Q1.

Two ways to solve this.

a) We assume that the implicit form eq. of the plane is \( Ax + By + Cz + D = 0 \)
where \( \begin{pmatrix} A \\ B \\ C \end{pmatrix} \) denotes a vector perpendicular to the plane.
So we choose \( A = 1, \ B = 2, \ C = 2 \), then all we need is to determine \( D \), which can be obtained from the fact that \( P = (1,1,1) \) is a point on the plane. I.e., \( 1 \times 1 + 2 \times 1 + 2 \times 1 + D = 0 \) \( \Rightarrow \) \( D = -5 \)

So the eqn. of the plane is

\[
\begin{align*}
  x + 2y + 2z - 5 &= 0 \\
  x + 2y + 2z - 5 &= 0
\end{align*}
\]

b) Alternatively, consider a point \( \mathbf{Q} = (x, y, z) \) on the plane. The vectors \( \overrightarrow{PQ} = (x-1, y-1, z-1) \) and \( \overrightarrow{v} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \) are perpendicular to each other. I.e., \( \overrightarrow{PQ} \cdot \overrightarrow{v} = 0 \)

\[
\begin{align*}
  (x-1)1 + (y-1)2 + (z-1)2 &= 0 \\
  x + 2y + 2z - 5 &= 0
\end{align*}
\]

Q4. The parametric eqn. of a line starting from \((8, 4, 0)\) along the vector \( \overrightarrow{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \) is

\[
\begin{align*}
  \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 8 \\ 4 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}
\end{align*}
\]

The points of intersection satisfy the eqn.

\[
\begin{align*}
  (x-3)^2 + (y-4)^2 + z^2 &= 25 \\
  (t+5)^2 + t^2 &= 25 \\
  t \ (t+5) &= 0 \ \Rightarrow \ t = 0 \ or \ -5.
\end{align*}
\]

I.e., the intersection points are \((8, 4, 0)\) and \((8-5, 4-5)\)

\[
\begin{align*}
  \text{Point } P_1 & \rightarrow (8, 4, 0) \\
  \text{Point } P_2 & \rightarrow (3, 4, -5)
\end{align*}
\]
The centre of the sphere is located at \((3, 4, 0)\). So by construction the vectors \(\overrightarrow{OP_1}\) and \(\overrightarrow{OP_2}\) are the outward normals.

\[
\begin{pmatrix}
8 - 3 \\
4 - 4 \\
0 - 0
\end{pmatrix} = \begin{pmatrix}
5 \\
0 \\
0
\end{pmatrix} \quad \begin{pmatrix}
3 - 3 \\
4 - 4 \\
-5 - 0
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
-5
\end{pmatrix}
\]

So the outward unit normal vectors are \(\begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix}\) and \(\begin{pmatrix}
0 \\
0 \\
-1
\end{pmatrix}\).

26. \(\overrightarrow{u} \times \overrightarrow{w}\) is perpendicular to \(\overrightarrow{u}\)

\[\Rightarrow \overrightarrow{u} \cdot (\overrightarrow{u} \times \overrightarrow{w}) = 0\]

310. Although this problem looks like a problem in 3 dimensions, note that all points \((A, B, P)\) have zero \(y\)-coordinates. So, the triangles \(EA'B'\) and \(EAP\) are confined to the \(x-z\) plane. We use this information to simplify the solution, as we confine the solution below fully to the \(x-z\) plane.

The unit vector along \(\overrightarrow{AB} = \frac{1}{\sqrt{(20 - 8)^2 + (12 - 3)^2}} \begin{pmatrix}
20 - 8 \\
12 - 3
\end{pmatrix}\)

\[\frac{1}{5} \begin{pmatrix}
4 \\
3
\end{pmatrix}\]

So, the location of \(P\) is \(\left(8 + \frac{4l}{5}, \ 3 + \frac{3l}{5}\right)\) on the \(x-z\) plane.

The slope-intercept form equations for \(EA\) and \(EP\) are

\[Z = \frac{7}{8}x - 4\] and

\[Z = \frac{7 + \frac{3l}{5}}{8 + \frac{4l}{5}}x - 4\] respectively.

These lines intersect the screen (\(xy\)-plane), for which \(Z = 0\), at

\[x = \frac{32}{7} \quad (\text{at } A)\] and \(\frac{32 + 16l/5}{7 + 3l/5} \quad (\text{at } P')\) respectively.
So, \( t = \frac{32 + 16l/5}{7 + 3l/5} \times \frac{32}{7} \) 

\[ = \frac{112l/5 - 96l/5}{7(7 + 3l/5)} = \frac{16l}{7(3l + 35)} \]

8.11. We have 3 points \( A, B, C \) on the plane; take any two vectors, e.g. \( \overrightarrow{AB} = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix} \) and \( \overrightarrow{AC} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} \)

The cross-product of these two vectors is perpendicular to the triangular plane. The cross product is

\[ \overrightarrow{u} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 1 \\ 2 & -1 & 0 \end{vmatrix} \]

So the answer is

\[ \pm \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} \]

Note: a surface has only two normal vectors differing in sign, so the answer is independent of whether you choose \( \overrightarrow{BC} \) \& \( \overrightarrow{AC} \) or \( \overrightarrow{AB} \) \& \( \overrightarrow{BC} \) pairs to work with in terms of cross products.

8.12. This problem is in the same vein as 8.10.

The unit vector along \( \overrightarrow{AB} = (3/5) \)

location of \( P = (2 + 3l/5, 4 + 4l/5) \)

Slope-intercept forms of the straight lines EA and EP are

\[ y = 4x - 4 \quad \text{and} \quad y = \frac{8 + 4l/5}{2 + 3l/5} x - 4 \]

These lines intersect the \( x \)-axis at

\[ x = \frac{2 + 3l/5}{2 + l/5} \quad \text{and} \quad x = \frac{8 + 12l/5}{8 + 4l/5} = \frac{2 + 3l/5}{2 + l/5} \]

So, \( t = \frac{2 + 3l/5}{2 + l/5} - 1 = \frac{2l}{l + 10} \)
2.14. The vector \( \overrightarrow{AB} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \). A vector perpendicular to that is \( \overrightarrow{c} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \), from which we know that the implicit form looks like

\[-x + y + C = 0\]

Since the line passes through \((1, 2)\),

\[C = -1\]

i.e., the equation is \( x - y + 1 = 0 \).

We let \( \overrightarrow{c} \) pass through \( P = (1, 5) \).

The parametric eqn. of the line passing through \( P \) along \( \overrightarrow{c} \) is

\[
\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \end{pmatrix}
\]

and it intersects the line \( x - y + 1 = 0 \) at \( S \), for which

\[(1 - t) - (5 + t) + 1 = 0 \quad \Rightarrow \quad t = -\frac{3}{2}\]

i.e., the co-ordinates of \( S \) are \((1 + \frac{3}{2}, 5 - \frac{3}{2}) = \left(\frac{5}{2}, \frac{7}{2}\right)\).