Welcome!

Lecture 4 - “Light Transport”

\[ I(x, x') = g(x, x') \left[ \epsilon(x, x') + \int_s \rho(x, x', x'') I(x', x'') dx'' \right] \]
Today's Agenda:

- Introduction
- The Rendering Equation
- Light Transport
Introduction

Whitted
Introduction

Whitted
Introduction

Whitted

Missing:

- Area lights
- Glossy reflections
- Caustics
- Diffuse interreflections
- Diffraction
- Polarization
- Phosphorescence
- Temporal effects
- Motion blur
- Depth of field
- Anti-aliasing
Introduction

Anti-aliasing

Adding anti-aliasing to a Whitted-style ray tracer:

Send multiple primary rays through each pixel, and average their result.

Problem:

- How do we aim those rays?
- What if all rays return the same color?
Anti-aliasing – Sampling Patterns

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More info: https://mynameismjp.wordpress.com/2012/10/24/msaa-overview
Introduction

Whitted

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✓ Anti-aliasing
Introduction

Distribution Ray Tracing*

*: Distributed Ray Tracing, Cook et al., 1984
Introduction

Distribution Ray Tracing*

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Glossy reflections
Distribution Ray Tracing*

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**Distribution Ray Tracing***

*: Distributed Ray Tracing, Cook et al., 1984
Introduction

Distribution Ray Tracing

Whitted-style ray tracing is a *point sampling* algorithm:

- **We may miss small features**
- **We cannot sample areas**

Area sampling:

- **Anti-aliasing**: one pixel
- **Soft shadows**: one area light source
- **Glossy reflection**: directions in a cone
- **Diffuse reflection**: directions on the hemisphere
Introduction

Area Lights

Visibility of an area light source:

\[ V_A = \int_A V(x, \omega_l) \, d\omega_l \]

Analytical solution case 1:

\[ V_A = A_{\text{light}} - A_{\text{light} \cap \text{sphere}} \]

Analytical solution case 2:

\[ V_A = ? \]
Introduction

Approximating Integrals

An integral can be approximated as a Riemann sum:

\[ V_A = \int_A^B f(x) \, dx \approx \sum_{i=1}^{N} f(t_i) \Delta_i, \text{ where } \sum_{i=1}^{N} \Delta_i = B - A \]

Note that the intervals do not need to be uniform, as long as we sample the full interval. If the intervals are uniform, then

\[ \sum_{i=1}^{N} f(t_i) \Delta_i = \Delta_i \sum_{i=1}^{N} f(t_i) = \frac{B - A}{N} \sum_{i=1}^{N} f(t_i). \]

Regardless of uniformity, restrictions apply to \( N \) when sampling multi-dimensional functions (ideally, \( N = M^d, M \in \mathbb{N} \)). Also note that aliasing may occur if the intervals are uniform.
Introduction

Monte Carlo Integration

Alternatively, we can approximate an integral by taking random samples:

\[ V_A = \int_A^B f(x) \, dx \approx \frac{B - A}{N} \sum_{i=1}^N f(X_i) \]

Here, \( X_1 \ldots X_N \in [A, B] \).

As \( N \) approaches infinity, \( V_A \) approaches the expected value of \( f(X) \).

Unlike in Riemann sums, we can use arbitrary \( N \) for Monte Carlo integration, regardless of dimension.
Monte Carlo Integration of Area Light Visibility

To estimate the visibility of an area light source, we take $N$ random point samples.

In this case, 5 out of 6 samples are unoccluded:

$$V \approx \frac{1}{6} (1 + 1 + 1 + 0 + 1 + 1) = \frac{5}{6}$$

Properly formulated using a MC integrator:

$$V = \int_{S^2} V(p) \, dp \approx \frac{1}{N} \sum_{i=1}^{N} V(P)$$

With a small number of samples, the variance in the estimate shows up as noise in the image.

Q: Where did the $\frac{B-A}{N}$ go?
A: the domain of the visibility function is [0..1], so $B - A = 1$. 
Introduction

Monte Carlo Integration of Area Light Visibility

We can also use Monte Carlo to estimate the contribution of multiple lights:

1. Take the average of $N$ samples from each light source;
2. Sum the averages.

$$E(x) = \frac{1}{N} \sum_{j=0}^{N} \sum_{i=1}^{2} L_i V(x \leftrightarrow l_i)$$

No averaging here: multiple lights are additive.
Monte Carlo Integration of Area Light Visibility

Alternatively, we can just take $N$ samples, and pick a random light source for each sample.

$$E(x \leftarrow) = \frac{2}{N} \sum_{i=1}^{N} L_Q V_Q(P), \quad Q \in \{1,2\}$$

$$= \frac{1}{N} \sum_{i=1}^{N} \frac{L_Q V_Q(P)}{0.5}$$
Monte Carlo Integration of Area Light Visibility

We obtain a better estimate with fewer samples if we do not treat each light equally.

In the previous example, each light had a 50% probability of being sampled. We can use an arbitrary probability, by dividing the sample by this probability.

\[
E(\mathbf{x} \leftarrow) = \frac{1}{N} \sum_{i=1}^{N} \frac{L_Q \ V_Q(P)}{\rho_Q}, \quad \sum \rho_Q = 1, \rho_Q > 0
\]
Introduction

Distribution Ray Tracing

Key concept of distribution ray tracing:

We estimate integrals using Monte Carlo integration.

Integrals in rendering:

- Area of a pixel
- Lens area (aperture)
- Frame time
- Light source area
- Cones for glossy reflections
- Wavelengths
- ...
Introduction

Open Issues

Remaining issues:

- Energy distribution in the ray tree / efficiency
- Diffuse interreflections
Today's Agenda:

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Whitted, Cook & Beyond

Missing in Whitted:

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- Diffraction
- Polarization
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Cook:

- Area lights
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Whitted, Cook & Beyond

Cook’s solution to rendering:

Sample the many-dimensional integral using Monte Carlo integration.

\[
\int_{A_{\text{pixel}}} \int_{A_{\text{lens}}} \int_{F_{\text{frame}}} \int_{\Omega_{\text{glossy}}} \int_{A_{\text{light}}} \ldots
\]

Ray optics are still used for specular reflections and refractions:
The ray tree is not eliminated.
God’s Algorithm

1 room
1 bulb
100 watts
$10^{20}$ photons per second

Photon behavior:

- Travel in straight lines
- Get absorbed, or change direction:
  - Bounce (random / deterministic)
  - Get transmitted
- Leave into the void
- Get detected
God’s Algorithm - Mathematically

A photon may arrive at a sensor after travelling in a straight line from a light source to the sensor:

\[ L(s \leftarrow x) = L_E(s \leftarrow x) \]

Or, it may be reflected by a surface towards the sensor:

\[ L(s \leftarrow x) = \int_A f_r(s \leftarrow x \leftarrow x') L(x \leftarrow x') G(x \leftrightarrow x') \, dA(x') \]

Those are the options.

Adding direct and indirect illumination together:

\[ L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_r(s \leftarrow x \leftarrow x') L(x \leftarrow x') G(x \leftrightarrow x') \, dA(x') \]
Rendering Equation

God's Algorithm - Mathematically

\[ L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_r(s \leftarrow x \leftarrow x') L(x \leftarrow x') G(x \leftrightarrow x') \, dA(x') \]

- **Emission**
- **Reflection**
- **Hemisphere**
- **Indirect**
- **Geometry factor**
The Rendering Equation:

\[ L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_T(s \leftarrow x \leftarrow x') L(x \leftarrow x') G(x \leftarrow x') \, dA(x') \]

The Rendering Equation*:

- Light transport from lights to sensor
- Recursive
- Physically based

The equation allows us to determine to which extent rendering algorithms approximate real-world light transport.

*: The Rendering Equation, Kajiya, 1986
Today's Agenda:

- Introduction
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Light Transport

Light Transport Quantities

Radiant flux - $\Phi$:

"Radiant energy emitted, reflected, transmitted or received, per unit time."

Units: watts = joules per second

$$W = J \, s^{-1}.$$

Simplified particle analogy: number of photons.

*Note: photon energy depends on electromagnetic wavelength:*

$$E = \frac{hc}{\lambda}, \text{ where } h \text{ is Planck's constant, } c \text{ is the speed of light, and } \lambda \text{ is wavelength. At } \lambda = 550 \text{nm (yellow), a single photon carries } 3.6 \times 10^{-19} \text{ joules.}$$

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Light Transport

Light Transport Quantities

In a vacuum, radiant flux emitted by a point light source remains constant over distance:

A point light emitting 100W delivers 100W to the surface of a sphere of radius \( r \) around the light. This sphere has an area of \( 4\pi r^2 \); energy per surface area thus decreases by \( 1/r^2 \).

In terms of photons: the density of the photon distribution decreases by \( 1/r^2 \).
Light Transport

Light Transport Quantities

A surface receives an amount of light energy proportional to its solid angle: the two-dimensional space that an object subtends at a point.

Solid angle units: steradians (sr).

Corresponding concept in 2D: radians; the length of the arc on the unit sphere subtended by an angle.
Light Transport Quantities

Radiance - \( L \):

“The power of electromagnetic radiation emitted, reflected, transmitted or received per unit projected area per unit solid angle.”

Units: \( W \, sr^{-1} \, m^{-2} \)

Simplified particle analogy:
Amount of particles passing through a pipe with unit diameter, per unit time.

Note: radiance is a continuous value: while flux at a point is 0 (since both area and solid angle are 0), we can still define flux per area per solid angle for that point.
Light Transport

Light Transport Quantities

Irradiance - $E$:

“The power of electromagnetic radiation per unit area incident on a surface.”

Units: Watts per $m^2 = \text{joules per second per } m^2$

$W m^{-2} = J m^{-2} s^{-1}$.

Simplified particle analogy: number of photons arriving per unit area per unit time, from all directions.
Light Transport

Light Transport Quantities

Converting radiance to irradiance:

\[ E = L \cos \theta \]
Light Transport

\[ L(s \leftarrow x) = L_E(s \leftarrow x) + \int_A f_r(s \leftarrow x \leftarrow x') L(x \leftarrow x') G(x \leftarrow x') dA(x') \]

\[ L_o(x, \omega_o) = L_E(x, \omega_o) + \int_{\Omega} f_r(x, \omega_o, \omega_i) L_i(x, \omega_i) \cos \theta_i \ d\omega_i \]

Radiance Radiance Radiance Irradiance

BRDF
Bidirectional Reflectance Distribution Function

BRDF: function describing the relation between radiance emitted in direction $\omega_o$ and irradiance arriving from direction $\omega_i$:

$$f_r(\omega_o, \omega_i) = \frac{L_o(\omega_o)}{E_i(\omega_i)} = \frac{L_o(\omega_o)}{L_i(\omega_i) \cos \theta_i}$$

or, if spatially variant:

$$f_r(x, \omega_o, \omega_i) = \frac{L_o(x, \omega_o)}{E_i(x, \omega_i)} = \frac{L_o(x, \omega_o)}{L_i(x, \omega_i) \cos \theta_i}$$

Properties:

- Should be positive: $f_r(\omega_o, \omega_i) \geq 0$
- Helmholtz reciprocity should be obeyed: $f_r(\omega_o, \omega_i) = f_r(\omega_i, \omega_o)$
- Energy should be conserved: $\int_\Omega f_r(\omega_o, \omega_i) \cos \theta_o \, d\omega_o \leq 1$
Relation between real-world light transport and the RE:

1. Each sensor element registers an amount of photons arriving from the first surface visible though that pixel.
2. This surface may be emissive, in which case it produced the sensed photons.
3. This surface may also reflect photons, arriving from “other surfaces” in the scene.
4. For the “other surfaces”: goto 2.
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INFOMAGR – Advanced Graphics

Jacco Bikker - November 2021 - February 2022

END of “Light Transport”

next lecture: “The Perfect BVH”