Please write clearly. Please do not ask for clarification during the exam. If you find a question unclear or ambiguous: write down how you interpret the question, then answer it. For each question you can score up to 10 points, your grade is points * 9 / 70 + 1.

IMPORTANT

1. When using Russian roulette, the recommended formula for survival probability is:

   \[ P_{\text{survive}} = \min(1, \max(\text{red}, \text{green}, \text{blue})) \]

   An alternative survival probability is:

   \[ P_{\text{survive}} = \min(1, (\text{red} + \text{green} + \text{blue})/3) \]

   Explain why and how the first formula is better.

2. Explain why the following techniques can be seen as importance sampling techniques:

   a) Next Event Estimation  
   b) Russian Roulette  
   c) For dielectrics: basing the probability of generating a reflection on the Fresnel term  

   Note, to be sure: I am not asking for a description of the techniques.

ACCELERATION STRUCTURES

3. Why is it often advantageous to have empty leaves in a kD-tree, and why do we not have these in a BVH?  

   Rays traversing empty space do not test primitives, which is good. So splitting a leaf may prevent primitive tests. In a BVH, this happens implicitly, as the bounds enclose the primitives, leaving empty space between the bounds.

4. We know that a BVH constructed using the SAH can be traversed more efficiently than one constructed using midpoint splits. But, why is this so?  

   The SAH leads to a tree that consists of AABBs with a low summed surface area, which directly affects the probability of a ray hitting these nodes.

BIAS AND APPROXIMATIONS

5. The solid angle of a planar area light source over the hemisphere of point \( p \) is approximately

   \[ \frac{A_{\text{light}} \cos \theta_{\text{light}}}{d^2}, \]

   where \( d \) is the distance between \( p \) and the light, \( A_{\text{light}} \) is the surface area of the light source and \( \cos \theta_{\text{light}} \) is the dot product between the light normal and a normalized vector from the light to \( p \).

   By now, it is well-known that this is an approximation. Why is this an approximation?
LIGT TRANSPORT

6. A programmer optimizes a path tracer for a GPU in the following unconventional manner.

The path tracer evaluates one path for each pixel. For these paths, a single light source is
randomly selected out of a large set of lights. This randomly chosen light source is the same
for all pixels.

a) After evaluating one sample per pixel, is the output of the path tracer unbiased?
   Motivate your answer.

b) After taking (and averaging) many samples, is the output of the path tracer unbiased?
   Motivate your answer.  The output is unbiased in both cases. Even though the result will look silly for a long
time, the expected value of the estimator equals the correct result, even after 1 sample.

7. Consider the following path tracer pseudo code:

Ray ray = generatePrimaryRay()
vec3 throughput = { 1, 1, 1 } 
vec3 energy = { 0, 0, 0 }
loop:
    IntersectionInfo intersection = scene.Intersect( ray )
    if (intersection.NoHit()) return energy
    if (intersection.HitLight())
        // we hit a light; set path energy and terminate
        energy += throughput * intersection.EmissiveColor
        return energy
    else
        // continue the random path
        vec3 R = NewRandomDirection( intersection.normal )
        throughput *= intersection.diffuse_color
        ray = new Ray( intersection.position, R ) 

Assume that the code produces correct output, and that the scene consists of pure
Lambertian (i.e. diffuse) surfaces, plus area lights. The distribution of the random directions
produced by ‘NewRandomDirection’ may or may not be uniform.

Under these assumptions:

Write down the pdf that is implicitly used in the random bounce code.

dot(R,N)/π. This is the pdf we used for cosine-weighted random directions. The dot is countered by the dot for the
conversion of radiance to irradiance and the division by π in the BRDF, so in this specific situation, things get really simple.

May the Light be with you!

Don’t forget to feed the Caracal.