

Condorcet's Paradox

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Marquis de Condorcet

Marie Jean Antoine Nicolas Caritat, marquis de Condorcet (September 17, 1743 – March 28, 1794) was a French philosopher, mathematician, and early political scientist who devised the concept of a Condorcet method. Unlike many of his contemporaries, he advocated a liberal economy, free and equal public education, constitutionalism, and equal rights for women and people of all races. His ideas and writings were said to embody the ideals of the Age of Enlightenment and rationalism, and remain influential to this day. He died a mysterious death in prison after a period of being a fugitive from French Revolutionary authorities.



Paradox Definition

The **voting paradox** (also known as **Condorcet's paradox** or the paradox of voting) is a situation noted by the Marquis de Condorcet in the late 18th century, in which collective preferences can be **cyclic** (i.e. not transitive), even if the preferences of individual voters are not. This is paradoxical, because it means that majority wishes can be in conflict with each other. When this occurs, it is because the conflicting majorities are each made up of different groups of individuals.

Simple Paradox Example

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3 Candidates: A, B, C
3 Voters: X, Y, Z

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Y	C	A	B
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Pairwise:

- $A > B$ (X, Y - Z)
- $B > C$ (X, Z - Y)
- $C > A$ (Y, Z - X)

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for instance: $A > B > C$, but $C > A$!!
(transitivity rule broken)

Condorcet Method

A **Condorcet method** is a voting system that will always elect the Condorcet winner; this is the candidate whom voters prefer to each other candidate, when compared to them one at a time. This candidate can easily be found by conducting a series of pairwise comparisons, using the basic procedure described in this article. The family of Condorcet methods is also referred to collectively as Condorcet's method. A voting system that always elects the **Condorcet winner** when there is one is described by electoral scientists as a system that satisfies the **Condorcet criterion**.

In certain circumstances an election has no Condorcet winner. This occurs as a result of a kind of tie known as a 'majority rule cycle', described by Condorcet's paradox. The manner in which a winner is then chosen varies from one Condorcet method to another. Some Condorcet methods involve the basic procedure described below, coupled with a Condorcet completion method— a special method used to find a winner when there is no Condorcet winner. Other Condorcet methods involve an entirely different system of counting, but are classified as Condorcet methods because they will still elect the Condorcet winner if there is one.

Condorcet Winner

If there is an alternative x that defeats every other alternative in pairwise comparison, this alternative x must win. Such an alternative is called a **Condorcet winner**.

Use of Condorcet Voting

Condorcet methods are not currently in use in government elections anywhere in the world, but a Condorcet method known as Nanson's method was used in city elections in the U.S. town of Marquette, Michigan in the 1920s [4], and today Condorcet methods are used by a number of private organisations. Organizations which currently use some variant of the Condorcet method are:

- The Debian project uses the Schulze method for internal referendums and to elect its leader
- The Software in the Public Interest corporation uses the Schulze method to elect members of its board of directors
- The Gentoo Linux project uses the Schulze method
- The Free State Project used Minimax for choosing its target state
- The voting procedure for the uk.* hierarchy of Usenet
- Five-Second Crossword Competition
- Babe Vs Babe people rating
- Kingman Hall, a student housing co-operative, uses the Schulze method for its elections

Arrow's Impossibility Theorem

“The **Arrow's Paradox**, pointed out in economics by Kenneth Arrow over two centuries after its discovery by Condorcet, is a more expanded version of **Condorcet's Paradox**, with multiple voters and policies (up to infinite), and the logic still holds. The phenomenon that results from it is cycling...”

Questions?

References

- Course website
 - @ <http://www.cs.uu.nl/docs/vakken/ig/> > Literature
- External Links
 - http://en.wikipedia.org/wiki/Voting_paradox
 - <http://www-rohan.sdsu.edu/~jwingram/condorcet.html> (simple dem. & example)
 - http://en.wikipedia.org/wiki/Arrow's_impossibility_theorem
 - <http://www.sjsu.edu/faculty/watkins/arrow.htm> (simple dem. & example)
 - http://en.wikipedia.org/wiki/Marquis_de_Condorcet
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