Multi-Agent Logics

- Coalition Logic
- Alternating-time Logic (ATL)
- Epistemic Variants (ATEL, ATEL-A)

Coalition Logic

- We repeat some slides from MAIR...

Non-normal modal logic

- We have seen that in normal modal logic (based on Kripke semantics) it holds that:
  $\vdash (\Box \phi \land \Box (\phi \rightarrow \psi)) \rightarrow \Box \psi$
- If one does not want this property one should use non-normal modal logic (based on neighborhood semantics)

“Minimal” Models

- $\mathcal{M} = (S, \pi, N)$
- Where
  - $S$ is a set of worlds
  - $\pi$ is a truth assignment function
  - $N$ is a function $S \rightarrow P(P(S))$

Neighborhood semantics

- Accessibility relation between a world and its accessible worlds is generalized to a relation between a world and a collection of sets of worlds

Interpretation

- The sets in the collection are called neighborhoods and represent propositions that are necessary in world $w$
- So semantics:
  - $\mathcal{M}, w \models \Box \phi \iff \|\phi\|^\mathcal{M} \in N(w)$
  - where $\|\phi\|^\mathcal{M}$ is the truth set of $\phi$ in $\mathcal{M}$:
  - $\|\phi\|^\mathcal{M} = \{w \mid \mathcal{M}, w \models \phi\}$
The following system is sound & complete w.r.t. the class of minimal models: PC (incl MP) +

\[ \phi \leftrightarrow \psi \n \square \phi \leftrightarrow \square \psi \]

Of course, this is a very weak logic, but it is possible again to put constraints on the models in order to get again properties such as D, T, 4, 5! (Cf. Chellas, Ch. 7)

Operator \([C]\phi\)
- “coalition C is able to ensure \(\phi\)”
- Semantics based on neighborhood models, with extra properties
- \(\mathcal{M} = (\mathcal{S}, \pi, E)\)
  - \(\mathcal{S}\) is a nonempty set of worlds/states
  - \(\pi\) is a truth assignment function
  - \(E : \mathcal{S} \rightarrow (2^{\text{P}(\text{AGT})})\) effectivity function

\(\mathcal{M}, s \models [C]\phi \iff \|\phi\|^\mathcal{M} \in E(w)(C)\)

\(\mathcal{M}, s \models [C]\phi \iff \|\phi\|^\mathcal{M} \in E(w)(C)\)

Intuition: \(E(w)(C)\) is the collection of sets \(X \subseteq \mathcal{S}\) such that C can force the world to be in some state of \(X\) (where \(X\) represents a proposition)
Towards ATL and ATEL

Alternating time temporal logic (ATL)
- Combination of CTL and Coalition Logic

Alternating time epistemic logic (ATEL)
- Combination of CTL, Coalition Logic and epistemic logic

First we look at CTL (a restricted version of CTL*)

CTL (Restricted CTL*)

- CTL is restricted version of CTL*
- The wff's of CTL are given by:
  \[ \phi ::= T \mid p \mid \neg \phi \mid \phi_1 \vee \phi_2 \mid E \Box \phi \mid E \Diamond \phi \mid E (\phi_1 U \phi_2) \]
- So as compared to CTL* only combinations of path quantifiers and linear time temporal operators allowed

ATL (Alternating-time Temporal Logic)

- The syntax of ATL is a generalization of that of CTL: (C ⊆ AGT, the set of agents)
  \[ \psi ::= T \mid p \mid \neg \psi \mid \psi_1 \vee \psi_2 \mid (\langle C \rangle) \Box \psi \mid (\langle C \rangle) \Diamond \psi \mid (\langle C \rangle) (\psi_1 U \psi_2) \]
  \[ (\langle C \rangle) \psi \] is a special operator stating that the set of agents C (coalition) has a strategy to ensure that \( \psi \) is true.
- Note: \( \langle \text{AGT} \rangle = E \) and \( \langle \emptyset \rangle = A \).

Semantics: Concurrent Game Structures

- Although ATL is similar to CTL, different models are used to interpret wff's:
  - Concurrent Game Structures.

Concurrent Game Structures

- S = <k, Q, At, π, d, δ> with
  - \( k \geq 1 \): number of agents/players
  - \( Q \): finite set of states
  - \( \text{At} \): finite set of atomic propositions ('observables')
  - \( \pi \): (labeling / observation) function denoting the true propositions per state
  - \( d(q) \geq 1 \): denoting the number of moves available at state q to player a

Concurrent Game Structures, ctd.

- for each \( q \in Q \), a move vector at q is a tuple \( \langle j_1, \ldots, j_k \rangle \) s.t. 1 \( \leq j_a \leq d(q) \) for each player a.
- Given \( q \in Q \), we write \( D(q) \) for the set \( \{1, \ldots, d(q)\} \times \{1, \ldots, d(q)\} \) of move vectors at q. \( D \) is called move function.
- \( \delta \): transition function: for each \( q \in Q \) and move vector \( \langle j_1, \ldots, j_k \rangle \in D(q) \), \( \delta(q, \langle j_1, \ldots, j_k \rangle) \in Q \) denotes the state resulting from state q if every player a chooses move \( j_a \).
Concurrent Game Structures, ctd.

- $q'$ is a successor of $q$ if there is a move vector $<j_1, \ldots, j_k> \in \mathcal{D}(q)$ s.t. $q' = \delta(q, j_1, \ldots, j_k)$
- A computation of $S$ is an infinite sequence $\lambda = q_0, q_1, q_2, \ldots$ such that every $q_{i+1}$ is a successor of $q_i$.
- A $q$-computation is a computation starting at $q$.
- Notations $\lambda[i], \lambda[0,i], \lambda[i, \infty]$.

Kripke structures seen as CGS

- A Kripke structure can be viewed as a special case of a CGS for a single player $(k = 1)$.

Turn-based game structures

- In a turn-based game structure, at every state, only a single player has a choice of moves.
- Formally, if for every $q \in Q$ there exists a player $a_q \in \{1, \ldots, k\}$ s.t. $d_b(q) = 1$ for all players $b \in \{1, \ldots, k\} \setminus \{a_q\}$.
- We say that in $q$ it is the turn of player $a_q$.

ATL Semantics

- ATL formulas are interpreted over the states of a concurrent game structure $S$.
- In particular, to evaluate formulas of the form $\langle C \rangle \psi$ at a state $q$ of $S$, we consider the following game between a protagonist and an antagonist, proceeding in an infinite sequence of rounds, after each round the game is in a state of $S$. Initial state is $q$.

ATL Semantics, ctd.

- Consider the state of the game is $u$. To update this state, first the protagonist chooses for every $a \in C$ a move $j_a \in \{1, \ldots, d_a(u)\}$.
- Next the antagonist chooses for every player $b$ in $\text{AGT} \setminus C$ a move $j_b \in \{1, \ldots, d_b(u)\}$, and the state is updated to $\delta(u, j_1, \ldots, j_k)$.
- This produces a computation.

ATL Semantics, ctd.

- Protagonist wins if the resulting computation satisfies the LTL formula $\psi$.
- Antagonist wins otherwise.
- The ATL formula $\langle C \rangle \psi$ is satisfied at state $q$ iff the protagonist has a winning strategy.
Formal semantics

- Given a game structure $S = \langle k, Q, A, \pi, d, \delta \rangle$, $\text{AGT} = \{1, \ldots, k\}$.
- Strategy for player $a \in \text{AGT}$ is a function $f_a$ mapping every finite state sequence $\lambda \in Q^+$ to a natural number s.t. if the last state of $\lambda$ is $q$, then $f_a(\lambda) \leq d_a(q)$. I.e. the strategy $f_a$ determines for every finite prefix $\lambda$ of a computation a move $f_a(\lambda)$ for player $a$.

Formal semantics (ctd)

- Each strategy $f_a$ for player $a$ induces a set of computations that player $a$ can enforce.
- Given $q$, $C \in \text{AGT}$, set $F_C = \{f_a | a \in C\}$ of strategies, one for each player in $C$, we define the set of outcomes $\text{out}(q, F_C)$ of $q$-computations that coalition $C$ can enforce when following the strategies in $F_C$:

Computation $\lambda = q_0, q_1, q_2, \ldots$ is in $\text{out}(q, F_C)$ if $q_0 = q$ and for all $i \geq 0$ there is a move vector $<j_1, \ldots, j_k>$ in $D(q_i)$ s.t.

- $(1) j_a = f_a(\lambda[0,i])$ for all $a \in C$, and
- $(2) \delta(q_i, j_1, \ldots, j_k) = q_{i+1}$

Interpretation of ATL wff’s

- Given a CGS $S$ and state $q$, the interpretation $S, q \models \psi$ is defined as:
  - $S, q \models p$ iff $p \in \pi(q)$ (if $p$ is atomic)
  - $S, q \models \neg \phi$ iff $S, q \not\models \phi$
  - $S, q \models \phi_1 \lor \phi_2$ iff $S, q \models \phi_1$ or $S, q \models \phi_2$
  - $S, q \models (\langle C \rangle) \psi$ iff there exists $f_c \in \text{Str}(C)$ such that for all $\lambda \in \text{out}(q, f_c)$, $\lambda[1] \models \psi$

ATL*: syntax

- State formulas ($C \subseteq \text{AGT}$):
  - propositional atoms in a set $A\text{t}$
  - $\phi_1, \phi_2$ state formulas ⇒ $\neg \phi_1, \phi_1 \lor \phi_2, \ldots$ state formulas
  - $\psi$ path formula ⇒ $\langle \langle C \rangle \rangle \phi$ state formula
**ATL*: syntax**

- Path formulas:
  - any state formula
  - $\psi_1, \psi_2$ path formulas
    $\Rightarrow$ $\neg \psi_1, \psi_1 \lor \psi_2, \psi_1 \cup \psi_2, \Box \psi_1, O \psi_1$ path formulas

**ATL*: semantics of state forms**

- $S,q \models p$ iff $p \in \pi(q)$ (if $p$ is atomic)
- $S,q \models \neg \psi \iff S,q \not\models \psi$
- $S,q \models \psi_1 \lor \psi_2 \iff S,q \models \psi_1$ or $S,q \models \psi_2$
- $S,q \models (C)\psi$ iff there exists $F_C \in \text{Str}(C)$ such that for all $\lambda \in \text{out}(q,F_C)$, $S,\lambda \models \psi$

**ATL*: semantics of path forms**

- $S,\lambda \models \psi$ iff $S,\lambda[0] \models \psi$ (a state formula)
- $S,\lambda \models \neg \psi \iff S,\lambda \not\models \psi$
- $S,\lambda \models \psi_1 \lor \psi_2 \iff S,\lambda \models \psi_1$ or $S,\lambda \models \psi_2$
- $S,\lambda \models O \psi \iff S,\lambda[1,\infty] \models \psi$
- $S,\lambda \models \Box \psi \iff S,\lambda[n,\infty] \models \psi$ for some $n \geq 0$

**ATL*: semantics path forms, ctd**

- $S,\lambda \models \psi_1 \cup \psi_2 \iff$
  - (a) exists $k \geq 0$ s.t. $S,\lambda[k,\infty] \models \psi_2$ and for all $0 \leq j < k$: $S,\lambda[j] \models \psi_1$, or
  - (b) for all $k \geq 0$: $S,\lambda[k,\infty] \models \psi_1$

*Note: clause (b) not in Alur et al. They define a strong until.*

**Towards an epistemic variant**

- ATEL (Alternating-time epistemic logic)
- Based on Concurrent Epistemic Game Structures (CEGSs)
- Problems with finding the right formalization
- A possible solution: ATL-A, ATEL-A (Ågotnes)

**Motivation**

- ATL is a logic about what agents can do, alone or in groups.
- Already Moore (1984) pointed out that agents generally have incomplete information so that a proper logic of action should incorporate knowledge.
Motivation, ctd.

Moore identified two main interactions between action and knowledge:
1. That knowledge is required to taking action
2. That actions may change knowledge

As to 1: a particular instance is knowledge about all available actions: i.e. the same actions are available in indiscernible states! (¶)

ATEL

Syntax is that of ATL together with a knowledge operator $K_i$ for every agent $i \in AGT$

Semantics: based on concurrent epistemic game structures which are of the form $<k, Q, At, \pi, d, \delta, \sim_1, \ldots, \sim_k>$, where $<k, Q, At, \pi, d, \delta>$ is a CGS and $\sim_i \subseteq Q \times Q$ is an equivalence relation, for each agent $i$.

Interpretation of $K_i$

$S, q \models K_i \varphi$ iff for all $q' \sim_i q$: $S, q' \models \varphi$

Problems

The property (¶) can be formalized in CEGSs as:

(3) $q \sim_i q' \implies d_i(q) = d_i(q')$

However, this property appears to be inexpressible in ATEL!

Proof: Ågotnes provides two models (actually frames, i.e. models without labelling function), in which one of them is satisfying (3) while in the other: the same but an ‘equivalent’ action is deleted (so not satisfying (3)), while they still make the same ATEL formulas true.

Possible solution: AT(E)L-A

Extending ATL and ATEL with actions (not unlike dynamic logic) thus extending expressibility

C(E)GSs are extended with ACT

Instead of $\langle C \rangle \Omega \varphi$ we now have $\langle \langle A, C \rangle \rangle \Omega \varphi$

with $C \subseteq AGT$, $A \in A^x$

where $A^x = (\mathcal{P}(ACT) \setminus \{\emptyset\})^k$

AT(E)L-A

Notation: given an $A \in A^x$, we use $A_j$ for the $j$-th component of $A$, and $A^x = \prod_{i \in AGT} A_j$

Notation: given $\bar{a} = (a_1, \ldots, a_k) \in \text{ACT}^k$ and $\bar{e} = (e_1, \ldots, e_k) \in \text{ACT}^k$ then $\bar{e}[\bar{a}/C] = (c_1, \ldots, c_k)$ where $c_i = a_i$ ($i \in C$) and $c_i = e_i$ (not $i \in C$)
ATEL-A

- \( S, q \models \langle \langle A, C \rangle \rangle \phi_q \) iff
- exists \( \bar{a} \in A \cap D(q) : \) for all \( \bar{u} \in A \cap D(q) : \)
- \( S, \delta(q, \bar{u}[\bar{a}/C]) \models \phi \)
- very succinct; equivalently: iff
- exists \( e_1 \in d_1(q) \cap A_1 \) ... \( e_m \in d_m(q) \cap A_m \) for all \( e_{z_1} \in d_{z_1}(q) \cap A_{z_1} \)
- ... for all \( e_{z_1} \in d_{z_1}(q) \cap A_{z_1} S, \delta(q, \bar{e}) \models \phi \)
- where \( C = \{ g_1, \ldots, g_m \}, \text{AGT} \setminus C = \{ s_1, \ldots, s_p \} \)

ATEL-A solves the problem

- The schema
  \[ \langle \langle a, \{i\} \rangle \rangle \to K_i \langle \langle a, \{i\} \rangle \rangle \]
- defines the property
  \( (3) \ q \sim q' \Rightarrow d(q) = d(q') \)
- Proof: see Ágotnes.
- Exercise: prove that (3) turns the schema into a validity.

ATEL and its variants:

- The paper by Ágotnes is quite technical, and we need not go into all details, but it shows that ‘obvious solutions’ to rather commonsense problems are sometimes hard to find.
- Undoubtedly, the story does not end here...! But this course does... ;-)