2007/2008 Graphics Tutorial 2

Problem 1 *What is the relation between the length of a vector and the dot product of the vector with itself?*

Problem 2 *What do we know if* $v \cdot w = 0$ *for two non-null vectors* v *and* w*?*

Problem 3 *Show that the cross product is not commutative..*

Problem 4 *let* ℓ *be a line through the origin in* $\mathbb{R}^2$*, and let* $n = (x_n, y_n)$ *be a normal vector for* ℓ*. Show* geometrically *that all points* $p = (x_p, y_p)$ *that lie on the line satisfy* $n \cdot p = 0$.

Problem 5 *let* ℓ *be a line in* $\mathbb{R}^2$ *that does* not *go through the origin, and let* $n = (x_n, y_n)$ *be a normal vector for* ℓ*. Show* geometrically *that all points* $p = (x_p, y_p)$ *that lie on the line satisfy* $n \cdot p - n \cdot p' = 0$*, where* p' *is an arbitrary point on the line.*

Problem 6 *Let* ℓ *be a line in* $\mathbb{R}^2$ *with implicit equation* $f(x, y) = ax + by + c = 0$*. For points that do* not *lie on the line,* $f(x, y) \neq 0$*. The line* ℓ *splits* $\mathbb{R}^2$ *in two* half-spaces*: the* positive *half-space* ℓ^+*, containing the points in* $\mathbb{R}^2$ *that lie on the side of* ℓ *to which the normal vector* (a, b) *points, and the* negative *half-space* ℓ^-*, containing the remaining points not on* ℓ*. Show that the name* positive half-space *is appropriate by arguing that* $f(x_p, y_p) > 0$ *for any point* $p = (x_p, y_p)$ *that lies in the halfspace to which the normal vector* (a, b) *points.*

Problem 7 *Give a* parametric equation *for the plane* V *in* $\mathbb{R}^3$ *through the points* $p_1 = (0, 7, 6)$*,* $p_2 = (8, 0, 8)$*, and* $p_3 = (12, 10, 0)$*.*

Problem 8 *Give an* implicit equation *for the plane* V *from the previous problem.*

Problem 9 *The line* ℓ *in* $\mathbb{R}^3$ *passes through the points* $q_1 = (7, 8, 9)$ *and* $q_2 = (10, 11, 12)$*. Determine the intersection of* ℓ *and the plane* V *from the previous two problems.*

Problem 10 *Does the line* ℓ *from the previous problem intersect the* triangle *formed by the points* p_1*,* p_2*, and* p_3*?*