Retake Homework Exam 2018

Deadline: 22 March 2019, 23:59

This homework exam has 5 questions for a total of 90 points. You can earn an additional 10 points by a careful preparation of your hand-in: using a good layout, good spelling, good figures, no sloppy notation, no statements like “The algorithm runs in $n \log n$.” (forgetting the $O(\cdot)$ and forgetting to say that it concerns time), etc. Use lemmas, theorems, and figures where appropriate. Your final grade will be the number of points divided by 10 (with a maximum of a 10).

Question 1 (10 points)
Let $S$ be a planar subdivision with $n$ vertices, represented as a DCEL. Give pseudo-code for an algorithm that, given a pointer to a face $f$, computes all faces of $S$ adjacent to $f$. Your algorithm should use the ‘Twin’, ‘NextEdge’, ‘PrevEdge’, etc. fields to navigate (i.e. you cannot assume you can directly access a list of vertices). Describe in a few sentences the main idea of your algorithm and give its running time.
Hint: You can use the data fields of vertices, half-edges, and faces to store marks/flags.

Question 2 (15 points)
Given a set $P$ of $n$ points in $\mathbb{R}^3$, and a real number $h$. You can assume that all coordinates are unique. Consider a horizontal slab $S$ of height $h$ (i.e. a region bounded by two horizontal planes that are distance $h$ apart), and let $A(S)$ denote the area of the axis-aligned bounding box of $P \cap S$. Develop an $O(n \log n)$ time algorithm that can compute the maximum value of $A(S)$ over all horizontal slabs of height $h$.
Argue that your algorithm is correct and achieves the desired running time.

Question 3
A simple polygon $P$ is star-shaped if and only if it contains a point $q \in P$ such that for any point $p \in P$, the line segment $pq$ lies inside $P$.

(a) (10 points)
We say that $P$ is strictly star-shaped if (and only if) the point $q$ is also a vertex of $P$. Show that there are polygons that are star-shaped but not strictly star-shaped.

(b) (10 points)
Design an algorithm that can decide if a simple polygon $P$ with $n$ vertices is star-shaped in $O(n)$ expected time. You can again assume no three vertices of $P$ are colinear.

Question 4
Let $S$ be a set of $n$ line segments in $\mathbb{R}^2$. You can assume no three endpoints in $S$ are colinear, and all coordinates are unique.

(a) (10 points)
Prove that there exists a line segment $z^*$ that intersects the maximum number of segments in $S$ and contains two endpoints of segments in $S$.

(b) (15 points)
Describe an $O(n^2 \log n)$ expected time algorithm to compute the maximum number of segments in $S$ intersected by a line segment.

Question 5
Let $P$ be a set of $n$ points in $\mathbb{R}^2$. You can assume that no three points are colinear, and all coordinates are unique. We would like to store $P$ such that we can efficiently report all points in a query wedge $Q$. A query wedge $Q$ is represented by a pair of half-lines $(\ell, m)$ that have the same endpoint, and consists of the points that lie above (the supporting line of) $\ell$ and below (the supporting line of) $m$. 

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(a) (10 points)
Describe a data structure that can solve these queries in $O(\sqrt{n} + k)$ time, where $k$ is the output size, provided that the slope of $\ell$ is zero, and the slope of $m$ is one. (i.e. your data structure has to support only wedges of this particular shape). Argue that your solution is correct and analyze the space usage and query time of your data structure.

(b) (10 points)
Describe a data structure that can solve these queries in $O(\log c n + k)$ time, where $k$ is again the output size and $c$ is some constant, even in case the half-lines $\ell$ and $m$ may have arbitrary slopes. Argue that your solution is correct and analyze the space usage and query time of your data structure.