Multi-Agent Logics

- Coalition Logic
- Alternating-time Logic (ATL)
- Epistemic Variants (ATEL, ATEL-A)
Coalition Logic

We repeat some slides from MAIR…
Non-normal modal logic

We have seen that in normal modal logic (based on Kripke semantics) it holds that:

\[ \vdash (\square \varphi \land \square (\varphi \rightarrow \psi)) \rightarrow \square \psi \]

If one does *not* want this property one should use non-normal modal logic (based on neighborhood semantics)
Neighborhood semantics

Accessibility relation between a world and its accessible worlds is generalized to a relation between a world and a collection of sets of worlds.
“Minimal” Models

\[ \mathcal{M} = \langle S, \pi, N \rangle \]

Where
- \( S \) is a set of worlds
- \( \pi \) is a truth assignment function
- \( N \) is a function \( S \to \mathcal{P}(\mathcal{P}(S)) \)
Interpretation

The sets in the collection are called neighborhoods and represent propositions that are necessary in world w.

So semantics:

\[ \mathcal{M}, w \models \Box \phi \iff \| \phi \|^{\mathcal{M}} \in N(w) \]

where \( \| \phi \|^{\mathcal{M}} \) is the truth set of \( \phi \) in \( \mathcal{M} \):

\[ \| \phi \|^{\mathcal{M}} = \{ w \mid \mathcal{M}, w \models \phi \} \]
The following system is sound & complete w.r.t. the class of minimal models: PC (incl MP) +

\[ \phi \leftrightarrow \psi \]

\[ \Box \phi \leftrightarrow \Box \psi \]
Extensions

Of course, this is a very weak logic, but it is possible again to put constraints on the models in order to get again properties such as D, T, 4, 5! (Cf. Chellas, Ch. 7)
Coalition Logic

- Operator $[\text{C}]\phi$
  - “coalition C is able to ensure $\phi$”
  - Semantics based on neighborhood models, with extra properties
  - $\mathcal{M} = \langle S, \pi, E \rangle$
    where
    - S is a nonempty set of worlds/states
    - $\pi$ is a truth assignment function
    - $E : S \rightarrow (\mathcal{P}(\text{AGT}) \rightarrow \mathcal{P}(\mathcal{P}(S)))$ effectivity function
(Playable) effectivity function

- E satisfies
  - not Ø ∈ E(w)(C)
  - S ∈ E(w)(C)
  - (not S\X ∈ E(w)(Ø)) ⇒ X ∈ E(w)(AGT)
  - X ∈ E(w)(C) & X ⊆ Y ⇒ Y ∈ E(w)(C)
  - X ∈ E(w)(C₁) & Y ∈ E(w)(C₂) &
    C₁ ∩ C₂ = Ø ⇒ X ∩ Y ∈ E(w)(C₁ ∪ C₂)
**Interpretation**

\[ \mathcal{M}, s \models [C] \phi \iff \| \phi \|^{\mathcal{M}} \in E(w)(C) \]

**Intuition:** \( E(w)(C) \) is the collection of sets \( X \subseteq S \) such that \( C \) can force the world to be in some state of \( X \) (where \( X \) represents a proposition).
Sound & complete axiomatization

- PC (incl. (MP))
- $\neg [C] \bot$
- $[C] \top$
- $\neg [\emptyset] \neg \phi \rightarrow [\text{AGT}] \phi$
- $[C](\phi \land \psi) \rightarrow [C] \phi \land [C] \psi$
- $[C_1] \phi \land [C_2] \psi \rightarrow [C_1 \cup C_2] (\phi \land \psi)$
- $\phi \leftrightarrow \psi / [C] \phi \leftrightarrow [C] \psi$

if $C_1 \cap C_2 = \emptyset$