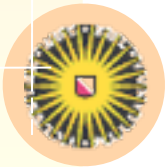


# Logica voor Artificiële Intelligentie

Part II, Lecture 7

Product Update with Action Models

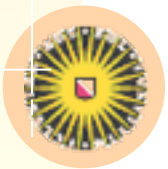


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# Updates not dealt with by public announcement logic

1. Public announcements that a **sub-group** of agents have learned whether or not  $\phi$ .
  2. Individuals suddenly getting suspicious that other agents **might** have learned something (e.g., did that muddy child not touch its forehead?)
- We describe the solution given in: **A. Baltag, L.S. Moss, and S. Solecki. *The logic of public announcements, common knowledge, and private suspicions*. Technical report, Dept of Cognitive Science, Indiana University and Dept of Computing, Oxford University, 2003.**



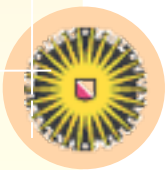
# Public announcement of a private update

**Example:** A and B are sitting at a bar. A flips a coin and immediately covers it with its hand. Neither of them knows whether the result is heads or tails.

How does the epistemic model look like? (blackboard)

Now A privately takes a look at the coin, B sees that A does this, A sees that B sees that he does this, etc.

How does the epistemic model look like? (blackboard)



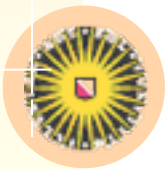
# Raising suspicion of a private update

**Example:** A and B are sitting at a bar. A flips a coin and immediately covers it with his hand. Neither of them knows whether the result is heads or tails.

We thus have the same starting model as before

Now B leaves the bar to go to the toilet. Coming back, he starts to suspect that A might have taken a look at the coin.

How does the epistemic model look like? (blackboard)



# Restricted Product and Action Models

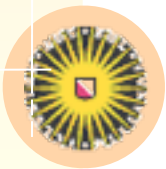
How do we, in general, construct the resulting models from the original models?

**Answer:** by constructing an **action model** representing the epistemic action, and taking a **restricted product** with the epistemic model.

Action models resemble Kripke models, but with the valuation function replaced by a **precondition function**.

Their ‘possible worlds’ are called **actions**.

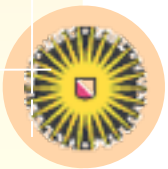
If an action has precondition  $\phi$  then the action can be applied to worlds where  $\phi$  holds.



# Updating with an Action Model

The result of updating with an action model is defined as the **product** of the epistemic model and the action model, **restricted** to the pairs  $(w, u)$  such that:

1.  $w$  satisfies the precondition of action  $u$ ,
2. the accessibility relations hold between pairs  $(w, u)$  and  $(w', u')$  just in case they hold **both** between  $w$  and  $w'$  and between  $u$  and  $u'$ , and
3. the valuation of propositional atoms in a world  $(w, u)$  in the product model is that of  $w$  in the original epistemic model.



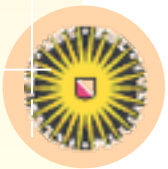
# Update Execution: Formal Definition

Given an epistemic model  $M = (W, V, R)$ , and an action model  $A = (E, \text{pre}, C)$ , we say that the result of executing  $A$  in  $M$  is the model  $M \otimes A = (W', V', R')$  where

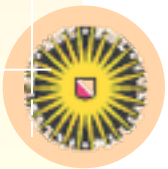
$$W' = \{(v, t) \mid v \in W, t \in E, \text{ and } M, v \models \text{pre}(t)\},$$

$$V'(v, t) = V(v), \text{ for } v \in W \text{ and } t \in E,$$

$$R'(a) = \{((v, t), (u, w)) \mid (v, u) \in R(a) \text{ and } (t, w) \in C(a)\},$$



To the extra slides



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# Public announcement logic is a special case

How do the action models for public announcements look like?

1. Only one action (possible world) in the action model: the public announcement
2. The precondition of the action is that the announcement has to be true
3. All agents know that this is the action taking place

Then taking the restricted product is exactly like **deleting worlds** as in PA and PAC.

So, the logics of PA and PAC are **a special case of product update**.



# Open questions

Are action models semantics or syntax?

Question: what about private communications whose **occurrence** is not made public and that **nobody suspects**?

Everything that is ‘not seen not to change’ can be / should be expected to change?

