Graphics 2008/2009

T1

Midterm exam

Thu, Oct 02, 2008, 15:00–17:00

- **Do not open this exam until instructed to do so.**
- **Read the instructions on this page carefully.**

- You may write your answers in English, Dutch, or German.
  Use a pen, not a pencil. Avoid usage of the color red.
- You may not use books, notes, or any electronic equipment
  (including your cellphone, even if you just want to use it as a clock).
- Please put your student ID on the table so we can walk around and check it during the exam.
  You also have to show it to the instructor when you turn your exams in before leaving the room.
- Write down your name and student number on every paper you want to turn in.
  Additional paper is provided by us. You are not allowed to use your own paper.

- The exam should be doable in less than 1.5 hours. You have max. 2 hours to work on the questions.
  If you finish early, you may hand in your work and leave, except for the first half hour of the exam.
- The exam consists of 4 problems printed on 4 pages (including this one).
  It is your responsibility to check if you have a complete printout.
  If you have the impression that anything is missing, let us know.
- The maximum number of points you can score is 20.
  You need at least 18 points to get the best possible grade.

Good luck!

Problem 1: Vectors

Subproblem 1.1 [2 pt] Assume two vectors $\mathbf{a} = (3,8)^T$ and $\mathbf{b} = (6,16)^T$.

(a) Which of the following statements are correct?
(shortly explain your answer, multiple answers may apply)

1. **a** and **b** are parallel.
2. **a** and **b** are linearly independent.
3. **a** and **b** form a 2D basis.
4. **a** is a scalar multiple of **b**.

(b) Which of the statements 1. - 4. are correct if $\mathbf{a} = (8,3)^T$?
(shortly explain your answer, multiple answers may apply)

Subproblem 1.2 [2 pt]

(a) What do we know about two random vectors **v** and **w** if their scalar product is zero, i.e. if $\mathbf{v} \cdot \mathbf{w} = 0$? Shortly explain your answer. (Note: think of *all* possible options.)

(b) What do we know about the value of the scalar product of two unit vectors **v** and **w** if the angle ϕ between them is between zero and 90°, i.e. if $0 < \phi < 90°$? Shortly explain your answer.

Problem 2: Basic geometric entities

Subproblem 2.1 [5 pt]

(a) What is the general form of a parametric equation of a plane in 3D? Write it down and explain the geometric interpretation of its components.

(b) Assume the following three points in $\mathbb{R}^3$:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Calculate a normal vector **n** for the plane defined by **a**, **b**, and **c**

(c) What is the general form of an implicit representation of a plane in 3D? Write it down and explain the geometric interpretation of its components.

(d) Create an implicit representation of the plane defined by the three vectors **a**, **b**, and **c** given in (b).
Do not just write down the solution but explain it (e.g. by using references to (c)).

Subproblem 2.2 [1 pt]

Assume you are calculating the intersection of a line and a plane in 3D. What are the possible number of solutions you can get (note: write down *all* possibilities)? What is the geometric interpretation of each case?

Problem 3: Matrices

Subproblem 3.1 [1.5 pt] Which of the following answers is correct if s is a scalar value, $\mathbf{v}$ is a vector in $\mathbb{R}^3$, and $\mathbf{A}$ is a 2x3 matrix? (no explanation required)

(a) s$\mathbf{A}$ is (a1) a scalar, (a2) a vector in $\mathbb{R}^2$, (a3) a vector in $\mathbb{R}^3$, (a4) a 2x3 matrix, (a5) a 3x3 matrix, or (a6) undefined?

(b) $\mathbf{Av}$ is (b1) a scalar, (b2) a vector in $\mathbb{R}^2$, (b3) a vector in $\mathbb{R}^3$, (b4) a 2x3 matrix, (b5) a 3x3 matrix, or (b6) undefined?

(c) $\mathbf{vA}$ is (c1) a scalar, (c2) a vector in $\mathbb{R}^2$, (c3) a vector in $\mathbb{R}^3$, (c4) a 2x3 matrix, (c5) a 3x3 matrix, or (c6) undefined?

Subproblem 3.2 [3.5 pt]

Assume we have three planes P_1, P_2, and P_3 which are defined by the following equations:

- $P_1 : 2x+2y+4z = 18$
- $P_2 : 1x+3y+4z = 19$
- $P_3 : 1x+2y+5z = 20$

(a) Write this down as a system of linear equations in matrix notation and solve using Gaussian elimination.

(b) What is the geometric interpretation of your solution?

(c) Assume we are replacing plane P_1 with a new plane P_1',

- $P_1' : 2x+4y+10z = 40$

Can the related system of linear equations still be solved? Explain why or why not. What is the geometric interpretation of this?

Problem 4: Transformations

Subproblem 4.1 [1 pt]

In $\mathbb{R}^2$, the matrix $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ defines a counterclockwise rotation about the angle α about the origin. Give a 3x3 transformation matrix for a similar rotation about the x-axis in $\mathbb{R}^3$.

Subproblem 4.2 [2.5 pt]

(a) Describe in your own words what happens to a point **p** in $\mathbb{R}^3$ if you apply the following transformation matrix to it:

$$\begin{pmatrix} 3 & 0 & 0 & x_m \\ 0 & 3 & 0 & y_m \\ 0 & 0 & 3 & z_m \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

How are the values in the last row of this matrix called and why do we need them?

(b) What happens if you apply the matrix from (a) to a vector **v** in $\mathbb{R}^3$. Explain, why your answer differs from the one given in (a).

Subproblem 4.3 [1.5 pt]

Assume we have a vector **v** and a rotation matrix $\mathbf{M}_{rot}$, a scaling matrix $\mathbf{M}_{scale}$, and a reflection matrix $\mathbf{M}_{ref}$. Now, we first want to rotate the vector **v** using $\mathbf{M}_{rot}$. Then, we want to reflect the resulting vector using $\mathbf{M}_{ref}$. Finally, we want to scale this result using $\mathbf{M}_{scale}$.

(a) Write down the order in which we have to multiply the original vector with the three matrices to do this (i.e. replace *a*, *b*, *c* in $\mathbf{M}_a\mathbf{M}_b\mathbf{M}_c\mathbf{v}$ with the correct indices *rot*, *scale*, and *ref*.)

(b) Why is the order important, i.e. why might we get a different result if we use a different order?

(c) What characteristic of matrix multiplication is the reason why we can replace the matrices $\mathbf{M}_a\mathbf{M}_b\mathbf{M}_c$ in the transformations above with a single matrix $\mathbf{M} = \mathbf{M}_a\mathbf{M}_b\mathbf{M}_c$?