INFOMAGR – Advanced Graphics

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Lecture 16 - “Big Picture”

Welcome!

\[
I(x, x') = g(x, x') \left[ \epsilon(x, x') + \int_s \rho(x, x', x'') I(x', x'') dx'' \right]
\]
Today’s Agenda: Monte Carlo

- Sampling an Area Light with One Ray
- Sampling Multiple Area Lights with One Ray
- Difficult Cases: Spherical Lights, Occluded Lights
- The Random Walk
- Random Walk with Next Event Estimation
- Russian Roulette
- Example Exam Questions
Case 1: Point Light

Flux leaving e: $I$ joules per second.
Flux arriving at a sphere, radius $r$, surface $4\pi r^2$ around e: $I$

Irradiance arriving on that sphere: $\frac{I}{4\pi r^2}$ (W/m$^2$)

Flux arriving per steradian: $\frac{I}{4\pi}$

Steradians for a unit area surface patch at location p: $\sim \frac{\vec{N} \cdot \vec{L}}{d^2}$

This is the solid angle of the unit area surface patch as seen from e, or:
The area of the patch projected on the unit sphere around e.

Light arriving at p from a point light at distance d:

$\frac{\vec{N} \cdot \vec{L}}{4\pi d^2}$ per unit surface area.

This is the irradiance from the light at e arriving at point p.

The contribution of multiple lights is summed.
Case 2: Area Light

Situation:

- surface point, location \( \mathbf{p} \), normal \( \mathbf{N}_p \);
- single-sided area light, intensity \( I \), area \( A \), normal \( \mathbf{N}_e \).

Steradians for the area light, as seen from \( \mathbf{p} \):

\[
\text{Steradians} = A \cdot \mathbf{N}_e \cdot \mathbf{L} \cdot d^2 \text{ (approximately)}.
\]

The energy arriving at \( \mathbf{p} \) is:

\[
I \cdot \mathbf{S} \cdot \mathbf{A} \approx A \cdot (\mathbf{N}_e \cdot \mathbf{L}) \cdot d^2.
\]

The irradiance (joules per second per unit area) is:

\[
I \cdot \frac{A (\mathbf{N}_e \cdot \mathbf{L}) (\mathbf{N}_p \cdot \mathbf{L})}{d^2}.
\]
Sampling an Area Light

The irradiance (joules per second per unit area) is:

\[ I_{\text{visible}} \frac{(\vec{N}_e \cdot \vec{L})(\vec{N}_p \cdot \vec{L})}{d^2} \]

Here, \( A_{\text{visible}} \) is the visible area of the light source. \( A_{\text{visible}} \) may be smaller than \( A \) in the presence of occluders.

We send 1 million rays to the light source. \( N \) rays reach the light source. The visible area is estimated as 

\[ A_{\text{visible}} = A \frac{N}{1,000,000} \]

Now, we send a single ray to the light source. The probability of a ray reaching the light source is \( \rho \). Now, \( A_{\text{visible}} = A \rho \).

For this single ray, the answer is usually wrong. However, on average the answer is correct.
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Sampling Multiple Lights

“To sample \( N \) lights with a single ray, chose a random light, and multiply whatever the ray returns by \( N \).”

Situation: *two lights.*

Mental steps:

- If the lights would have been point lights, we would have sampled both and summed the results.
- We can sample an area light with a single ray. So, we sample both using a single ray, and sum the results.
- Using one ray, we could sample alternating lights. Since each light is now sampled in half the cases, we should increase the result we get each time by 2.
- Or, we can sample a randomly selected light. On average, each light is again sampled in half of the cases, so we scale by 2.
- In other words, we scale by 1/50%=2, where 50% is the probability of selecting a light.

Generalized:

If we have \( N \) lights, and we sample each with a probability \( \rho_i \), we scale the contribution by \( \frac{1}{\rho_i} \) to get an unbiased sample of the set of \( N \) lights.

Any \( \rho_i \) is valid, as long as \( \sum \rho_i = 1 \) and \( \rho_i > 0 \) unless we know the sample will yield 0.
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Lights

Sampling a Spherical Light

“Any $\rho_i$ is valid, as long as $\sum \rho_i = 1$ and $\rho_i > 0$ unless we know that the sample will yield 0.”

Situation: *spherical light source.*

Selecting points on the sphere:

- We can skip points on one hemisphere (i.e., $\rho = 0$)
- This does not affect $\sum \rho_i$
- Therefore it doesn’t affect the other probabilities

Similar situation: when evaluating the Lambertian BRDF, we the hemisphere below the surface without accounting for the omission in any way.
Lights

Sampling Occluded Lights

Situation 1:

*We have no information about occlusion.*

**NEE probes each light with 50% probability.**

- samples are scaled up by $1/50\% = 2$;
- rays to light 2 always yields 0;

→ **point $p$ receives energy from light 1 in 50% of the cases, but the light is multiplied by 2.**

Situation 2:

*We know light 2 is occluded.*

**NEE probes light 1 with 100% probability.**

- samples are scaled by 1;

→ **point $p$ receives energy from light 1 in 100% of the cases, multiplier is 1.**

The only difference between situation 1 and 2 is variance: in situation 1, we get twice the energy each time we sample light 1, but it gets sampled in only 50% of the cases. In situation 2, we get a much more even amount of energy for each sample.
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The Random Walk

How much light gets transported to the eye?

1. The light that \( p \) emits towards the eye (typically: nothing);
2. The light that \( p \) reflects towards the eye.

The answer to 2:

Light (irradiance) coming from all directions, reflected in a single direction; i.e:

\[
\int \omega f_r(p, \theta_o, \theta_i) \cos \theta_i
\]
The Random Walk

How much light gets transported to the eye?

1. The light that \( p \) emits towards the eye (typically: nothing);
2. The light that \( p \) reflects towards the eye:
   a) That is: the light that \( q, r, s \) emit towards \( p \), plus
   b) The light that \( q, r, s \) (and all other scene surface points) reflect towards \( p \).

Regarding 2b:
- The further away a point, the lower the probability that a random ray from \( p \) strikes it.
- The probability is also proportional to \( \hat{N}_{s, p, q} \cdot -\vec{L} \).
- At \( p \), we scale by \( \hat{N}_p \cdot \vec{L} \) to compensate for the fact that we sample radiance, while in fact we gather irradiance.
Sampling the Hemisphere using a Single Ray

The light being reflected towards the eye is the light arriving from all directions over the hemisphere, scaled by the BRDF: $\int_{\Omega} f_r(p, \theta_o, \theta_i)$. 

Sampling the integral using a single random ray: \textit{Scale up by} $2\pi$. 
Walk

Random Walk

Point $p$ reflects what point $q$ reflects, which is what point $r$ emits.
Random Walk

If we leave the scene, the path returns no energy.
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Next Event Estimation

At each vertex, we sample the light source using an explicit light ray.
Next Event Estimation

Why does this work?

“The light arriving via point \( p \) is the light reflected by point \( p \), plus the light emitted by point \( p \).”

And thus:

The light reflected by point \( p \) is the light arriving at \( p \) originating from light sources (1), plus the light reflected towards \( p \) (2).

1: Direct light at point \( p \).

2: Indirect light at point \( p \).
Next Event Estimation

If we send out a ray in a random direction over the hemisphere of \( p \), this ray may return two types of illumination:

1: Direct: the ray hit the light source;
2: Indirect: the ray missed the light source.

If we ignore all random rays that hit a light source (as in: terminate them, return 0), we remove the direct light arriving at \( p \).

If we sample just the lights, we remove the indirect light arriving at \( p \).

Since the contributions show no overlap, we can sample them individually, using two rays, and sum the result.
Next Event Estimation

Direct and indirect illumination can be sampled separately, *as long as we guarantee that there will not be overlap.*

This works for any point, not just the primary hit:

E.g., the light that point \( q \) reflects towards point \( p \) is the direct lighting reflected by \( q \) towards \( p \), plus the indirect lighting reflected by \( q \).
Next Event Estimation

1. Why don’t we use next event estimation for a specular surface?

Explicit light sampling still requires evaluation of the BRDF. For a specular surface, the BRDF for $\theta_o$ is $\infty$ for a single $\theta_i$, which is why we continue the (not so) random walk in that direction. All other directions yield 0.

Consequence:

Since we do not send out an explicit light ray in this case, the random walk may now return direct illumination: there is no overlap.

In fact, if we didn’t accept direct illumination, we would be missing energy.
Next Event Estimation

2. Why should we return direct illumination for the primary ray?

The eye vertex did not send out an explicit light ray. Since direct illumination is not sampled separately, the random walk may return this illumination.

*The eye is thus considered a specular vertex.*
Next Event Estimation

Also think about it like this:

The eye looks directly at a light source.

*If we terminate those paths, the light will look black.*

The eye looks at a light in a mirror.

*If we terminate those paths, we see a black light in the mirror.*

In all other cases, we send out an explicit light ray. To compensate for that extra ray:

- The extra ray may only sample direct illumination. It doesn’t bounce.
- Any other way of sampling direct illumination is blocked.
Next Event Estimation

Important:

Earlier slides discussed the mathematical foundation of NEE. Make sure you understand it from the theoretical point of view as well.
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**Important Notice**

In a path tracer, every technique works independently from others.

**E.g.:**

Importance sampling works with and without next event estimation; Russian roulette works with every BRDF, and also with Next Event Estimation.

**Hint:**

Try a new algorithm in the simplest case possible, after that enable more advanced features.
Russian Roulette

Core idea:

The longer a path becomes, the less energy it transports.

Killing half of 16 rays is easy; what do we do with a single path?

\[ \text{➔ Kill it with a probability of 50%.} \]
Russian Roulette

Russian roulette is applied to the random walk.

Most basic implementation: just before you start calculating the next random direction, you decide if the path lives or dies.
Better Russian Roulette

The termination probability of 50% is arbitrary. 

*Any probability is statistically correct.*

However: for 50% survival rate, survivors scale up by $2^\left(\frac{1}{50}\right)$.

➔ In general, for a survival probability $\rho$, survivors scale up by $\frac{1}{\rho}$.

We can chose the survival probability *per path*. It is typically linked to albedo: the color of the last vertex. A good survival probability is:

$$\rho_{\text{survive}} = \text{clamp} \left( \frac{\text{red} + \text{green} + \text{blue}}{3}, 0.1, 0.9 \right)$$

Note that $\rho_{\text{survive}} > 0$ to prevent bias. Also note that $\rho = 1$ is never a good idea.
RR and Next Event Estimation

A path that gets terminated gets to keep the energy accumulated with Next Event Estimation.

We are applying Russian roulette to indirect illumination only.
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1. In streaming path tracing (or wavefront path tracing) we split the path tracing process in multiple kernels. Why?

2. How is the stencil buffer used in early GPU ray tracers for flow control?

3. Why did early work on GPU ray tracing focus on stackless traversal?

4. Stackless methods visit more nodes in the acceleration structure. Why?

5. Why did the first BVH GPU ray tracing paper use a ray packet?
1. In streaming path tracing (or wavefront path tracing) we split the path tracing process in multiple kernels. Why?

Two reasons: the first and most important is that (thanks to compaction) at the start of each kernel the hardware is utilized optimally. The second is that complex code blocks no longer affect the simpler blocks in terms of register pressure.

2. How is the stencil buffer used in early GPU ray tracers for flow control?

The stencil determines which pixels are active, and thus for which pixels the pixel shader is executed. We run the desired shader for all enabled pixels by drawing a screen-filling quad. The stencil only allows to do or not do all work; repeating the process with different shaders allows us to run specific functionality conditionally.

3. Why did early work on GPU ray tracing focus on stackless traversal?

A stack requires a significant amount of space for each ray. When thousands of rays are processed concurrently, this exceeds the memory available on early GPUs.

4. Stackless methods visit more nodes in the acceleration structure. Why?

Stackless methods typically restart traversal, and thus revisit nodes until they reach the point where a different decision is made then before.

5. Why did the first BVH GPU ray tracing paper use a ray packet?

A packet requires a single stack for the entire packet, instead of a stack per ray. Also, on the CPU packets have been effective in reducing bandwidth; it was expected that GPUs would also benefit from this.
6. Why did 5-faces use a brickmap instead of e.g. a BVH? Under what circumstances is this a good idea / not a good idea?

7. In the paper “Understanding the Efficiency of Ray Traversal on GPUs”, Aila and Laine conclude that memory bandwidth is not the main bottleneck in GPU ray tracing. What is the bottleneck?

8. Explain speculative traversal.

9. How do we convert a 2-way BVH to a 4-way BVH?

10. If we ignore BVH traversal, why is the path tracing algorithm inefficient on the GPU? In other words: which problem is solved by streaming path tracing?
Exam

A BVH builds in $O(n \log n)$ time; a grid builds in $O(n)$ time. This is also true for a nested grid. Also, the construction can be done using rasterization hardware. The brickmap is great if all triangles (can) change every frame. Traversal is not fully robust to varying local scene detail, so this is only practical for demos.

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11. The Phong illumination model, as used in OpenGL, is not physically plausible. Why not?

12. Microfacet BRDFs use a geometry factor. What does this do?

13. What is the halfway vector, and what is it used for?

14. What is the solid angle of a light source with area $A$ and normal $\vec{N}_L$ located at distance $d$ from a shading point with normal $\vec{N}$?
11. The Phong illumination model, as used in OpenGL, is not physically plausible. Why not?

It doesn't normalize the diffuse and the specular terms, it doesn't take distance attenuation into account, and the brightness of the reflection is reduced when the specular exponent increases.

12. Microfacet BRDFs use a geometry factor. What does this do?

This vector is the normalized average of a vector to the eye and a vector to the light, and thus the only possible normal that yields a specular reflection of the light to the eye. The probability density of a particular halfway vector is the amount of light transport via the microfacet material.

13. What is the halfway vector, and what is it used for?

It accounts for self-shadowing of the microfacets for both incoming and outgoing light.

14. What is the solid angle of a light source with area $A$ and normal $\vec{N}_L$ located at distance $d$ from a shading point with normal $\vec{N}$?

$$\frac{A \cdot \vec{N}_L \cdot \vec{L}}{d^2},$$

where $\vec{L}$ is the normalized vector from the light to the shading point. Note that normal $\vec{N}$ at the shading point is irrelevant here.
15. How does next event estimation reduce variance?

16. Is next event estimation an unbiased approach?

17. Terminating a path after 20 bounces introduces bias in the stochastic experiment. What is bias?
15. How does next event estimation reduce variance?

16. Is next event estimation an unbiased approach?

17. Terminating a path after 20 bounces introduces bias in the stochastic experiment. What is bias?

   Bias is any deviation of the expected value from the correct integral.
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Upcoming attractions:

- Working college (Thursday)
- Assignment 3: January 27th, 23.59
- Exam: January 30th, 17.00, RUPPERT-BLAUW

(Dis)Liked the course? Suggestions?

➡ CARACAL EVALUATION ➡
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END of “Big Picture”
next up: “Big Exam”