Lecture 13 - “BRDFs”

Welcome!
Today's Agenda:

- Exam Questions: Sampler (3)
- Phong BRDF
- Microfacets
- Demo Time
- Quo Vadis
Exam Questions

In path tracing, we can reduce variance by using *cosine weighted* sampling of the hemisphere, rather than uniform sampling, for diffuse surfaces.

a) Why does this reduce variance?

b) When using cosine weighted sampling, the result remains unbiased. What does unbiased mean?
Today's Agenda:

- Exam Questions: Sampler (3)
- Phong BRDF
- Microfacets
- Demo Time
- Quo Vadis
BRDFs, Recap

Recall that a BRDF defines the relation between incoming and outgoing radiance for directions and a surface point:

\[ f_r(x, \theta_i, \theta_o) = \frac{L_o(x, \theta_o)}{E_i(x, \theta_i)} = \frac{L_o(x, \theta_o)}{L_i(x, \theta_i) \cos \theta_i} \]

What about materials that are not purely specular, nor diffuse?
Phong

BRDFs, Recap

We already know how to do materials that are diffuse and shiny.

But that gets us good looking marble floors, not glossy reflections.

50% diffuse, 50% specular,

50% diffuse, 50% glossy (or: 100% glossy)
Glossy Reflection

Glossy reflections:
- sending out rays in random directions close to the reflected vector.

Simple solution:
\[ \vec{R} = \text{reflect}(\vec{V}, \vec{N}); \]
\[ P = I + \vec{R} + \text{scale}(\text{randomPointInSphere}(), \text{specularity}) \]
\[ \vec{R} = \text{normalize}(P - I); \]

Or:
\[ \vec{R} = \text{reflect}(\vec{V}, \vec{N}); \]
\[ P = I + \vec{R} + \text{scale}(\text{randomDirectionInHemisphereCosineWeighted}(\vec{R}), \text{specularity}) \]
\[ \vec{R} = \text{normalize}(P - I); \]
Phong

Glossy Reflection

In OpenGL, shading is defined as follows*:

\[ I_x = k_{ambient}l_{ambient} + \sum_{m \in \text{lights}} (k_{diffuse} (\vec{N} \cdot \vec{L}_m) + k_{specular} (\vec{R}_m \cdot \vec{V})^{\text{exponent}}) l_m \]

where

- \( k_{ambient}, k_{diffuse} \) and \( k_{specular} \) are material properties (typically: rgb);
- \( \vec{R}_m \) is the vector \( \vec{L}_m \) reflected in the normal \( \vec{N} \);
- \( l_m \) is the illumination from light \( m \).

Phong

Blinn-Phong BRDF (images: Disney BRDF Explorer)
Phong

Modified Phong BRDF
Phong

Fixing Phong

Recall the requirements for a proper BRDF:

- Should be positive: \( f_r(\omega_o, \omega_i) \geq 0 \)
- Helmholtz reciprocity should be obeyed: \( f_r(\omega_o, \omega_i) = f_r(\omega_i, \omega_o) \)
- Energy should be conserved: \( \int_\Omega f_r(\omega_o, \omega_i) \cos \theta_o \, d\omega_o \leq 1 \)

BRDFs obeying these rules are called \textit{physically plausible}.

For a path tracer, we have additional requirements:

1. It ‘would be nice’ if we could generate a random direction proportional to the BRDF (IS)
2. We need to be able to calculate the probability density (importance) for a given direction (for MIS).
Fixing Phong

1. Sampling the specular lobe proportional to the BRDF:

Sampling proportional to $N \cdot L$, according to the G.I.C.:

$$
x = \cos(2\pi r_1) \sqrt{1 - r_2}
$$
$$
y = \sin(2\pi r_1) \sqrt{1 - r_2}
$$
$$
z = \sqrt{r_2}
$$

Sampling proportional to $(R \cdot V)^\alpha$, according to the G.I.C.:

$$
t = r_2^{\alpha + 1}
$$
$$
x = \cos(2\pi r_1) \sqrt{1 - t}
$$
$$
y = \sin(2\pi r_1) \sqrt{1 - t}
$$
$$
z = \sqrt{t}$$
### Tangent / Local Space

#### Setting up a local coordinate system in 2D:
- First axis is the normal;
- Second axis is perpendicular to normal.

\[
\vec{N} = n \\
\vec{T} = (-n, y) \\
\]

#### Setting up a local coordinate system in 3D:

\[
\vec{N} = n \\
\vec{T} = \text{normalize}(\vec{N} \times \vec{W}) \\
\vec{B} = \vec{T} \times \vec{N}
\]

where \(\vec{W}\) is a random unit vector; \(\vec{W} \neq \vec{N}\).
Phong

Tangent / Local Space

Converting a vector from world space to local space:

\[
\begin{pmatrix}
P_x \\
P_y \\
P_z
\end{pmatrix} = \begin{pmatrix}
P \cdot \vec{T} \\
P \cdot \vec{B} \\
P \cdot \vec{N}
\end{pmatrix}
\]

Local space to world space:

\[
P_{\text{world}} = M \times P_{\text{local}}
= P_x \vec{T} + P_y \vec{B} + P_z \vec{N}
\]
Normalizing the Lobe

A material cannot reflect more energy than it receives:

\[ \Rightarrow \text{We thus scale the BRDF by the inverse of its integral over the hemisphere.} \]

For the Lambertian BRDF: \( \text{scale} = \frac{1}{\pi} \) (because \( \cos \theta \) integrates to \( \pi \))

For the cosine lobe, the scale is \( \frac{\alpha + 1}{2\pi} \) (*). However, there is a problem:

*: Physically Based Rendering, page 969; also see: http://www.farbrausch.de/~fg/stuff/phong.pdf
Phong

The Modified Phong BRDF

The Phong BRDF has problems:

1. *it does not obey the Helmholtz reciprocity rule.*
2. *We can only estimate the integral of the lobe, since it intersects the surface.*

Compensating for the intersection: *Modified Phong BRDF.*

\[ f_r(x, \theta_i, \theta_o) = k_d \frac{1}{\pi} + k_s \frac{\alpha + 2}{2\pi} \cos^\alpha \varphi \]

where \( \varphi \) is the angle between the view vector and the reflected light vector.

For \( k_d + k_s < 1 \), this guarantees energy preservation*.

The Modified Phong BRDF

Despite the kludges, we now have a decent BRDF for glossy materials.

We can sample it:

\[
t = r_1^{\frac{\alpha + 1}{2}}, \quad x = \cos(2\pi r_1) \sqrt{1 - t} \\
y = \sin(2\pi r_1) \sqrt{1 - t} \\
z = \sqrt{t}
\]

We have a PDF:

\[
\frac{\alpha + 2}{2\pi} \cos^\alpha \varphi
\]

...Which is normalized, or at least preserves energy.

And finally, we can blend it with the Lambertian BRDF:

- Define a probability \( p \) of sampling Phong;
- Draw a random number \( r_0 \);
- Sample Phong if \( r_0 < p \), Lambert otherwise;
- Combine PDFs: \( PDF = p \ PDF_{phong} + (1 - p) PDF_{diff} \)
Today's Agenda:

- Exam Questions: Sampler (3)
- Phong BRDF
- Microfacets
- Demo Time
- Quo Vadis
BRDFs Without Issues

We now have two BRDFs without problems:

1. The Lambertian BRDF
2. The pure specular BRDF

These are physically plausible and can be sampled. The PDF is also clear.

And, we have the somewhat kludged modified Phong BRDF.
Microfacet

Microfacet BRDFs*

We can simulate a broad range of materials if we assume:

- at a microscopic level, the material consists of tiny specular fragments.

- If the fragment orientations are chaotic, the material appears diffuse.
- If the fragment orientations are all the same, the material appears specular.
- Different but similar orientations yield glossy materials.

---

* Torrance & Sparrow, Theory for Off-Specular Reflection from Roughened Surfaces. 1967.
Microfacet

Microfacet BRDFs*

The Microfacet BRDF:

\[ f_r(\mathbf{L}, \mathbf{V}) = \frac{F(\mathbf{L}, \mathbf{V})G(\mathbf{L}, \mathbf{V}, \mathbf{H})D(\mathbf{H})}{4(\mathbf{N} \cdot \mathbf{L})(\mathbf{N} \cdot \mathbf{V})} \]

Ingredients:

1. Normal distribution D
2. Geometry term G
3. Fresnel term F
4. Normalization
Microfacet

Normal Distribution

Microfacet BRDF, ingredient 1: \( D(\vec{H}) \) \( \Rightarrow \) the normal distribution function.

Parameter \( \vec{H} \): the halfway vector:

\[
f_r(\vec{V}, \vec{L}) = \ldots
\]

\[
f_r(x, \theta_i, \theta_o) = \frac{L_o(x, \theta_o)}{L_i(x, \theta_i) \cos \theta_i}
\]

A microfacet that reflects \( \vec{L} \) towards \( \vec{V} \) must have a normal halfway \( \vec{V} \) and \( \vec{L} \):

\[
H = \text{normalize}(V + L).
\]
Microfacet

Normal Distribution

Intuitive choices for $D$:

$$D \left( \overrightarrow{H} \right) = C: \text{microfacet normals are equally distributed} \Rightarrow \text{diffuse material.}$$

$$D \left( \overrightarrow{H} \right) = \begin{cases} \infty, & \text{for } \overrightarrow{H} = (0,0,1) \\ 0, & \text{otherwise} \end{cases}: \text{all microfacet normals are } (0,0,1) \Rightarrow \text{pure specular.}$$

Good practical choice for $D$: the Blinn-Phong distribution;

$$D \left( \overrightarrow{H} \right) = \frac{\alpha + 2}{2\pi} \left( \overrightarrow{N} \cdot \overrightarrow{H} \right)^{\alpha}$$
Microfacet

Geometry Term

Microfacet BRDF, ingredient 2: $G(\vec{V}, \vec{L}, \vec{H}) \Rightarrow$ the geometry term.
It describes what fraction of a microsurface with normal $\vec{H}$ is visible in both directions $\vec{L}$ and $\vec{V}$. 
Microfacet

Geometry Term

Intuitive choice for $G$:

$$G(\mathbf{V}, \mathbf{L}, \mathbf{H}) = 1: \text{no occlusion.}$$

Good practical choice for $G^*$:

$$G(\mathbf{V}, \mathbf{L}, \mathbf{H}) = \min(1, \min\left(\frac{2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{V})}{\mathbf{V} \cdot \mathbf{H}}, \frac{2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{L})}{\mathbf{V} \cdot \mathbf{H}}\right))$$

*: Physically Based Rendering, page 455
Microfacet

Fresnel Term

Microfacet BRDF, ingredient 3: $F(\vec{L}, \vec{H}) \Rightarrow$ the Fresnel term.

So far, we assumed that the light reflected by a specular surface is only modulated by the material color.

This is not true for dielectrics: here we use the Fresnel equations to determine reflection.

In nature, Fresnel does not just apply to dielectrics.
Microfacet

Fresnel Term

Iron is specular, but reflectivity differs depending on incident angle.

Aluminum is even more interesting: reflectivity depends on wavelength. The three lines in the graph:
Top: blue, middle: green, bottom: red.

Copper takes this to extremes: at grazing angles, it appears white. The lines in the graph:
Top: red, middle: green, bottom: blue.

(hence its reddish appearance)
Microfacet

Fresnel Term

For Fresnel, we once again use Schlick’s approximation:

\[ F_r = k_{specular} + (1 - k_{specular})(1 - (\vec{L} \cdot \vec{H}))^5 \]

Note that this is calculated per color channel (\(k_{specular}\) is an rgb triplet).

Values for \(k_{specular}\) for various materials:

- **Iron** 0.56, 0.57, 0.58
- **Copper** 0.95, 0.64, 0.54
- **Gold** 1.00, 0.71, 0.29
- **Aluminum** 0.91, 0.92, 0.92
- **Silver** 0.95, 0.93, 0.88
Bringing it All Together

The Microfacet BRDF:

\[ f_r(\mathbf{L}, \mathbf{V}) = \frac{F(\mathbf{L}, \mathbf{V}) G(\mathbf{L}, \mathbf{V}, \mathbf{H}) D(\mathbf{H})}{4(\mathbf{N} \cdot \mathbf{L})(\mathbf{N} \cdot \mathbf{V})} \]

\[ D(\mathbf{H}) = \frac{\alpha + 2}{2\pi} (\mathbf{N} \cdot \mathbf{H})^{\alpha} \]

\[ G(\mathbf{V}, \mathbf{L}, \mathbf{H}) = \min(1, \min \left( \frac{2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{V})}{\mathbf{V} \cdot \mathbf{H}}, \frac{2(\mathbf{N} \cdot \mathbf{H})(\mathbf{N} \cdot \mathbf{L})}{\mathbf{V} \cdot \mathbf{H}} \right)) \]

\[ F_r = k_{\text{specular}} + (1 - k_{\text{specular}})(1 - (\mathbf{L} \cdot \mathbf{H}))^5 \]

For a full derivation of the denominator of the BRDF, see Physically Based Rendering, section 8.4.2.
Microfacet

Lambertian BRDF

Advanced Graphics – BRDFs
Microfacet

Blinn-Phong Microfacet BRDF, $\alpha=1$
Microfacet

Blinn-Phong Microfacet BRDF, \( \alpha=10 \)
Microfacet

Blinn-Phong Microfacet BRDF, $\alpha=50$
Microfacet

Blinn-Phong Microfacet BRDF, $\alpha=500$
Microfacet

Blinn-Phong Microfacet BRDF, $\alpha=50000$
Microfacet

Specular BRDF

Survive = SurviveProbability( diffuse; estimation = doing it properly, closer) / pdf;
Radiance = SampleLight( Brand, I, AL, E Light;
+x + radiance; y + radiance,x,y) > O) & (pdf > 0);
V = true;
Ext BRDF = EvaluateBRDF( L, N ) * (pdf * Ext weight = M( direct, brent ),
Ext factor = diffuse = BRDF;
Ext coefficients = dot( N, L );
E (weight * cosThetaOut) / directRad) * (radial);
Shadow walk = done properly, closely following Surface
Survive = SampleDiffuse( diffuse, N, r1, r2, & pdf, pdf;
+ E = E * dot( N, R ) / pdf);
Today's Agenda:

- Exam Questions: Sampler (3)
- Phong BRDF
- Microfacets
- Demo Time
- Quo Vadis
Today's Agenda:

- Exam Questions: Sampler (3)
- Phong BRDF
- Microfacets
- Demo Time
- Quo Vadis
Quo Vadis

Anisotropic materials:
Quo Vadis

Quo Vadis

The Other Side: BSSRDF*

Quo Vadis

Quo Vadis

Measured BRDFs:
Today’s Agenda:

- Exam Questions: Sampler (3)
- Phong BRDF
- Microfacets
- Demo Time
- Quo Vadis
INFOMAGR – Advanced Graphics

Jacco Bikker  - November 2018 - February 2019

END of “BRDFs”

next lecture: “Bidirectional”