PART 1 – MATH - max 36 points

1. [2+5=7 points] Given are two points: \( P = (1,2,3) \) and \( Q = (5,10,11) \) in \( \mathbb{R}^3 \), which lie on line \( L \).
   a. Write down the general implicit equation of a plane perpendicular to line \( L \).
      \[ x + 2y + 2z + D = 0 \]
   b. We draw a line from point \( R = (3,8,5) \) that is perpendicular to line \( L \),
      intersecting it at point \( S \). Calculate the length of line segment \( RS \).
      \[ \sqrt{8} = 2 \sqrt{2} \]

2. [3+3=6 points] Consider three points in \( \mathbb{R}^2 \): \( A = (1,1) \), \( B = (-3,4) \) and \( C = (1,7) \).
   a. We place a light at point \( C \). What is the length of the
      shadow of the line segment \( AB \) on the x-axis?
      \[ \frac{28}{3} = 9 \frac{1}{3} \]
   b. We place a camera at point \( B \), viewing line segment \( AC \), rendering
      it on the y-axis as the one-dimensional ‘screen’ as \( A'C' \). What is the
      length of the line segment \( A'C' \)?
      \[ 9/2 = 4.5 \]

3. [1+5+3=9 points] Given: a sphere in \( \mathbb{R}^3 \), with centre \( C = (3,3,3) \) and a point on the surface of the sphere: \( P = (2,5,1) \).
   a. Write down the implicit eq. for the sphere.
      \[ (x-3)^2 + (y-3)^2 + (z-3)^2 = 9 \]
   b. Calculate the point on the surface of the sphere closest to \( (6,9,1) \).
      \( (30/7,39/7,15/7) \)
   c. Unit vector \( \hat{u} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \) is a tangent vector of the sphere at point \( P \). Calculate the
      bitangent vector of the sphere at point \( P \).
      \[ \begin{bmatrix} 2\sqrt{2} \\ 3 \\ 3\sqrt{2} \end{bmatrix} \]

4. [4 points] We define a coordinate system in \( \mathbb{R}^2 \) (i.e., x- and y-axes and the origin). Draw this coordinate system and shade the region for which two conditions hold: \( x + y > 1 \) and \( x + 1 < y \).
5. [3 points] Write down the implicit equation of the tangent plane to the sphere \((x - 3)^2 + (y - 4)^2 + z^2 = 9\) at point \(P = (5,5,2)\).
\[2x + y + 2z - 19 = 0\]

6. [2+1+4=7 points] Consider Figure 1 below, which depicts a situation in \(\mathbb{R}^2\). Given:

- Line \(P\), defined as \(x - 2y + 1 = 0\) and line \(Q\), defined as \(y - 2x - 3 = 0\)
- Points \(A\) and \(B\) on line \(Q\). The location of \(A\) is \((0,3)\). The length of line \(AB\) is \(w\).
- The points \(A\) and \(B\) are projected onto line \(P\) at \(A'\) and \(B'\) respectively, i.e. \(AA'\) and \(BB'\) are both perpendicular to line \(P\).

![Diagram of line P and Q](image)

a. Calculate the length of line segment \(AA'\).
\[\sqrt{5}\]

b. Determine the location of point \(A'\).
\[(1,1)\]

c. Express the length of \(A'B'\) as a function of \(w\).
\[4w/5\]

PART 2 – THEORY - max 10 points

7. [6 points] A texture is stored as a paletized image. The dimensions of the texture are 512 x 512 pixels, and it uses exactly 256 unique colors. How much memory (in bytes) is needed to store this texture?
\[512^2 + 1024 \text{ or } 512^2 + 768\]

8. [4 points] Complete the following sentence. Write down the four terms that complete the sentence on your answer sheet.

“The flickering and Moiré-patterns we see on distant textured objects are symptoms of **UNDERSAMPLING**. This problem can be reduced by using **MIPMAPPING**. When a textured object is close to the camera, the texture may appear blocky. This is caused by **OVERSAMPLING**. We can smooth out the blocky texture using **BILINEAR INTERPOLATION**.

Note: only the actual terms are allowed, descriptions score no points.