Exercise 1.
Given: a $3 \times 2$ matrix $A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{pmatrix}$.

a) Calculate the sum of $A$ with itself.
b) Can you multiply $A$ with itself? If so, do this; otherwise explain why not.
c) Determine the transpose of $A^\top$ of $A$.
d) Multiply $A$ with $A^\top$.

Exercise 2.
Given: matrix $A = \begin{pmatrix} 3 & 1 \\ 1 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ 6 & 1 \end{pmatrix}$.

a) Calculate the determinant of $A$.
b) Calculate the determinant of $B$.
c) What is the geometric interpretation of these two results?

Exercise 3.
Given: vector $\vec{a} = (1,5)$ and $\vec{b} = (7, -1)$.
Calculate the area of the parallelogram defined by the two vectors.

Exercise 4.
Given: matrix $A = \begin{pmatrix} -2 & 2 & 3 \\ -1 & 1 & 3 \\ 2 & 0 & -1 \end{pmatrix}$.

a) Calculate the determinant of $A$ using Laplace’s expansion.
b) Verify your answer using the Rule of Sarrus.
Matrix characteristics

Exercise 5.

Given: matrix \( A = \begin{pmatrix} 1 & 4 & -2 \\ 3 & 2 & -6 \\ -2 & 5 & 4 \end{pmatrix} \).

a) Calculate the determinant of matrix \( A \) using the Rule of Sarrus.

b) Explain the result for a) geometrically.

Exercise 6.

Given:

- \( n_A \times m_A \) matrix \( A \);
- \( m_A \times m_B \) matrix \( B \);
- \( m_A \times m_C \) matrix \( C \).

Prove, that if \( AB = AC \), it does not necessarily follow that \( B = C \) (even if \( A \) is not the null matrix).

Exercise 7.

A matrix is called orthogonal if all column vectors are mutually perpendicular. A matrix is called orthonormal if it is orthogonal and all column vectors have unit length. Show that the inverse of an \( n \times n \) orthonormal matrix is its transpose. Hint: \( A^{-1}A = I \), so you can solve this by proving that \( A^TA = I \).

Matrix inversion

Exercise 8.

Given: matrix \( A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 0 & 2 & 5 \end{pmatrix} \).

a) Calculate the cofactor matrix \( C \).

b) Calculate the adjoint matrix \( A^T \).

c) Calculate the determinant for \( A \).

d) Calculate the inverse \( A^{-1} \) for \( A \) using the results for a), b) and c).
Coordinate systems

Exercise 9.

a) Write down a matrix that scales 2D vectors by a factor of 2.
b) Determine the inverse of this matrix.
c) Write down the matrix that projects 3D vectors on the $x = 0$ plane.
d) If possible, determine the inverse of this matrix. Otherwise, explain why this is not possible.

Exercise 10.

Determine the matrix that rotates vectors along an ellipse centered around the origin, with a height of 4 and a width of 1.

Exercise 11.

Normals must be transformed using $(A^{-1})^T$.

a) Explain why (the solution was given in the lecture, but try to do this yourself).
b) Verify using the example of shearing and a few normals that the transform is correct.
c) Will be proposed transform maintain the magnitude of normals after transformation?

Exercise 12.

Determine a matrix that translates points in $\mathbb{R}^2$ by $x_t, y_t$.

The End

(for now)