(Prop.) Branching temporal logic

- **CTL***
  - Formulas:
    - *State* formulas: pertaining to *states* in time tree
    - *Path* formulas: pertaining to *paths* in time tree

**CTL*** syntax

- **State** formulas
  - propositional atoms in a set At
  - \( \Box_i, \Diamond_i \) *state formulas* \( i \)
  - \( \neg \Box_i, \Diamond_i, \Box_i \Diamond_i, \Box_i \Diamond_i \) *state formulas*
  - \( \Box \) *path formula* \( i \) \( E \), \( A \) *state formula*

**CTL*** semantics

- **Models** are temporal structures \( M = (S, R, p) \) where
  - \( S \) is a set of states
  - \( R \subseteq S \times S \), a serial binary relation, a *tree*
  - \( p : S \to P(At) \) is a truth assignment function
- **A fullpath** is an *infinite* sequence \( s_0, s_1, \ldots \) of states such that \( \Box(s_0, s_1) R \).

**CTL*** semantics

Given \( M = (S, R, p) \):
- \( M, s_0 \models P \) if \( P \in s_0 \) (P \in At)
- \( M, s_0 \models \neg \Box_i \) if \( \neg M, s_0 \models \Box_i \)
- \( M, s_0 \models \Box_i \) if \( M, s_0 \models \Box_i \)
- \( M, s_0 \models \Box_i \) or \( M, s_0 \models \Diamond_i \)
- \( M, s_0 \models \Box_i \Diamond_i \) if exists fullpath \( s_0, s_1, \ldots \) in \( M \) s.t. \( M, s_0 \models \Box_i \)
- \( M, s_0 \models A \) if for all fullpath \( s_0, s_1, \ldots \) in \( M \):
  - \( M, s_0 \models \Box_i \)
- \( M, s_0 \models \Box_i \Diamond_i \) for some \( n \geq 0 \)

**CTL*** semantics

- \( M, s_0 \models \Box_i \) if \( s_0 \models \Box_i \) (state form)
- \( M, s_0 \models \neg \Box_i \) if \( \neg M, s_0 \models \Box_i \)
- \( M, s_0 \models \Box_i \) or \( M, s_0 \models \Box_i \)
- \( M, s_0 \models \Box_i \Diamond_i \) if \( s_0 \models \Box_i \) or \( M, s_0 \models \Box_i \)
- \( M, s_0 \models \Box_i \Diamond_i \) if \( M, s_0 \models \Box_i \)
- \( M, s_0 \models \Box_i \Diamond_i \) for some \( n \geq 0 \)
**CTL* semantics**

  - (a) exists \( k \geq 0 \) s.t. \( M, [] \models [1] \), and for all \( 0 \leq j < k \), \( M, [] \models [j] \), or
  - (b) for all \( j \geq 0 \), \( M, [] \models [j] \).

**BDI logic**

- **combination of:**
  - branching time temporal logic (CTL*)
  - modal logic(s) of belief, desires & goals (intentions)

**BDI logic: syntax**

- **extension of the (first-order version of) branching-time temporal logic CTL***

  - **temporal operators**
    - \( U, [j], [O], \) optional, inevitable

  - **Modalities:**
    - \( \text{BEL}([j]), \text{GOAL}([j]), \text{INTEND}([j]) \)

**BDI logic: syntax**

- **State formulas**
  - any first-order formula
  - \( [1], [j], [O] \) state formulas
  - \( [O, [1], [O]] \) (k) state formulas
  - event type \( [succeeded(e)], [failed(e)] \) state formulas
  - \( [1] \) state formula \( [\text{BEL}([j]), \text{GOAL}([j]), \text{INTEND}([j])] \) state formulas
  - \( [1] \) path formula \( [\text{optional}()] \) state formula

**BDI logic: syntax**

- **Path formulas**
  - any state formula
  - \( [1], [O] \) path formulas
  - \( [O, [1], [O], [O], [1], [O], [O]] \) path formulas

**BDI logic: syntax**

- **abbreviations**
  - \( [] = ~[O] \)
  - inevitable([j]) = ~optional(~[j])
  - done(e) = succeeded(e) \& failed(e)
  - succeeds(e) = inevitable(O(succeeded(e)))
  - fails(e) = inevitable(O(failed(e)))
  - does(e) = inevitable(O(done(e)))
BDI logic

\[ M = (W, E, T, <, U, B, G, I, []) \]

- \( W \) set of possible worlds
- \( E \) set of primitive event types
- \( T \) set of time points
- \( < \) a binary relation on time points, serial, transitive, backward-linear
- \( U \) universe of discourse
- \( \Delta \) mapping of first-order entities to \( U \), for any world and time point
- \( B, G, I \) \( W \to T \to W \) accessibility relations for \( \text{BEL, GOAL, INTEND} \)

BDI logic: semantics

\[ R(w,t) = \{ w' \mid R(w,t,w') \} \]

- \( \Delta R_B, \Delta R_G, \Delta R_I \) for any world and time point
- \( B, G, I \) \( W \to T \to W \) accessibility relations for \( \text{BEL, GOAL, INTEND} \)

BDI logic: possible worlds

\[ w = (T_w, A_w, S_w, F_w) \]

- \( T_w \) set of time points in world \( w \)
- \( A_w \) restriction of \( < \) to \( T_w \)
- \( S_w : T_w \to T_w \) E maps adjacent time points to (successful) events
- \( F_w : T_w \to T_w \) E maps adjacent time points to (failing) events
- the domains of the functions \( S_w \) and \( F_w \) are disjoint

BDI logic: fullpaths in worlds

- **Fullpath** in world \( w \) is an infinite sequence of time points \( (t_0, t_1, \ldots) \) such that \( (t_0, t_1) \in A_w \)
- fullpath \( (t_0, t_1, \ldots) \) in world \( w \) is denoted as: \( (w_{t_0}, w_{t_1}, \ldots) \)
BDI logic: semantics of state f.

- $M, v, w_1 \models q(y_1, ..., y_n) \land (v(y_1), ..., v(y_n)) \models (q, w, t)$
- $M, v, w_1 \models \Box \models M, v, w_1 \models \Box
- M, v, w_1 \models \Box \models M, v, w_2 \models \Box
- M, v, w_1 \models \Box \models M, v, w_1 \models \Box
- M, v, w_1 \models \Box \models M, v, w_1 \models \Box
- M, v, w_1 \models [x \Box \models M, v(d/x), w_1 \models []$ for some $d \in U$

Semantics of state formulas (ctd)

- $M, v, w_1 \models \text{optional}(\Box)$ \ exists fullpath $(w_{i_0}, w_{i_1}, ...) \ s.t. M, v, (w_{i_0}, w_{i_1}, ...) \models []$
- $M, v, w_1 \models \text{BEL}(\Box)$ \ for all $w' \in B(w, t)$: $M, v, w_1 \models []$
- $M, v, w_1 \models \text{GOAL}(\Box)$ \ for all $w' \in G(w, t)$: $M, v, w_1 \models []$
- $M, v, w_1 \models \text{INTEND}(\Box)$ \ for all $w' \in I(w, t)$: $M, v, w_1 \models []$

Semantics of path formulas

- $M, v, (w_{i_0}, w_{i_1}, ...) \models \Box \models M, v, w_{i_0} \models \Box$

(state form)
- $M, v, (w_{i_0}, w_{i_1}, ...) \models \Box \models M, v, (w_{i_0}, w_{i_1}, ...) \models []$
- $M, v, (w_{i_0}, w_{i_1}, ...) \models \Box \models M, v, (w_{i_0}, w_{i_1}, ...) \models []$
- $M, v, (w_{i_0}, w_{i_1}, ...) \models \Box \models M, v, (w_{i_0}, w_{i_1}, ...) \models []$
- $M, v, (w_{i_0}, w_{i_1}, ...) \models []$ for some $k \geq 0$

Semantics of path formulas (ctd)

- $M, v, (w_{i_0}, w_{i_1}, ...) \models \Box \models M, v, w_{i_0} \models \Box$

- (a) exists $k \leq 0$ s.t. $M, v, (w_{i_0}, ...) \models []$ and
- for all $0 \leq j < k$: $M, v, (w_{i_j}, ..., w_{i_k}) \models []$, or
- (b) for all $j \geq 0$: $M, v, (w_{i_j}, ...) \models []$

State formulas pertaining to events

- $M, v, w_1 \models \text{succeeded}(e) \models$ there exists $t_{10}$ s.t.
  $S_{10}(0, 11) = e$
- $M, v, w_1 \models \text{failed}(e) \models$ there exists $t_{10}$
  s.t. $F_{10}(0, 11) = e$