1a. \( dx = 4 - (-2) = 6, \ dy = 1 - 0 = 1 \) \( \Rightarrow a = \frac{1}{6} \)
\( c = 0 + 2 \times \frac{1}{6} = \frac{1}{3} \)
\( \Rightarrow y = \frac{1}{6}x + \frac{1}{3}. \)

c. \( A = -1, \ B = 6 \)
\( C = -\vec{N} \cdot P = -((-1,6) \cdot (4,1)) = -1 \times (-4 + 6) = -2 \)
\( -1x + 6y - 2 = 0. \)

d. \((-1,6)\) and \((1,-6)\) (and all scaled versions of these)

e. See c

f. ‘C’ is the distance of the line to the normal, times the magnitude of the normal. In this case, the magnitude of the normal is \( \sqrt{1 \times 1 + 6 \times 6} = \sqrt{37}; \) the line is thus at a distance of \( \frac{2}{\sqrt{37}} \) from the origin (~0.329).

2a. Draw the line and the normal \( \vec{n}. \) A vector \( \vec{p} \) from the origin to an arbitrary point \( p \) on the line is parallel to the line, and thus perpendicular to the normal, hence \( \vec{n} \cdot \vec{p} = 0. \)

b. Draw a line (not through the origin). Pick two points on the line, \( p \) and \( p' \) \((p \neq p')\). The vector \( p - p' \) is perpendicular to \( \vec{n} \), and thus \( \vec{n} \cdot (p - p') = 0. \) Rewriting this gives \( \vec{n} \cdot p = \vec{n} \cdot p'. \)

3a. \( dx = 4 - (-2) = 6, \ dy = 1 - 0 = 1, \ dz = 5 - 1 = 4; \)
length = \( \sqrt{dx^2 + dy^2 + dz^2} = \sqrt{53} = \sim 7.28. \)

b. \( p(t) = (-2,0,1) + t \times (6, 1, 4). \)

c. e.g. \(-1, 6, 0\) (swapping \( x/-y \), zeroing \( z \)) and \(-4, 0, 6\) (swapping \( x/-z \), zeroing \( y \)).

d. Unlimited, in a disc of radius 1 around the line.

Note: vectors do not have a position, so a disc or cylinder ‘around the line’ is a bit misleading.

4a. \( dx = 6, \ dy = 1, \ dz = 4, \ dw = -2. \)
Normals: e.g. \((-1, 6, 0, 0\) (swapping \( x/-y \), zeroing \( y \) and \( z \)) and \((0, 0, 2, 4\) (swapping \( z/-w \), zeroing \( x \) and \( y \)).

b. Unlimited, in a sphere of radius 1 around the 4D line.

5. Slope-intersect to implicit: shuffle to isolate 0;
Implicit to slope-intersect: shuffle to isolate \( y; \)
Implicit to parametric: determine two points on the line: after normalization, one point is the origin plus the normal times \( C; \) the second point is the first point plus \((B, -A).\)

6. Consider \( l \) and a point \( p \) that lies in the halfplane to which \((a, b)\) points. Let \( p_0 \) be the perpendicular projection of \( p \) onto \( l \). We can write: \( p = p_0 + \lambda(a, b), \) with \( \lambda > 0. \) The distance \( f(p_x, p_y) \) of \( p \) to \( l \) is then \( f(p_0) + \lambda \sqrt{a^2 + b^2}. \) Since \( f(p_0) = 0 \) (it is on the line), \( \lambda > 0, \) and the square root must be greater than zero, we have that \( f(p_x, p_y) > 0. \)

7a. \((x - 3)^2 + (y - 5)^2 - 9 = 0. \)
b. \( p(\varphi) = \left(3 + 3\cos \varphi \over 5 + 3\sin \varphi \right)\)
8a. \( p(a, b) = \begin{pmatrix} 3 + 3.14 \cos a \sin b \\ 5 + 3.14 \sin a \sin b \\ 1 + 3.14 \cos b \end{pmatrix} \)

b. using the result from a, use e.g. \( a = 0, b = 0 \) and \( a = 0, b = \pi \).

9a. E.g., \((0,1,0)\) and \((0,0,1)\).

b. \((1,0,0)\).

c. 4.

d. An infinite number: all planes parallel to the x-axis (such as \( y = 0 \) and \( z = 0 \)).

10a. \( f(u, v) = p_1 + u(p_2 - p_1) + v(p_3 - p_1) \).

b-d. Solve this by calculating a vector from \( p_1 \) to \( p_2 \), and one from \( p_1 \) to \( p_3 \); determine the cross product between these vectors to find the normal of the plane. This yields A,B and C for the general implicit plane equation. Determine D by filling in \( p_1 \).

11a,b. \( f(t) = p_1 + t(p_2 - p_1) \).

c. \((0,7,6)\); the lines are linearly independent, but share a point, which must therefore be the only intersection point.

12a. \((x - 3)^2 + (y - 3)^2 + (z - 3)^2 - r^2 = 0\)

\( r = ||p - c|| = \sqrt{1 + 4 + 4} = 3 \).

b. \( d = ||q - c|| - r \)

c. The normal of the plane is the vector \( p - c \); use \( p \) itself to find D.

d. Swap e.g. \( x/y \) and \( y/z \) to obtain two tangent vectors.