2007/2008 Graphics Tutorial 3

Problem 1 *Let the following planes in* $\mathbb{R}^3$ *be given:*

- $V_1 : 3x + 2y + z = 10$
- $V_2 : 2x + y + 3z = 13$
- $V_3 : x + 3y + 2z = 13$

1. *Compute the coordinates of the intersection point* p *of the three planes with* Gaussian elimination. *Check that* p *indeed satisfies all three plane equations.*
2. *Express each coordinate of* p *as a fraction of two determinants, and work out the* x *coordinate.*

The system of linear equations of the above problem has a unique solution. Other systems may have no solutions at all, or an infinite number of solutions. In the next problem, we consider a system that has no solutions (such a system is called *inconsistent*):

Problem 2 *Let the following planes in* $\mathbb{R}^3$ *be given:*

- $V_1 : 3x + 2y + z = 10$
- $V_2 : 6x + 4y + 2z = 26$
- $V_3 : x + 3y + 2z = 13$

1. *What is the geometric interpretation of the (empty) set of solutions of this system?*
2. *What happens of we try to solve this system with Gaussian elimination?*
3. *We can write the system in matrix notation as* $Ax = b$. *What is the determinant of* A *in this case?*

In the next problem, we consider a system that has an infinite number of solutions:

Problem 3 *Let the following planes in* $\mathbb{R}^3$ *be given:*

- $V_1 : 3x + 2y + z = 10$
- $V_2 : 6x + 4y + 2z = 20$
- $V_3 : x + 3y + 2z = 13$

1. *What is the geometric interpretation of the set of solutions of this system?*
2. *What happens of we try to solve this system with Gaussian elimination?*
3. *We can write the system in matrix notation as* $Ax = b$*. What is the determinant of* A *in this case?*
4. *Although we "get stuck" with Gaussian elimination, we can get the matrix in* upper diagonal form*: this is a situation where the matrix (or more precisely, the left (square) part of the augmented matrix) only has non-zero entries on or above the diagonal. We can use this to find a parametric equation that satisfies the three plane equations. In our case, for example, we could state* $z = t$*, and work our way up the matrix to find* y *and* x *expressed in the parameter* t *as well. Use this technique to find a parametrix equation for the line that is the intersection of the three planes.*

Problem 4 *A matrix is* orthogonal *if all colunm vectors are mutually perpendicular. A matrix is* orthonormal *if it is orhogonal and all column vectors have unit length.*

Show that the inverse of an $n \times n$ *orthonormal matrix is its transpose. Hint: show that for a* 2×2 *matrix* A*, we have* $A^T A = I$*, and then generalize.*