Elementary Maths for GMT - 2016 - Homework Exam 1

This homework exam consists of 3 questions. You can earn a total of 100 points. You may discuss questions and approaches on a high level with your fellow students, but must write down your own solutions. The first 10 points can be obtained by following these rules:

- Hand in your solutions on paper.
- Write clearly and legibly, or use a type setting system (such as \LaTeX).
- Write your name and student number on each sheet you hand in, and use page numbers.

Be concise in your answers. An explanation of how you obtained your answer is not mandatory unless specified in the question; however, a short derivation may be helpful in case of arithmetic mistakes.

**Question 1**

Let $f : (0, 1) \times (0, 1) \to \mathbb{R}$ be defined as $f(x, y) = x/y$.

(a) Is $f$ injective, surjective, bijective, or none of these?
(b) Compute the gradient of $f$.
(c) What is the limit of $f$ at $(0, 0)$?

**Question 2**

The *regular octahedron*, one of the five Platonic solids, is a 3-dimensional shape bounded by eight regular triangles such that four triangles meet in each vertex.

(a) Describe the symmetry group of the octahedron.
(b) Give a minimal set of generators for this group.
Assume the edges of the octahedron have length 1, and let $d$ be the length of a long diagonal (the distance between two opposite vertices).
(c) Calculate $d$.
(d) Is $d$ constructible? If so, give a construction.
Let $H$ be a plane through four of the vertices of the octahedron. Let $f : \mathbb{R}^2 \to \mathbb{R}$ describe the height of the octahedron above $H$.
(e) Give a formula for $f$.
(f) Calculate the volume of the octahedron by integrating $f$.

**Question 3**

An embedding of a graph is called $k$-planar if each edge is intersected by at most $k$ other edges. A graph is $k$-planar if it has a $k$-planar embedding.

(a) Draw an example of a graph that is 1-planar, but not planar.
(b) What is the maximum number of edges in a 1-planar graph with $n$ vertices?