $p$ is the direct lighting reflected by $q$ towards $p$, plus the indirect lighting reflected by $q$. 
Next Event Estimation

1. Why don’t we use next event estimation for a specular surface?

Explicit light sampling still requires evaluation of the BRDF. For a specular surface, the BRDF for $\theta_o$ is $\infty$ for a single $\theta_i$, which is why we continue the (not so) random walk in that direction. All other directions yield 0.

Consequence:

Since we do not send out an explicit light ray in this case, the random walk may now return direct illumination: there is no overlap.

In fact, if we didn’t accept direct illumination, we would be missing energy.
Next Event Estimation

2. Why should we return direct illumination for the primary ray?

The eye vertex did not send out an explicit light ray. Since direct illumination is not sampled separately, the random walk may return this illumination.

*The eye is thus considered a specular vertex.*
Next Event Estimation

Also think about it like this:

The eye looks directly at a light source.  
*If we terminate those paths, the light will look black.*

The eye looks at a light in a mirror.  
*If we terminate those paths, we see a black light in the mirror.*

In all other cases, we send out an explicit light ray. To compensate for that extra ray:

- The extra ray may only sample direct illumination. It doesn’t bounce.
- Any other way of sampling direct illumination is blocked.
http://sjbrown.co.uk/2011/01/03/two-way-path-tracing
Today’s Agenda: Monte Carlo

- Sampling an Area Light with One Ray
- Sampling Multiple Area Lights with One Ray
- Difficult Cases: Spherical Lights, Occluded Lights
- The Random Walk
- Random Walk with Next Event Estimation
- Digest
- P3 Topics
Efficiency

Efficiency in Monte-Carlo:

- We must use importance sampling.
  - We are not tracing photons backwards, we are establishing importance.
  - A path that cannot be importance sampled is a noisy path.
  - We can still trace from the light to the camera (‘forward path tracing’ aka light tracing).
  - Consider using Multiple Importance Sampling.
  - *(it works for NEE, but also when combining forward and backward path tracing)*
  - Any pdf is valid, as long as ... and ....
  - We can play with this stuff.
  - We can learn importance.
Today’s Agenda: Monte Carlo

- Sampling an Area Light with One Ray
- Sampling Multiple Area Lights with One Ray
- Difficult Cases: Spherical Lights, Occluded Lights
- The Random Walk
- Random Walk with Next Event Estimation
- Digest
- P3 Topics
Future Work

Materials

1 Introduction

We introduced a new physically based shading model on 'Wreck-It Ralph' [Bur12], and we used this single, general-purpose BRDF on all materials (except hair). This model, which has come to be known as the Disney BRDF, is able to reproduce a wide range of materials with only a few parameters. For our next film, 'Frozen', we continued to use this BRDF unmodified, but effects like refraction and subsurface scattering were computed separately from the BRDF, and indirect illumination was approximated using point clouds. All of these effects were combined through ad hoc shading in an additive way.

Starting with 'Big Hero 6' in 2014, we switched from ad hoc lighting and shading to path-traced global illumination that factored the surface scattering and indirect illumination, all integrated.

Other resources:

Implementing the Disney BSDF
schuttejoe.github.io/post/disneybsdf


Eric Heitz's Research Page:
eheitzresearch.wordpress.com/research
(all his work is good)
Future Work

Materials

“thin film”

“consistent normal mapping”
Future Work

Spectral Rendering

Stratified Wavelength Clusters for Efficient Spectral Monte Carlo Rendering

Glenn F. Evans  Michael D. McCool
Computer Graphics Laboratory
Department of Computer Science
University of Waterloo

Abstract
Wavelength dependent Monte Carlo rendering can correctly and generally capture effects such as spectral caustics (rainbows) and chromatic aberration. It also improves the color accuracy of reflectance models and of illumination effects such as color bleeding and metamerism.

The stratified wavelength clustering (SWC) strategy carries several wavelength-stratified radiance samples along each light transport path. The cluster is split into several paths or degraded into a single path only if a specular reflection at the surface of a dispersive material is encountered along the path. The overall efficiency of this strategy is high since the fraction of clusters that need to be split or degraded in a typical scene is low, and also because specular dispersion tends to decrease the source color variance, offsetting the increased unstratified cost of generating each path.

Keywords: Monte Carlo methods, wavelength dependent (spectral) rendering, caustics, rainbows, refraction, reflectance, global illumination.

1 Introduction
The benefits of using three component color models for composite reflectance, absorption, and dispersion are well known. However, we demonstrated not only that this is feasible, but also that with a strategy we call stratified wavelength clustering (SWC) the marginal cost is negligible for Monte Carlo ray tracing and bidirectional path tracing.

2 Outline
Prior work on wavelength dependent and spectral rendering is surveyed in Section 3. Section 4 presents the stratified wavelength cluster strategy. Three splitting strategies are also presented and compared. In Section 5 a bidirectional path tracer that uses stratified wavelength clusters is described. Finally, in Section 6 we present our results, comparing and contrasting each Monte Carlo integration over wavelength, quasi Monte Carlo integration over wavelength using Blotin sequences, and the stratified wavelength cluster strategy.

3 Background
Before presenting our approach for extending Monte Carlo global illumination algorithms to wavelength-dependent (spectral) rendering, we first review the relationship of spectral power densities to perceived color and review wavelengths-dependent phenomena that have a significant effect on the generated im-

Other resources:


Physically Meaningful Rendering using Tristimulus Colours. Meng et al., 2015.

PBRT. Pharr et al., 2004-2018.
https://www.pbrt.org

https://www.mitsuba-renderer.org
Future Work

Spectral Rendering
Future Work

Participating Media

Other resources:


Area Light Equi-Angular Sampling on ShaderToy:

https://www.shadertoy.com/view/ldXGzS
Future Work

Participating Media
Future Work

Light Transport

Other resources:
Bidirectional Path Tracing. Michal Vlnas, 2018 (student paper).
(article on) Vertex Connection and Merging, Georgev et al., 2012.
https://schuttejoe.github.io/post/vertexconnectionandmerging
Future Work

Light Transport
Future Work

Primitives

Other resources:


Two-Level Ray Tracing with Reordering for Highly Complex Scenes. Hanika et al., 2010.
Future Work

Production

Arnold: A Brute-Force Production Path Tracer

ILIYAN GEORGEV, THIAGO IZE, MIKE FARNSWORTH, RAMÓN MONTOYA-VOZMEDIANO, ALAN KING, BRECHT VAN LOMMEL, ANGEL JIMENEZ, OSCAR ANSON, SHINJI OGAKI, ERIC JOHNSTON, ADRIEN HENRUBEL, DECLAN RUSSELL, FRÉDÉRIC SERVANT, and MARCOS FAJARDO. Solid Angle

Arnold is a physically based renderer for feature-length animation and visual effects. Conceived in an era of complex multi-pass rasterization-based workflows struggling to keep up with growing demands for complexity and realism, Arnold was created to take on the challenge of making the simple and elegant approach of brute-force Monte Carlo path tracing practical for production rendering. Achieving this required building a robust piece of ray-tracing software that can ingest large amounts of geometry with detailed shading and lighting and produce images with high fidelity, while scaling well with the available memory and processing power.

Arnold’s guiding principles are to expose as few controls as possible, provide rapid feedback to artists, and adapt to various production workflows. In this article, we describe its architecture with a focus on the design of its path tracing kernel.

Other resources:


The Design and Evolution of Disney’s Hyperion Renderer. Burley et al., 2018.

Manuka: A batch-shading architecture for spectral path tracing in movie production. Fascione et al., 2018.

Future Work

Real-time

Other resources:


REAL-TIME RAYTRACING WITH NVIDIA RTX. Stich, 2018.

(not a lot of research so far...)

Ray Tracing Gems 1 & 2,
https://research.nvidia.com/publication/2020-07_Spatiotemporal-reservoir-resampling,
https://research.nvidia.com/publication/2019-07_Temporally-Dense-Ray,
https://research.nvidia.com/publication/2019-03_Improving-Temporal-Antialiasing,
Other resources:

REAL-TIME RAYTRACING WITH NVIDIA RTX. Stich, 2018.

(not a lot of research so far...)

Ray Tracing Gems,
https://research.nvidia.com/publication/2020-07_Spatiotemporal-reservoir-resampling,
https://research.nvidia.com/publication/2019-07_Temporally-Dense-Ray,
https://research.nvidia.com/publication/2019-03_Improving-Temporal-Antialiasing, ...
Future Work

Massive Scenes


https://pharr.org/matt/blog/2018/07/16/moana-island-pbrt-all.html
Today’s Agenda: **Monte Carlo**

- Sampling an Area Light with One Ray
- Sampling Multiple Area Lights with One Ray
- Difficult Cases: Spherical Lights, Occluded Lights
- The Random Walk
- Random Walk with Next Event Estimation
- Digest
- P3 Topics
INFOMAGR – Advanced Graphics

Jacco Bikker  -  November 2022 - February 2023

END of “Probability”

next up: “Bidirectional”